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# A Study of Resonance in a Geocentric Satellite Including its Latitude and Second Order Tesseral Harmonic Due to Earth's Equatorial Ellipticity

Rajiv Aggarwal[1], Sushil Yadav[2], Bhavneet Kaur[3]
[1]Sri Aurobindo College, Department of Mathematics, University of Delhi, New Delhi, INDIA
[2]Maharaja Agrasen College, Department of Mathematics, University of Delhi, Delhi, INDIA
[3]Lady Shri Ram College, Department of Mathematics, University of Delhi, New Delhi, INDIA

**ABSTRACT**

**The present paper deals with the study of resonance in a geocentric satellite including its latitude and the earth's equatorial ellipticity. We have studied the resonance between angular velocity of the satellite and earth's equatorial ellipticity parameter $\Gamma$ (angle between the projection of satellite in the plane of equator and the minor axis of the earth's equatorial section). The object of this study is to determine amplitude and time period of the resonance oscillation by using a method given in Brown and Shook (1933). We have shown the effect of earth's equatorial ellipticity parameter $\Gamma$ on amplitude and time period including $\phi$ (latitude of the satellite). We observed that for increasing values of the earth's equatorial ellipticity parameter $\Gamma$, the corresponding amplitude decreases and time period also decreases.**

***Keywords*-------** Geocentric satellite. Earth's equatorial ellipticity. Resonance

## I. INTRODUCTION

Resonance plays a fundamental role in any dynamical system. It can be the source of both instability and long-term stability. Resonance occurs between two satellites or planets if there is a repetition in the geometric configuration of their position with respect to their orbits within a small period of time. This happens when the ratio of the orbital period is close to small integers. Resonance is common in the satellite system due to the effect of tides. Several authors have worked on the resonance problems in the solar system but a very less attention is given on the second order tesseral harmonic due to earth's equatorial ellipticity.

Mohamad Radwan (2002) studied the resonance resulting from the commensurability between the mean motion of the satellite, the Moon and the Sun. A conditionally periodic solution is constructed for the motion of an Earth satellite taking into consideration the Oblateness of the Earth and the Luni-Solar attraction.

Callegari Jr. *et al.* (2004) analyzed the model of the dynamics of two planets near a first-order mean-motion resonance in the domain of the general three body problem. They studied the phase space of the resonance and near-resonance regions in Uranus-Neptune (2/1 resonance) system by means of surfaces of section and spectral analysis techniques.

Voyatzis *et al.* (2005) studied symmetric and nonsymmetrical periodic orbits in the exterior mean motion resonances 1:2, 1:3 and 1:4 with Neptune in the framework of the planar circular restricted three-body problem. They observed that in each resonance there exist two branches of symmetric elliptic periodic orbits with stable and unstable segments.

Vrbik, Jan (2013) analyzed the motion of a test particle of a planar, circular, restricted three body problem in resonance, using the Kustaanheimo-Stiefel formalism. Using 7/4 resonance as an example, he showed that a good qualitative description of the motion can be reduced to three simple equations for semi-major axis, eccentricity and resonance angle, which reveals the onset of chaos and sheds a new light on its weak nature.

Pourtakdoust *et al.* (2014) studied the gravitational effect of a fourth body on the resonance orbit defined in the restricted three-body problem (RTBP). Resonance Hamiltonian of the RTBP and the Hamiltonian associated with the fourth gravitational body that perturbs the resonance orbit are computed. The Melnikov approach is utilized as a mean for the detection of chaos in resonance orbit under the influence of the fourth gravitation body. They showed that inclusion of the fourth body gravitation leads the (3:1) as well as the (4:1) resonance orbits to chaos.

In our paper (2013), we have studied the resonance in a geocentric satellite due to earth's equatorial ellipticity. We have not taken into account the latitude of the satellite. We have also given a brief description of the method of Brown and Shook (1933) which we have used to investigate the resonance In the present work, we have investigated resonance in a geocentric satellite including its latitude and second order tesseral harmonic $J_2^{(2)}$ due to earth's equatorial ellipticity. We have used a method given in Brown and Shook (1933) to determine the amplitude and time period of the resonance oscillation. In this study, the gravitational attraction of the sun and the moon, the sun's radiation pressure and residual drag effects are neglected.

