

A Production Model for Time Dependent Decaying Rate with Probabilistic Demand

P.K. Gupta

Department of Statistics, D.A.V. (P.G.) College, Muzaffarnagar, UP, INDIA

ABSTRACT

In this study, we developed a production inventory system for decaying items with probabilistic demand under inflation. Decaying rate is dependent on time. Partially backlogged shortages are allowed with time dependent backloging rate. The backloging rate is exponential decreasing function of time. The expected total cost with uniform demand and discrete units and continuous units has been taken.

Keywords-- Probabilistic demand, inflation, partial backloging

I. INTRODUCTION

In business world, time is a phenomenon, which affects everything around it. Inflation very clearly is a concept closely related to time. Inflation is that state disequilibrium, in which an expansion of purchasing power tends to cause, or it is the effect of increase in the price level. During inflation, there is too much currency in relation to the physical volume of the business being done. Inflation is normally associated with high prices, which cause decline in the purchasing power or the value of money. Since the 1970's energy crisis, inflation has become an embedded feature of the economy throughout the world, and large scale inflation rates were not in many industries. In this direction, the pioneer research, Buzacott (1975) was the first proponent for developing an EOQ model with inflation subject to different types of pricing policy. Chandra and Baher (1985) discussed some inventory systems with the effects of inflation and time value of money. In 1991, Datta and Pal considered a model with linear time dependent demand rate and shortages to investigate the effects of inflation and time value of money on ordering policy over a finite time horizon. Harriga (1995) modified Datta and Pal's (1991) model by relaxing the assumption of equal inventory carrying time during each replenishment cycle and their mathematical formulation Hariga and Ben-Daya (1996) extended Hariga (1995) by removing the restriction of equal replenishment cycle and provided two solution procedures with or without shortages. Chang (2004)

suggested a model for deteriorating items with inflationary environment under a situation in which the supplier provides the purchaser, a permissible delay of payments if the purchaser orders a large quantity. Yang, H. L. (2004) considered a two warehouses inventory models for deteriorating items with shortages under inflation. Moon et al. (2005) presented the deterioration to amelioration when the environment was inflationary. Jaggi et al. (2006) discussed a optimal order policy for deteriorating items with inflation induced demand. In this paper, they presented the optimal inventory replenishment policy of deteriorating items under inflationary conditions using a DCF approach over a finite planning horizon. In the same year, Yang (2006) used a two warehouses inventory models for deteriorating items under inflation with partial backloging and constant demand rates. Singh, S.R. *et al.* (2009) discussed an inventory system for perishable items with stock dependent demand and time dependent partial backloging. In their model, constant holding cost has been taken. Tripathy, C.K. and Mishra, U. (2010) developed a deterministic inventory model for deteriorating items with constant demand. Shortages were allowed with constant partial backloging rate in their study. Singhal et al. (2014) presented a probabilistic inventory model for weibull deteriorating items with flexibility and reliability consideration. Singhal et al. (2016) developed a stochastic partial backloging inventory models for deteriorating items with time dependent demand and volume elasticity.

In the present paper attempts have been made to investigate an EPQ model assuming the existence of a probabilistic demand, time dependent deterioration rate. Shortages are allowed with partial backloging. The backloging rate is exponential decreasing function of time. Inflation is also taken in this study. We also considered the expected total cost with uniform demand and discrete units and continuous units.

II. ASSUMPTIONS AND NOTATIONS

ASSUMPTIONS

1. Model is developed on multi-item products.
2. Demand is uniformly over the period and a function of temperature that follows as probability distribution.

- 3. Shortages are allowed and partially backlogged.
- 4. Lead time is zero.
- 5. The time horizon is infinite.
- 6. Deterioration rate is taken as time dependent.

NOTATIONS

- 1. K – Production rate per unit time.
- 2. r_i – Demand over the period.
- 3. Q_i – Inventory level of i^{th} item.
- 4. p_i – Selling price per unit of i^{th} item.

