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An Application of Pentagonal Fuzzy Number Matrix in Decision Making

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ABSTRACT

Decision making is the method to find the best one among a set of selected alternatives. Under a fuzzy environment, the decision making enable the decision maker to choose their own opinion. In this paper, we have used the pentagonal fuzzy number and pentagonal fuzzy number matrix to select the winner of the singing competition among the top three contestants.

***Keywords*--** Pentagonal fuzzy number, pentagonal fuzzy number matrix, decision making

I. INTRODUCTION

In real world, People were overcoming lot of uncertainty in the day to day life. Fuzzy environment has a potential to solve such kind of uncertainty. Fuzzy set theory was introduced by L. A. Zadeh in the year 1965. Which plays an important role in predicting the solution for the problems. It involves in many fields such as medicine, engineering, etc. Fuzzy set is any set which allows to have a members in the interval [0,1] and it is known as membership function.

Decision making is a tool to find the best solution or better than that. There are various kinds of decision making. The decision taken by single person and several person which is individual decision making and multi person decision making respectively. It helps us to find the solution which is incomplete.

This paper have been summarized as basic definitions, decision making under fuzzy pentagonal number, procedure, illustrative example and finally solution.

II. MATHEMATICAL DEFINITIONS

1) Fuzzy number: A fuzzy number 'A' is a fuzzy set on the real line R must satisfy the following conditions

(i) $\mu_A(x_0)$ is piecewise continuous

(ii) There exist at least on $x_0 \in$ R with $\mu_{\tilde{A}}(x_0) = 1$

(iii) $\mu_{\tilde{A}}$must be normal and convex

2) Pentagonal fuzzy number: A pentagonal fuzzy number of a fuzzy set $\tilde{A}$is defined as $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ and its membership is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x \le a_1 \\ \left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ \left(\frac{x-a_2}{a_3-a_2}\right) & for\ a_2 \le x \le a_3 \\ 1 & for\ x = a_3 \\ \left(\frac{a_4-x}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ \left(\frac{a_5-x}{a_5-a_4}\right) & for\ a_4 \le x \le a_5 \\ 0 & for\ x \ge a_5 \end{cases}$$

3) Pentagonal fuzzy number matrix: The elements of pentagonal fuzzy number matrix is A = $(a_{ij})_{n\times n}$
$a_{ij} = (a_{ilL}, a_{ijM}, a_{ijN}, a_{ijR}, a_{ijS})$be the ij^{th} element of A.

4) Pentagonal fuzzy membership matrix: The membership function of $(a_{ij}) = (a_{ijL}, a_{ijM}, a_{ijN}, a_{ijR}, a_{ijS})$ is defined as $\left(\frac{a_{ijL}}{10}, \frac{a_{ijM}}{10}, \frac{a_{ijN}}{10}, \frac{a_{ijR}}{10}, \frac{a_{ijS}}{10}\right)$,
If $0 \le a_{ijL} \le a_{ijM} \le a_{ijN} \le a_{ijR} \le a_{ijS} \le 1$

Where $0 \le \frac{a_{ijL}}{10} \le \frac{a_{ijM}}{10} \le \frac{a_{ijN}}{10} \le \frac{a_{ijR}}{10} \le \frac{a_{ijS}}{10} \le 1$

III. ARITHMETIC OPERATION ON PENTAGONAL FUZZY NUMBER MATRIX

Let A = $(a_{ij})_{m\times n}$ and B = $(b_{ij})_{m\times n}$ are two pentagonal fuzzy number matrices of same order n× n

5) Addition: (A+B) = $(a_{ij} + b_{ij})_{m\times n}$

Where $(a_{ij} + b_{ij})$ = $(a_{ijL} + b_{ijL}, a_{ijM} + b_{ijM}, a_{ijN} + b_{ijN}, a_{ijR} + b_{ijR}, a_{ijS} + b_{ijS})$ is the ij^{th} element of (A+B).