## II. EQUATIONS OF MOTION

The equations of motion of the geocentric satellite $P(r,\theta,\phi)$ moving around the earth $E$ in the equatorial plane are given by (Frick and Garber (1962))

$$M_s\left(\ddot{r}-r\dot{\theta}^2\cos^2\phi-r\dot{\phi}^2\right)=\frac{\partial U}{\partial r}, \tag{1}$$

$$M_s\left(\frac{1}{r\cos\phi}\frac{d}{dt}\left(r^2\dot{\theta}\cos\phi\right)\right)=\frac{1}{r\cos\phi}\frac{\partial U}{\partial\theta}\ , \tag{2}$$

$$M_s\left(\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\phi}\right)+r\dot{\theta}^2\cos\phi\sin\phi\right)=\frac{1}{r\cos\phi}\frac{\partial U}{\partial\theta}\ , \tag{3}$$

where

$$U=\frac{g_0R_0^{\,2}}{r}\left\{1-\frac{J_2R_0^{\,2}}{r^2}\left(\frac{3\sin^2\phi-1}{2}\right)+3\frac{J_2^{(2)}R_0^{\,2}}{r^2}\cos^2\phi\cos 2\Gamma\right\}, \tag{4}$$

$U=$ earth's gravitational potential,

$g_0=$ gravitational aceleration on earth's surface,

$r=$ radial distance between satellite and the centre of the earth,

$M_s=$ mass of satellite,

$J_2=$ coefficient due to earth oblateness,

$R_0=$ radius of the earth,

$J_2^{(2)}=$ coefficient due to the earth's equatorial ellipticity,

$\phi=\angle PEM=$ latitude of the satellite, (Fig.1)

$\theta=\angle X_GEF=$ longitude of the satellite, (Fig.1)

$\Gamma = \angle MEF = \theta - \theta_E$

$=$ angle between the projection of satellite in the plane of equator and minor axis of the earth's equatorial section, (Fig.2)

$\theta_E = \angle X_G EF =$ angular position of the minor axis of the earth's equatorial section,

$\dot{\theta}_E =$ angular rate of earth's rotation,

$X_G, Y_G, Z_G =$ inertial coordinate system with origin at the centre of the earth

and $X_G Y_G$ plane in the earth's equatorial plane,

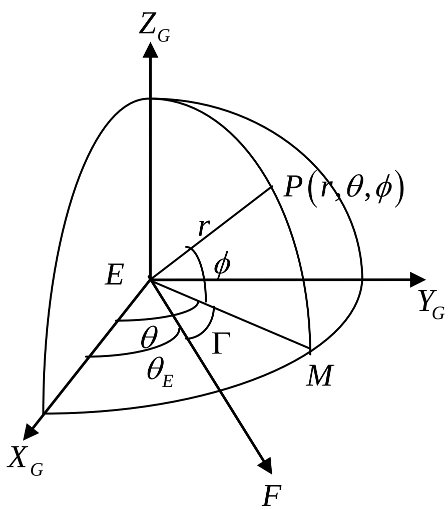


Fig.1 Configuration of the satellite $P$

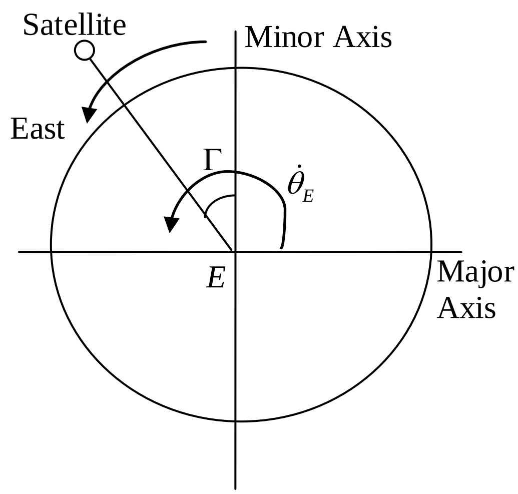


Fig.2 Earth's Equatorial Ellipticity parameter $\Gamma$

Substituting value of $U$ from (4) in Equations (1), (2) and (3), we obtain

$$\ddot{r} - r\dot{\theta}^2 \cos^2\phi - r\dot{\phi}^2 = -\frac{g_0 R_0^{\ 2}}{r^2} + 3\frac{J_2 g_0 R_0^{\ 4}}{2r^4}\left(3\sin^2\phi - 1\right) - 9\frac{J_2^{(2)} g_0 R_0^{\ 4}}{r^4}\cos^2\phi \cos 2\Gamma, \quad (5)$$