THE MODEL

In this model, we consider, demands are r_i ($i = 1, 2, \dots, n$) that depends upon temperature and selling price of i^{th} item. Temperature follows probability distribution over period. Here,

$$r_i = a_i \tau + \frac{c_i \sum_{j=1, j \neq i}^n p_j}{(n-1)p_i} \neq$$

Where,

$$a_i = \frac{\partial r_i}{\partial \tau} (\geq 0) = \text{marginal response of } i^{th} \text{ consumption to a change in } \tau. \quad (\text{temperature})$$

$$\left[\frac{\sum_{j=1, j \neq i}^n p_j}{(n-1)p_i} \text{ is constant} \right]$$

$$c_i = \frac{\partial r_i}{\partial \left(\frac{\sum_{j=1, j \neq i}^n p_j}{(n-1)p_i} \right)} (\geq 0) = \text{marginal response of } i^{th} \text{ consumption to a change in}$$

$\frac{\sum_{j=1, j \neq i}^n p_j}{(n-1)p_i}$ { The ratio of the average selling price of ($j = 1, 2, \dots, i-1, i+1, \dots, n$) items to the selling price of i^{th} item } (τ is constant) that depends upon the choice of the consumers.

III. MODELING AND ANALYSIS

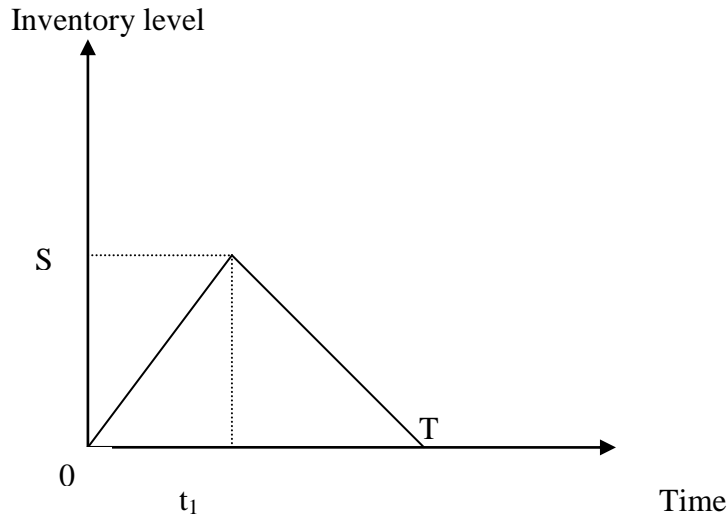
In this study, we have developed two cases:

Case I: When shortages do not occur:

In this case, the initial inventory of the cycle is zero and production starts at the very beginning of the cycle. As production continues, inventory begins to pile up continuously after meeting demand and deterioration.

- 5. T – Duration of the production cycle.
- 6. bt – Quadratic rate of deterioration.
- 7. $B = e^{-\delta t}$ - Backlogging rate.
- 8. C_{hi} – Holding cost per i^{th} item per unit time.
- 9. C_{si} – Shortage cost per i^{th} item per unit time.
- 10. r – Inflation rate
- C_{Lsi} – lost sale cost.

Production stops at time t_1 . The accumulated inventory is just sufficient enough to account for demand and deterioration over the interval $[t_1, T]$. The instantaneous inventory level $Q_i(t)$ at any time is governed by the following equations:



$$\frac{dQ_i(t)}{dt} + btQ_i(t) = K - \frac{r_i}{T}, 0 \leq t \leq t_1 \quad \dots (1)$$

$$\frac{dQ_i(t)}{dt} + btQ_i(t) = -\frac{r_i}{T}, t_1 \leq t \leq T \quad \dots (2)$$

With boundary conditions, $Q_i(0) = 0, Q_i(t_1) = S$

Solutions of equation (1), (2) are:

$$Q_i(t) = \left(K - \frac{r_i}{T}\right) \left[t + \frac{bt^3}{6} \right] e^{-\frac{bt^2}{2}} \quad 0 \leq t \leq t_1 \quad \dots (3)$$

$$Q_i(t) = S e^{\left(\frac{b}{2}(t_1^2 - t^2)\right)} + \frac{r_i}{T} \left[(t_1 - t) + \frac{b}{6}(t_1^3 - t^3) \right] e^{-\left(\frac{bt^2}{2}\right)} \quad (t_1 \leq t \leq T) \quad \dots(4)$$