6) Subtraction: (A-B) = $(a_{ij} + b_{ij})_{m\times n}$

Where $(a_{ij} - b_{ij})$ = $(a_{ijL} - b_{ijL}, a_{ijM} - b_{ijM}, a_{ijN} - b_{ijN}, a_{ijR} - b_{ijR}, a_{ijS} - b_{ijS})$ is the ij^{th} element of (A-B).

7) Maximum operation on pentagonal fuzzy number: The maximum operation is given by max(A,B) = $(\sup\{a_{ij}, b_{ij}\})$
Where sup (a_{ij}, b_{ij}) =
$(\sup(a_{ijL}, b_{ijL}), \sup(a_{ijM}, b_{ijM}), \sup(a_{ijN},$ $bijN$,sup $aijR, bijR$,sup $aijS, bijS)$ is the $ijth$ element of max (A,B).

8) Arithmetic mean (AM) for pentagonal fuzzy number: Let A = $(a_1, a_2, a_3, a_4, a_5)$ be pentagonal fuzzy number

AM(A) = $\frac{(a_1+a_2+a_3+a_4+a_5)}{5}$

IV. DECISION MAKING UNDER FUZZY PENTAGONAL NUMBER

9) Relativity function: Let x and y be a variables defined on a universal set X. The relativity function is denoted as $f\left(\frac{x}{y}\right) = \left\{\frac{\mu_y(x)-\mu_x(y)}{\max\{\mu_y(x),\mu_x(y)\}}\right\}$

Where $\mu_y(x)$ is the membership function of x with respect to 'y' for pentagonal fuzzy number and $\mu_x(y)$ is the membership function of y with respect to 'x' got pentagonal fuzzy number. Here $\mu_y(x) - \mu_x(y)$ is calculated using subtraction operation and max $\{\mu_y(x), \mu_x(y)\}$ is calculated by definition (7).

10) Comparison matrix: Let A = $\{x_1, x_2, x_3, \dots\dots\dots . x_{i-1}, x_i, x_{i+1}, \dots\dots\dots . . x_n\}$ be a set of n-variables defined on X. Form a matrix of relativity values $f\left(\frac{x_i}{x_j}\right)$ where x_i's for i=1 to n-variable defined on a matrix x. The matrix C=(c_{ij}) is a square matrix of order n is called comparison matrix (or) C-matrix with, $AM\left(f\left(\frac{x_i}{x_j}\right)\right) = \frac{AM\left(\mu_{x_j}(x_i)(-)\mu_{x_i}(x_j)\right)}{AM\left(\max\{\mu_{x_j}(x_i),\mu_{x_i}(x_j)\}\right)}$

Where AM represents arithmetic mean.

V. PROCEDURE

STEP 1

Consider the pentagonal fuzzy number matrix from the imprecise estimation needed for the problem using the definition of "Pentagonal fuzzy number matrix".

STEP 2

Convert the given matrix into membership function using the definition of "Pentagonal fuzzy membership matrix".

STEP 3

Calculate the relativity values $f\left(\frac{x_i}{x_j}\right)$ by the definition (9).

STEP 4

Calculate the comparison matrix from the values of $f\left(\frac{x_i}{x_j}\right)$ using the definition "Comparison matrix".

STEP 5

Find the minimum value from each row C_i.

STEP 6

The maximum value of the column C_i is the required solution.

VI. LUSTRATIVE EXAMPLE

Let us consider three students Jency, Praveen, Nancy participated in the singing competition. They are the top three contestants of this competition.

Vocalists will be judged based on the criteria listed below and will receive judge's brief comments immediately following their performance.

Criteria	**Possible points**
Vocal skills *(Pitch, Breath control, Dynamics, Phrasing, Enunciation, Tone)*	10
Presentation skills *(Eye contact, Body expression, Facial expression, Attire, Mic technique)*	10
Audience response to performance *(Are people smiling, nodding or ignoring the performance)*	10
Mastery of lyrics *(No errors or lapses in memory)*	10
Judge's overall impression	10

Let us consider the set $\{e_1, e_2, e_3, e_4, e_5\}$ as universal set, where e_1, e_2, e_3, e_4, e_5 denotes vocal skills, presentation skills, audience response to performance, mastery of lyrics, judge's overall impression respectively.