$$\frac{1}{r\cos\phi}\frac{d}{dt}\left(r^2\dot{\theta}\cos\phi\right) = -6\frac{J_2^{(2)} g_0 R_0^{\ 4}}{r^4}\cos\phi \sin 2\Gamma, \quad (6)$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\phi}\right) + r\dot{\theta}^2 \cos\phi \sin\phi = -3\frac{J_2 g_0 R_0^{\ 4}}{2r^4}\sin\phi\cos\phi - 6\frac{J_2^{(2)} g_0 R_0^{\ 4}}{r^4}\sin\phi\cos\phi\cos 2\Gamma. \quad (7)$$

For the unperturbed system, we have

$$J_2 = 0,\ J_2^{(2)} = 0.$$

Equations (5), (6) and (7) become

$$\ddot{r} - r\dot{\theta}^2\cos^2\phi - r\dot{\phi}^2 = -\frac{g_0 R_0^{\ 2}}{r^2}, \tag{8}$$

$$\frac{1}{r\cos\phi}\frac{d}{dt}\left(r^2\dot{\theta}\cos\phi\right) = 0, \tag{9}$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\phi}\right) + r\dot{\theta}^2\cos\phi\sin\phi = 0. \tag{10}$$

Using (9), we have

$$r^2\dot{\theta}\cos\phi = \text{constant} = h\,(\text{say}). \tag{11}$$

Substituting the values $r = \frac{1}{u}$, $\dot{r} = -\frac{h}{\cos^2\phi}\frac{du}{d\theta}$, $\ddot{r} = -\frac{hu^2}{\cos^4\phi}\frac{d^2u}{d\theta^2}$ and using (11) in Equation (5), we get

$$\frac{d^2u}{d\theta^2} + \left(\cos^2\phi\right)u = -\frac{\dot{\phi}^2}{\dot{\theta}^2}u^7 - \frac{u^2}{\dot{\theta}^2}\times$$
$$\left(-g_0R_0^{\ 2}u^2 + \frac{3}{2}J_2 g_0 R_0^{\ 4}u^4\left(3\sin^2\phi - 1\right) - 9J_2^{(2)}g_0R_0^{\ 4}u^4\cos^2\phi\cos 2\Gamma\right),$$

or

$$\frac{d^2u}{d\theta^2} + \left(\cos^2\phi\right)u = -\frac{\dot{\phi}^2}{\dot{\theta}^2}\frac{1}{r^7} - \frac{1}{\dot{\theta}^2 r^2}\times$$
$$\left(-\frac{g_0R_0^{\ 2}}{r^2} + \frac{3}{2}\frac{J_2 g_0 R_0^{\ 4}}{r^4}\left(3\sin^2\phi - 1\right) - 9\frac{J_2^{(2)}g_0R_0^{\ 4}}{r^4}\cos^2\phi\cos 2\Gamma\right).$$

Replacing $r$, $\dot{\theta}$ by their steady-state values $r_0$, $\dot{\theta}_0$, we get

$$\frac{d^2u}{d\theta^2} + \left(\cos^2\phi\right)u = -\frac{\dot{\phi}^2}{\dot{\theta}_0^{\ 2}}\frac{1}{r_0^{\ 7}} - \frac{1}{\dot{\theta}_0^{\ 2}r_0^{\ 2}}\times$$
$$\left(-\frac{g_0R_0^{\ 2}}{r_0^{\ 2}} + \frac{3}{2}\frac{J_2 g_0 R_0^{\ 4}}{r_0^{\ 4}}\left(3\sin^2\phi - 1\right) - 9\frac{J_2^{(2)}g_0R_0^{\ 4}}{r_0^{\ 4}}\cos^2\phi\cos 2\Gamma\right). \tag{12}$$

We may take

$$\theta = \dot{\theta}_0 t = nt\,(\text{say}),$$

$$\Gamma = \dot{\Gamma}t = mt\,(\text{say}).$$

 

Substituting these values in Equation (12), we get

$$\frac{d^2u}{dt^2}+\left(n^2\cos^2\phi\right)u=-\frac{\dot{\phi}^2}{r_0^{\;7}}-\frac{1}{r_0^{\;2}}\times \left(-\frac{g_0R_0^{\;2}}{r_0^{\;2}}+\frac{3}{2}\frac{J_2g_0R_0^{\;4}}{r_0^{\;4}}\left(3\sin^2\phi-1\right)-9\frac{J_2^{(2)}g_0R_0^{\;4}}{r_0^{\;4}}\cos^2\phi\cos 2\dot{\Gamma}t\right). \tag{13}$$