Present worth holding cost occurs during $(0, T)$ is:

$$\begin{aligned} H_i &= C_{hi} \left[\int_0^{t_1} e^{-rt} Q_i(t) dt + \int_{t_1}^T e^{-r(t_1+t)} Q_i(t) dt \right] \\ &= C_{hi} \left[\left(K - \frac{r_i}{T}\right) \left[\frac{t_1^2}{2} + \frac{bt_1^4}{24} - \frac{brt_1^5}{30} - \frac{bt_1^4}{8} - \frac{b^2 t_1^6}{72} \right] + S[(T - t_1) \right. \\ &\quad \left. + \frac{b}{2} \left\{ t_1^2(T - t_1) - \frac{1}{3}(T^3 - t_1^3) \right\} - r \left\{ t_1(T - t_1) + \frac{1}{2}(T^2 - t_1^2) \right\} \right] \\ &\quad + \frac{r_i}{T} \left[\left\{ t_1(T - t_1) - \frac{1}{2}(T^2 - t_1^2) \right\} + \frac{b}{6} \left\{ t_1^3(T - t_1) - \frac{1}{4}(T^4 - t_1^4) \right\} \right. \\ &\quad \left. - r \left\{ \frac{t_1}{2}(T^2 - t_1^2) - \frac{1}{3}(T^3 - t_1^3) \right\} - \frac{br}{6} \left\{ \frac{t_1^3}{2}(T^2 - t_1^2) - \frac{1}{5}(T^5 - t_1^5) \right\} \right. \\ &\quad \left. - \frac{b}{2} \left\{ \frac{t_1}{3}(T^3 - t_1^3) - \frac{1}{4}(T^4 - t_1^4) \right\} - \frac{b^2}{12} \left\{ \frac{t_1^3}{3}(T^3 - t_1^3) - \frac{1}{6}(T^6 - t_1^6) \right\} \right. \\ &\quad \left. - r \left\{ t_1^2(T - t_1) - \frac{t_1}{2}(T^2 - t_1^2) \right\} - \frac{br}{6} \left\{ t_1^4(T - t_1) - \frac{t_1}{4}(T^4 - t_1^4) \right\} \right] \dots (5) \end{aligned}$$

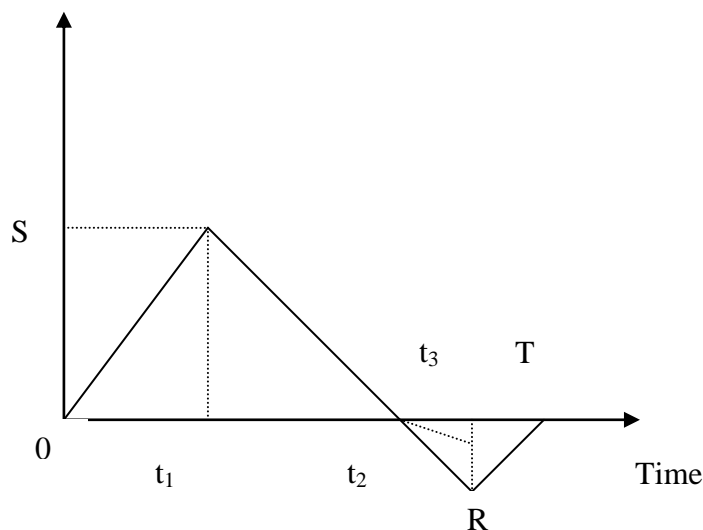
Present worth deterioration cost occurs during (0, T) is:

$$\begin{aligned}
 D_1 = & \left[\int_0^{t_1} bte^{-rt} Q_i(t) dt + \int_{t_1}^T bte^{-r(t_1+t)} Q_i(t) dt \right] \\
 = & \left(K - \frac{r_i}{T} \right) \left[\frac{bt_1^3}{3} - \frac{brt_1^4}{4} - \frac{b^2rt_1^6}{36} - \frac{b^2t_1^5}{10} - \frac{b^3t_1^7}{84} \right] \\
 & + \frac{r_i}{T} \left[b \left\{ \frac{t_1}{2}(T^2 - t_1^2) - \frac{1}{3}(T^3 - t_1^3) \right\} + \frac{b^2}{6} \left\{ \frac{t_1^3}{2}(T^2 - t_1^2) - \frac{1}{5}(T^5 - t_1^5) \right\} \right. \\
 & - br \left\{ \frac{t_1}{3}(T^3 - t_1^3) - \frac{1}{4}(T^4 - t_1^4) \right\} - \frac{b^2r}{6} \left\{ \frac{t_1^3}{3}(T^3 - t_1^3) - \frac{1}{6}(T^6 - t_1^6) \right\} \\
 & - \frac{b^2}{2} \left\{ \frac{t_1}{4}(T^4 - t_1^4) - \frac{1}{5}(T^5 - t_1^5) \right\} - \frac{b^3}{12} \left\{ \frac{t_1^3}{4}(T^4 - t_1^4) - \frac{1}{7}(T^7 - t_1^7) \right\} \\
 & - br \left\{ \frac{t_1^2}{2}(T^2 - t_1^2) - \frac{t_1}{3}(T^3 - t_1^3) \right\} - \frac{b^2r}{6} \left\{ \frac{t_1^4}{2}(T^2 - t_1^2) - \frac{t_1}{5}(T^5 - t_1^5) \right\} \\
 & \left. + S \left[\left\{ b - br \right\} t_1 \right] \frac{1}{2}(T^2 - t_1^2) - br \frac{1}{3}(T^3 - t_1^3) + \frac{b^2}{2} \left\{ \frac{t_1^2}{2}(T^2 - t_1^2) - \frac{1}{4}(T^4 - t_1^4) \right\} \right] \quad \dots (6)
 \end{aligned}$$