Since we are investigating the resonance in the satellite due to earth's equatorial ellipticity, we are ignoring the secular terms and terms due to the oblateness $J_2$ of the earth, we, therefore, write

$$\frac{d^2u}{dt^2}+\left(n^2\cos^2\phi\right)u=9\frac{J_2^{(2)}g_0R_0^{\;4}}{r_0^{\;6}}\cos^2\phi\cos 2mt. \tag{14}$$

The solution of Equation (14) is

$$u=A\cos(nt-B)+9\frac{J_2^{(2)}g_0R_0^{\;4}}{r_0^{\;6}}\cos^2\phi\left(\frac{1}{-4m^2+n^2\cos^2\phi}\right)\cos 2mt. \tag{15}$$

where $A$ and $B$ are constants.

We know that if any one of the denominator vanishes, the motion is indeterminate at that point. The resonance occurs at the point where $n\cos\phi=2m$.

## III. RESONANCE AT THE POINT WHERE $n\cos\phi=2m$

To study the motion of satellite at the point where $n\cos\phi=2m$, we will follow the procedure as given in Brown and Shook (1933).

It is proposed to find the solution of Equation (14) when that of

$$\frac{d^2u}{dt^2}+\left(n^2\cos^2\phi\right)u=0, \tag{16}$$

is periodic and is known.

The solution of Equation (16) is

$$u=c\cos l,$$

where

$$l=\left(n\cos\phi\right)t+\varepsilon',\; n=\frac{\sqrt{c_1}}{c\cos\phi}=\text{function of } c, \tag{17}$$

the arbitrary constants being $c$ and $\varepsilon'$.

Equation (14) can be written as

 

$$\frac{d^2u}{dt^2}+\left(n^2\cos^2\phi\right)u=NA_1\cos 2mt=N\psi'(\text{say}),,$$

where

$$N=9\frac{J_2^{(2)}g_0R_0^{\;4}}{r_0^{\;6}} \text{ and } A_1=\cos^2\phi.$$

$$\psi'=\frac{\partial\psi}{\partial u}=A_1\cos 2mt,\;\; \psi=uA_1\cos 2mt,$$

$$\psi=\frac{c}{2}A_1\left\{\cos(2mt+l)+\cos(2mt-l)\right\}. \tag{18}$$

Thus, we get

$$\frac{dc}{dt}=\frac{N}{K}\frac{\partial u}{\partial l}\psi'=\frac{N}{K}\frac{\partial\psi}{\partial l}, \tag{19}$$

$$\frac{dl}{dt}=(n\cos\phi)-\frac{N}{K}\frac{\partial u}{\partial c}\psi'=(n\cos\phi)-\frac{N}{K}\frac{\partial\psi}{\partial c}, \tag{20}$$

where

$$K=\frac{\partial}{\partial c}\left((n\cos\phi)\frac{\partial u}{\partial l}\right).\frac{\partial u}{\partial l}-(n\cos\phi)\frac{\partial^2 u}{\partial l^2}.\frac{\partial u}{\partial c}, \tag{21}$$

$$=\text{a function of } c \text{ only.}$$

Since $n,\ K$ are functions of $c$ only, we can put (19) and (20) into canonical form with new variables $c_1, B$ defined by

$$dc_1=Kdc, \tag{22}$$

$$dB=-(n\cos\phi)dc_1=-(n\cos\phi)Kdc. \tag{23}$$

Equations (24) and (25) can be put in the form

$$\frac{dc_1}{dt}=\frac{\partial}{\partial l}(B+M\psi),$$

$$\frac{dl}{dt}=-\frac{\partial}{\partial c_1}(B+M\psi).$$

Differentiating Equation (20) with respect to $t$ and substituting the expressions for $\frac{dl}{dt}$ and $\frac{dc}{dt}$ in the result, we obtain

 

$$\frac{d^2 l}{dt^2} = \frac{N}{K}\left\{\frac{\partial(n\cos\phi)}{\partial c}\frac{\partial\psi}{\partial l} - (n\cos\phi)\frac{\partial^2\psi}{\partial l\partial c} - \frac{\partial^2\psi}{\partial c\partial t}\right\}$$
$$+\frac{N^2}{K^2}\left\{\frac{\partial^2\psi}{\partial l\partial c}\frac{\partial\psi}{\partial c} - K\frac{\partial}{\partial c}\left(\frac{1}{K}\frac{\partial\psi}{\partial c}\right)\frac{\partial\psi}{\partial l}\right\}. \quad (24)$$

Since the last expression of Equation (24) has the factor $N^2$, it may, in general, be neglected in a first approximation.

In Equation (18), we find that $l$ and $t$ are present in $\psi$ as the sum of the periodic terms with argument $l' = l - 2mt$.

The affected term, in our case, is

$$\psi = \frac{1}{2} c A_1 \cos l'. \quad (25)$$

Equation (24) for $l'$ is then

$$\frac{d^2 l'}{dt^2} + \left((n\cos\phi) - 2m\right)^2 \frac{N}{K}\left\{\frac{\partial}{\partial c}\left(\frac{1}{(n\cos\phi) - 2m}\frac{\partial\psi}{\partial l'}\right)\right\} = 0.$$

or

$$\frac{d^2 l'}{dt^2} - \left((n\cos\phi) - 2m\right)^2 \frac{N}{2K}\left\{\frac{\partial}{\partial c}\left(\frac{cA_1}{(n\cos\phi) - 2m}\right)\right\}\sin l' = 0. \quad (26)$$

As a first approximation, we put

$c = c_0,\ n\cos\phi = n_0\cos\phi_0,\ K = K_0$ (all constants).

Then Equation (26) can be written as

$$\frac{d^2 l'}{dt^2} - \left(n_0\cos\phi_0 - 2m\right)^2 \frac{N}{2K_0}\left\{\frac{\partial}{\partial c}\left(\frac{cA_1}{n\cos\phi - 2m}\right)\right\}_0 \sin l' = 0. \quad (27)$$

If the oscillation be small, Equation (27) can be put in the form

$$\frac{d^2 l'}{dt^2} - \left(n_0\cos\phi_0 - 2m\right)^2 \frac{N}{2K_0}\left\{\frac{\partial}{\partial c}\left(\frac{cA_1}{n\cos\phi - 2m}\right)\right\}_0 l' = 0.$$

or

$$\frac{d^2 l'}{dt^2} + p^2 l' = 0, \quad (28)$$

where

$$p^2 = \left| \frac{NA_1}{2K_0} c_0 \left( \frac{\partial (n\cos\phi)}{\partial c} \right)_0 \right|,$$

$$= \left| \frac{9}{2} \frac{J_2^{(2)} g_0 R_0^{\,4}}{r_0^{\,6}} \cos^2\phi \frac{c_0}{K_0} \left( -\frac{\sqrt{c_1}}{c^2} \right)_0 \right|,$$

or

$$p = \frac{3\sqrt{g_0 J_2^{(2)}} R_0^{\,2}}{\sqrt{2r_0^{\,3}}} \cos\phi_0 \sqrt{\frac{\sqrt{c_1}}{K_0 c_0}}, \qquad (29)$$

and

$$K_0 = (K)_0,$$

$$= \left\{ \frac{\partial}{\partial c} \left( n\cos\phi \frac{\partial u}{\partial l} \right) \frac{\partial u}{\partial l} - n\cos\phi \frac{\partial^2 u}{\partial l^2} \frac{\partial u}{\partial c} \right\}_0,$$

Substituting $u = c\cos l$, we get

$$K_0 = \left\{ \frac{\partial}{\partial c} \left( n\cos\phi \frac{\partial}{\partial l}(c\cos l) \right) \frac{\partial}{\partial l}(c\cos l) - n\cos\phi \frac{\partial^2}{\partial l^2}(c\cos l) \frac{\partial}{\partial c}(c\cos l) \right\}_0,$$

Now we substitute $l = (n\cos\phi)t + \varepsilon'$ and using $n\cos\phi = \frac{\sqrt{c_1}}{c}$, we obtain

$$K_0 = \left\{ \sqrt{c_1} \cos^2 \left( (n\cos\phi)t + \varepsilon' \right) \right\}_0,$$

$$= \sqrt{c_1} \cos^2 \left( (n_0 \cos\phi_0)t + \varepsilon_0' \right) = \sqrt{c_1} \cos^2 (2mt + \varepsilon_0'),$$

$$= \sqrt{c_1} \cos^2 \left( 2\dot{\Gamma} t + \varepsilon_0' \right) == \sqrt{c_1} \cos^2 (2\Gamma + \varepsilon_0').$$