Case II: When shortages occur:

In this case, the initial inventory of the cycle is zero and production starts at the very beginning of the cycle. As production continues, inventory begins to pile up continuously after meeting demand and deterioration. Production stops at time t_1 . The accumulated inventory is just sufficient enough to account for demand and deterioration over the interval $[t_1, t_2]$. Shortage starts after t_2 with the concept of partial backlogging and reach to maximum shortage level at time t_3 . Production restarts after t_3 to fulfill the backlog demand and the cycle ends with zero inventory. The instantaneous inventory level $Q_i(t)$ at any time is governed by the following equations:

Inventory level



$$\frac{dQ_i(t)}{dt} + btQ_i(t) = K - \frac{r_i}{T}, \quad 0 \leq t \leq t_1 \quad \dots (7)$$

$$\frac{dQ_i(t)}{dt} + btQ_i(t) = -\frac{r_i}{T}, \quad t_1 \leq t \leq t_2 \quad \dots (8)$$

$$\frac{dQ_i(t)}{dt} = -B \frac{r_i}{T}, \quad t_2 \leq t \leq t_3 \quad \dots (9)$$

$$\frac{dQ_i(t)}{dt} = K - \frac{r_i}{T}, \quad t_3 \leq t \leq T \quad \dots (10)$$

With boundary conditions $Q_i(0) = 0$, $Q_i(t_1) = S$, $Q_i(t_2) = 0$, $Q_i(t_3) = -R$
Solutions of these equations are:

$$Q_i(t) = (K - \frac{r_i}{T}) \left[t + \frac{bt^3}{6} \right] e^{-\left(\frac{bt^2}{2}\right)} \quad 0 \leq t \leq t_1 \quad \dots (11)$$

$$Q_i(t) = S e^{\left\{ \frac{b}{2}(t_1^2 - t^2) \right\}} + \frac{r_i}{T} \left[(t_1 - t) + \frac{b}{6}(t_1^3 - t^3) \right] e^{-\left(\frac{bt^2}{2}\right)} \quad (t_1 \leq t \leq t_2) \quad \dots (12)$$

$$Q_i(t) = \frac{r_i}{T\delta} \left[e^{-\delta t} - e^{-\delta t_2} \right], \quad t_2 \leq t \leq t_3 \quad \dots (13)$$

$$Q_i(t) = -R + (K - \frac{r_i}{T})(t - t_3), \quad t_3 \leq t \leq T \quad \dots (14)$$

Present worth holding cost occur during $(0, t_1)$ is:

$$\begin{aligned} H_2 &= C_{hi} \left[\int_0^{t_1} e^{-rt} Q_i(t) dt + \int_{t_1}^{t_2} e^{-r(t_1+t)} Q_i(t) dt \right] \\ &= C_{hi} \left[\left(K - \frac{r_i}{T} \right) \left[\frac{t_1^2}{2} + \frac{bt_1^4}{24} - \frac{rt_1^3}{3} - \frac{brt_1^5}{30} - \frac{bt_1^4}{8} - \frac{b^2 t_1^6}{72} \right] \right. \\ &\quad + S(t_2 - t_1) + \frac{b}{2} \left\{ t_1^2(t_2 - t_1) - \frac{1}{3}(t_2^3 - t_1^3) \right\} \\ &\quad - r \left\{ t_1(t_2 - t_1) + \frac{1}{2}(t_2^2 - t_1^2) \right\} + \frac{r_i}{T} \left[\left\{ t_1(t_2 - t_1) - \frac{1}{2}(t_2^2 - t_1^2) \right\} \right. \\ &\quad + \frac{b}{6} \left\{ t_1^3(t_2 - t_1) - \frac{1}{4}(t_2^4 - t_1^4) \right\} - r \left\{ \frac{t_1}{2}(t_2^2 - t_1^2) - \frac{1}{3}(t_2^3 - t_1^3) \right\} \\ &\quad - \frac{br}{6} \left\{ \frac{t_1^3}{2}(t_2^2 - t_1^2) - \frac{1}{5}(t_2^5 - t_1^5) \right\} - \frac{b}{2} \left\{ \frac{t_1}{3}(t_2^3 - t_1^3) - \frac{1}{4}(t_2^4 - t_1^4) \right\} \\ &\quad - \frac{b^2}{12} \left\{ \frac{t_1^3}{3}(t_2^3 - t_1^3) - \frac{1}{6}(t_2^6 - t_1^6) \right\} - r \left\{ t_1^2(t_2 - t_1) - \frac{t_1}{2}(t_2^2 - t_1^2) \right\} \\ &\quad \left. - \frac{br}{6} \left\{ t_1^4(t_2 - t_1) - \frac{t_1}{4}(t_2^4 - t_1^4) \right\} \right] \quad \dots (15) \end{aligned}$$