The solution of Equation (28) is given by

$$l' = A\sin(pt + \lambda_0), \qquad (30)$$

where

$$A = \frac{\sqrt{c_2}}{p},$$

$c_2$ and $\lambda_0$ are constants of integration,

$$l' = l - 2mt.$$

 

The equation for $l$ gives

$$l = 2\Gamma + A\sin\left(pt + \lambda_0\right) \tag{31}$$

Using Equations (19), (25) and (30), the equation for $c$ gives

$$c = c_0 + \frac{NA_1}{2}\left(\frac{c}{K}\right)_0 \frac{A}{p}\cos\left(pt + \lambda_0\right), \tag{32}$$

where

$c_0$ is determined from $n_0 \cos\phi_0 = 2m,$

since $n_0$ is a known function of $c_0$.

The amplitude $A$ and the time period $T$ are given by

$$A = \frac{\sqrt{c_2}}{p}, \quad T = \frac{2\pi}{p}.$$

where

$$p = \frac{3\sqrt{g_0 J_2^{(2)} R_0^{\,2}}}{\sqrt{2r_0^{\,3}}}\cos\phi_0 \frac{1}{\sqrt{c_0}\cos 2\Gamma}$$

Using Equation (17), we have

$$c_0 = \frac{\sqrt{c_1}}{n_0 \cos\phi_0}.$$

We may choose the constants of integration

$c_1 = 1,\ c_2 = 1$ and $\varepsilon_0' = 0.$

Hence we get

$$A = \frac{\sqrt{2}}{3}\frac{r_0^{\,3}}{\sqrt{g_0 J_2^{(2)} R_0^{\,2}}\sqrt{n_0}\cos^{\frac{3}{2}}\phi_0}\cos 2\Gamma, \tag{33}$$

$$T = \frac{2\sqrt{2}\pi}{3}\frac{r_0^{\,3}}{\sqrt{g_0 J_2^{(2)} R_0^{\,2}}\sqrt{n_0}\cos^{\frac{3}{2}}\phi_0}\cos 2\Gamma. \tag{34}$$

## IV. APPLICATION

Using the following data of a satellite, we draw Amplitude $A$ versus $\Gamma$ (Fig.3) and time period $T$ versus $\Gamma$ (Fig.4).

$R_0 = 6392.1\times10^5$ cm,

$r_0 = 7822.49\times10^5$ cm,

$$n_0 = \left(0.0009 \times \frac{180}{\pi}\right) \text{deg/ sec},$$

$$J_2^{(2)} = 2.32 \times 10^{-6},$$

$$g_0 = 9.8 \text{ m / sec}^2,$$

$$\phi_0 = 0^\circ.$$

We make the above quantities dimensionless by taking

$$M_E + M_S = 1 \text{unit},$$

$$D_{ES} = \text{distance between the earth and the sun} = 1 \text{unit},$$

$$G = \text{universal gravitational constant=1unit.}$$

where

$$M_E = \text{mass of the earth=}5.9742 \times 10^{27} \text{ gm},$$

$$M_S = \text{mass of the sun=}1.9891 \times 10^{33} \text{ gm},$$

$$D_{ES} = 1.4959787061 \times 10^{13} \text{ cm},$$

$$G = 6.672 \times 10^{-8} \text{ cm}^3 \text{gm}^{-1} \text{sec}^{-2}.$$

We have drawn amplitude $A$ versus $\Gamma$ and time period $T$ versus $\Gamma$. Since amplitude $A$ and time period $T$ contain the term $\cos 2\Gamma$, we have drawn amplitude $A$ versus $\Gamma$ (Fig.3) and time period $T$ versus $\Gamma$ (Fig.4) for $0^\circ \le \Gamma \le 45^\circ$. It is observed that amplitude $A$ decreases and time period $T$ also decreases as $\Gamma$ increases in the above range.

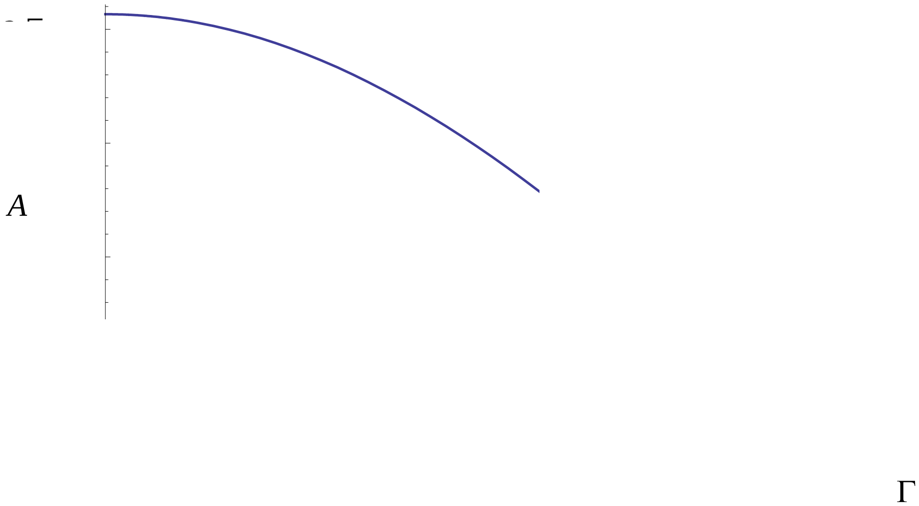


Fig.3 Amplitude $A$ versus $\Gamma$

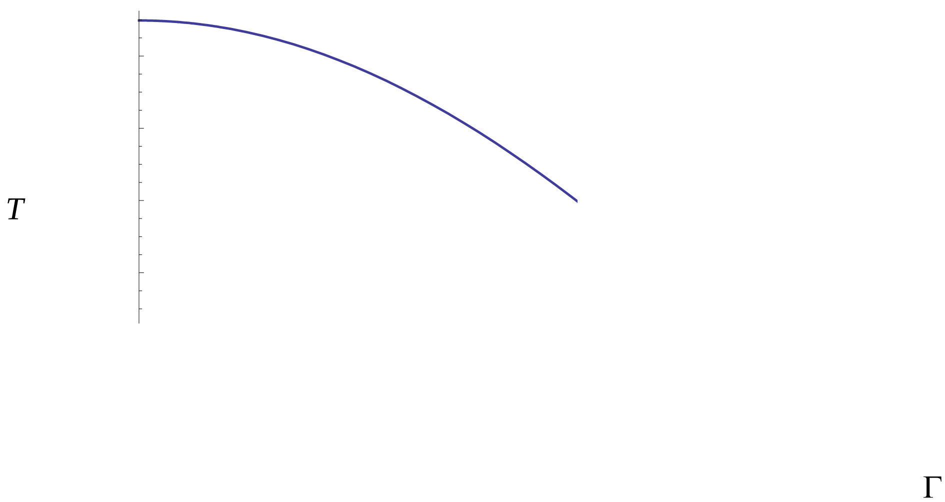


Fig.4 Time Period $T$ versus $\Gamma$

Effect of amplitude $A$ and time period $T$ on $\phi$ (latitude of the satellite) for $0^\circ \le \Gamma \le 45^\circ$ is shown in Fig.5 and Fig.6 respectively. It is observed that amplitude $A$ increases as $\Gamma$ increases for $0^\circ \le \phi \le 90^\circ$ (Fig.5) and time period $T$ also increases as $\Gamma$ increases for $0^\circ \le \phi \le 90^\circ$ (Fig.6).

 