Present worth deterioration cost occurs during $(0, t_2)$ is:

$$\begin{aligned} D_2 &= \left[\int_0^{t_1} bte^{-rt} Q_i(t) dt + \int_{t_1}^{t_2} bte^{-r(t_1+t)} Q_i(t) dt \right] \\ &= \left(K - \frac{r_i}{T} \right) \left[\frac{bt_1^3}{3} + \frac{b^2 t_1^5}{30} - b \frac{rt_1^4}{4} - \frac{b^2(a+r)t_1^6}{36} - \frac{b^2 t_1^5}{10} - \frac{b^3 t_1^7}{84} \right] \\ &\quad + \frac{r_i}{T} \left[b \left\{ \frac{t_1}{2}(t_2^2 - t_1^2) - \frac{1}{3}(t_2^3 - t_1^3) \right\} + \frac{b^2}{6} \left\{ \frac{t_1^3}{2}(t_2^2 - t_1^2) - \frac{1}{5}(t_2^5 - t_1^5) \right\} \right] \end{aligned}$$

$$\begin{aligned}
& -br \left\{ \frac{t_1}{3}(t_2^3 - t_1^3) - \frac{1}{4}(t_2^4 - t_1^4) \right\} - \frac{b^2 r}{6} \left\{ \frac{t_1^3}{3}(t_2^3 - t_1^3) - \frac{1}{6}(t_2^6 - t_1^6) \right\} \\
& - \frac{b^2}{2} \left\{ \frac{t_1}{4}(t_2^4 - t_1^4) - \frac{1}{5}(t_2^5 - t_1^5) \right\} - \frac{b^3}{12} \left\{ \frac{t_1^3}{4}(t_2^4 - t_1^4) - \frac{1}{7}(t_2^7 - t_1^7) \right\} \\
& - br \left\{ \frac{t_1^2}{2}(t_2^2 - t_1^2) - \frac{t_1}{3}(t_2^3 - t_1^3) \right\} - \frac{b^2 r}{6} \left\{ \frac{t_1^4}{2}(t_2^2 - t_1^2) - \frac{t_1}{5}(t_2^5 - t_1^5) \right\} \\
& + S \left[(b - br)t_1 \frac{1}{2}(t_2^2 - t_1^2) - br \frac{1}{3}(t_2^3 - t_1^3) + \frac{b^2}{2} \left\{ \frac{t_1^2}{2}(t_2^2 - t_1^2) - \frac{1}{4}(t_2^4 - t_1^4) \right\} \right] \\
& \dots (16)
\end{aligned}$$

Present worth shortages cost occurs during (t_2, T) is:

$$\begin{aligned}
S_1 &= -C_{si} \left[\int_{t_2}^{t_3} e^{-r(t_2+t)} Q_i(t) dt + \int_{t_3}^T e^{-r(t_3+t)} Q_i(t) dt \right] \\
&= C_{si} \left[\frac{r_i}{T\delta} \left\{ \frac{e^{-r t_2}}{(r+\delta)} \left\{ e^{-(r+\delta)t_3} - e^{-(r+\delta)t_2} \right\} - \frac{e^{-(r+\delta)t_2}}{r} \left\{ e^{-r t_3} - e^{-r t_2} \right\} \right\} \right. \\
&\quad \left. - e^{-r t_2} \left\{ \frac{e^{-r T}}{r} \left(K - \frac{r_i}{T} \right) (t_3 - T) + (e^{-r T} - e^{-r t_3}) \left(\frac{R}{r} - \frac{1}{r^2} \left(K - \frac{r_i}{T} \right) \right) \right\} \right] \dots (17)
\end{aligned}$$