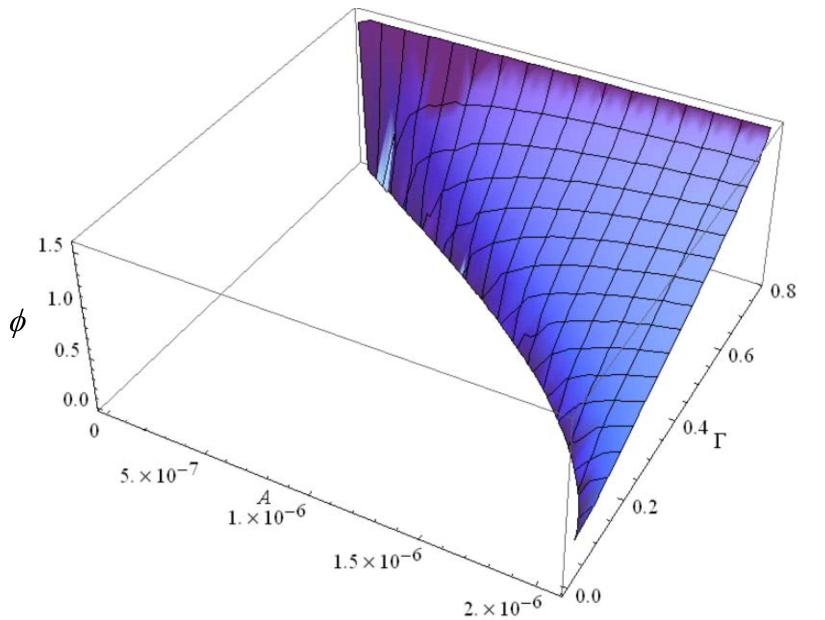


Fig.5 Effect of amplitude $A$ on $\phi$ (latitude of satellite) for $0^\circ \le \Gamma \le 45^\circ$

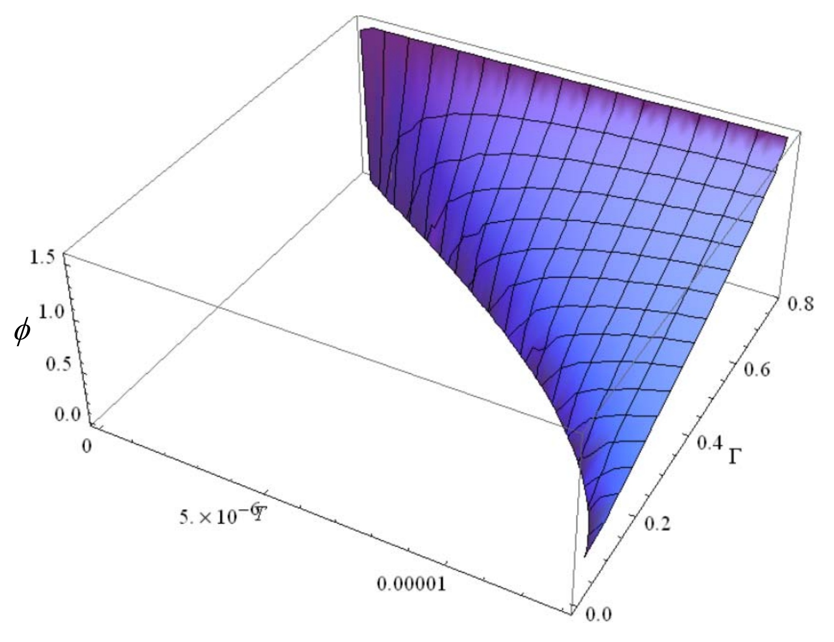


Fig.6 Effect of amplitude $T$ on $\phi$ (latitude of satellite) for $0^\circ \le \Gamma \le 45^\circ$

## V. CONCLUSION

It is observed that the denominator in Equation (15) vanishes at the point where $\dot{\theta}_0 \cos\phi = 2\dot{\Gamma}$, where $\dot{\theta}_0$ is the steady-state orbital angular rate of the satellite, $\phi$ is latitude of the satellite and $\dot{\Gamma}$ is the rate of change of the parameter due to earth's equatorial ellipticity. Resonance occurs at the point where $\dot{\theta}_0 \cos\phi = 2\dot{\Gamma}$. Amplitude and time period of resonance oscillation are determined by using the method as given in Brown and

 

Shook (1933). Using the data of a satellite, we have shown the effect of $\Gamma$ and $\phi$ on amplitude and time period. We conclude that as $\Gamma$ increases from $0^{\circ}$ to $45^{\circ}$, amplitude decreases (Fig.3) and time period also decreases (Fig.4). Finally, we have shown effect of amplitude $A$ and time period $T$ on $\phi$ for $0^{\circ} \leq \Gamma \leq 45^{\circ}$ in Fig.5 and Fig.6 respectively. It is observed that amplitude $A$ increases as $\Gamma$ increases for $0^{\circ} \leq \phi \leq 90^{\circ}$ (Fig.5) and time period $T$ also increases as $\Gamma$ increases for $0^{\circ} \leq \phi \leq 90^{\circ}$ (Fig.6).

The present study has tremendous applications in the motion of geo-stationary satellites. These satellites are widely used for mass media and telecommunications.

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