Present worth lost sale cost occurs during (t_2, t_3) is:

$$\begin{aligned}
L_1 &= C_{Lsi} \int_{t_2}^{t_3} (1-B) \frac{r_i}{T} e^{-r t_3} dt \\
&= C_{Lsi} \left[\frac{r_i}{T} e^{-r t_3} \left\{ (t_3 - t_2) + \frac{1}{\delta} (e^{-\delta t_3} - e^{-\delta t_2}) \right\} \right] \dots (18)
\end{aligned}$$

CASE 1:- UNIFORM DEMAND AND DISCRETE UNITS

τ is a. r.v. with probability $P(\tau)$, s.t. $\sum_{\tau=\tau_0}^{\infty} P(\tau) = 1$ and $P(\tau) \geq 0$

Therefore, the total expected average cost is:

$$\begin{aligned}
Ea c(\tau^*) &= \frac{1}{T} \sum_{i=1}^n \left[\sum_{\tau=\tau_0}^{\tau^*} H_1 P(\tau) d\tau + \sum_{\tau=\tau_0+1}^{\infty} H_2 P(\tau) d\tau + \sum_{\tau=\tau_0}^{\tau^*} D_1 P(\tau) d\tau \right. \\
&\quad \left. + \sum_{\tau=\tau_0+1}^{\infty} D_2 P(\tau) d\tau + \sum_{\tau=\tau_0+1}^{\infty} S_1 P(\tau) d\tau + \sum_{\tau=\tau_0+1}^{\infty} L_1 P(\tau) d\tau \right]
\end{aligned}$$

Where $H_1, H_2, D_1, D_2, S_1, L_1$ are given by equations (5), (15), (6), (16), (17) and (18) respectively.

Since, $Q_{i0} \geq r_i$

$$\begin{aligned}
Q_{i0} &\geq a_i \tau + \frac{c_i}{(n-1)} \frac{\sum_{j=1, j \neq i}^n p_j}{p_i} \\
\tau &\leq \frac{1}{a_i} \left[Q_{i0} - \frac{c_i}{(n-1)p_i} \sum_{j=1, j \neq i}^n p_j \right] = \tau^* \text{ (say)}
\end{aligned}$$

$$\text{Then, } Q_{i0} = a_i \tau^* + \frac{c_i \sum_{j=1, j \neq i}^n p_j}{(n-1)p_i}$$

Also, $Q_{i0} < r_i$

$$\Rightarrow \tau^* < \tau \Rightarrow \tau > \tau^* \text{ and } Q_{i0} \geq r_i, \tau \leq \tau^*$$

CASE 2: UNIFORM DEMAND AND CONTINUOUS UNITS

When uncertain demand is estimated as a continuous random variable, the cost equation of the inventory involves integrals instead of summation signs. The discrete point probability $P(\tau)$ are replaced by the probability differential $f(\tau)$

for small interval. In this case, $\int_0^\infty f(\tau) d\tau = 1$ and $f(\tau) \geq 0$. The total expected average cost during period $(0, T)$ is,

$$Ea c(\tau^*) = \frac{1}{T} \sum_{i=1}^n \left[\int_{\tau=\tau_0}^{\tau^*} H_1 f(\tau) d\tau + \int_{\tau=\tau^*}^{\infty} H_2 f(\tau) d\tau + \int_{\tau=\tau_0}^{\tau^*} D_1 f(\tau) d\tau \right. \\ \left. + \int_{\tau=\tau^*}^{\infty} D_2 f(\tau) d\tau + \int_{T=\tau^*}^{\infty} S_1 f(\tau) d\tau + \int_{T=\tau^*}^{\infty} L_1 f(\tau) d\tau \right]$$

where $P_1, P_2, H_1, H_2, D_1, D_2, S_1, L_1$ are given by equation (5), (15), (6), (16), (17) and (18) respectively.

IV. CONCLUSION

A production inventory model for time dependent deteriorating rate has been developed. The demand rate is stochastic because in real situation, there are many commodities whose demand is not certain. Shortages are allowing with partial backlogging. The environment of whole study has been taken as inflationary. We hope that this study has the scope of direct application in the very practical situations.

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