Choose performance based on the overall quality of performance.

The matrix A represents the scores in the form of pentagonal fuzzy number matrix.

STEP 1:

$$A = \begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{ccc} x & y & z \\ (1,3,6,7,9) & (3,6,7,8,10) & (4,5,7,8,9) \\ (3,5,6,8,10) & (2,3,7,8,8) & (1,3,6,8,10) \\ (1,2,6,6,7) & (4,6,7,7,10) & (2,4,5,78) \end{array}$$

STEP 2:

$$(A)_{(mem)} = \begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{ccc} x & y & z \\ (0.1,0.3,0.6,0.7,0.9) & (0.3,0.6,0.7,0.8,1) & (0.4,0.5,0.7,0.8,0.9) \\ (0.3,0.5,0.6,0.8,1) & (0.2,0.3,0.7,0.8,0.8) & (0.1,0.3,0.6,0.8,1) \\ (0.1,0.2,0.6,0.6,0.7) & (0.4,0.6,0.7,0.7,1) & (0.2,0.4,0.5,0.7,0.8) \end{array}$$

$\mu_x(x)$ = (0.1, 0.3, 0.6, 0.7, 0.9)

$\mu_y(x)$ = (0.3, 0.6, 0.7, 0.8, 1)

$\mu_z(x)$ = (0.4, 0.5, 0.7, 0.8, 0.9)

$\mu_x(y)$ = (0.3, 0.5, 0.6, 0.8, 1)

$\mu_y(y)$ = (0.2, 0.3, 0.7, 0.8, 0.8)

$\mu_z(y)$ = (0.1, 0.3, 0.6, 0.8, 1)

$\mu_x(z)$ = (0.1, 0.2, 0.6, 0.6, 0.7)

$\mu_y(z)$ = (0.4, 0.6, 0.7, 0.7, 1)

$\mu_z(z)$ = (0.2, 0.4, 0.5, 0.7, 0.8)

STEP 3

$$f\left(\frac{x}{x}\right) = \left\{\frac{\mu_x(x) - \mu_x(x)}{\max\{\mu_x(x), \mu_x(x)\}}\right\}$$

$$= \frac{(0.1, 0.3,0.6,0.7,0.9) - (0.1,0.3,0.6,0.7,0.9)}{\max\{(0.1,0.3,0.6,0.7,0.9), (0.1,0.3,0.6,0.7,0.9)\}}$$

$$= \frac{(0,0,0,0,0)}{(0.1,0.3,0.6,0.7,0.9)}$$

= 0

$$f\left(\frac{x}{y}\right) = \left\{\frac{\mu_y(x) - \mu_x(y)}{\max\{\mu_y(x), \mu_x(y)\}}\right\}$$

$$= \frac{(0.3,0.6,0.7,0.8,1) - (0.3,0.5,0.6,0.8,1)}{\max\{(0.3,0.6,0.7,0.8,1), (0.3,0.5,0.6,0.8,1)\}}$$

$$= \frac{(0,0.1,0.1,0,0)}{(0.3,0.6,0.7,0.8,1)}$$

$$= \frac{0.04}{0.68}$$

= 0.0588

$$f\left(\frac{x}{z}\right) = \left\{\frac{\mu_z(x) - \mu_x(z)}{\max\{\mu_z(x), \mu_x(z)\}}\right\}$$

$$= \frac{(0.4,0.5,0.7,0.8,0.9) - (0.1,0.2,0.6,0.6,0.7)}{\max\{(0.4,0.5,0.7,0.8,0.9), (0.1,0.2,0.6,0.6,0.7)\}}$$

$$= \frac{(0.3,0.3,0.1,0.2,0.2)}{(0.4,0.5,0.7,0.8,0.9)}$$

$$= \frac{0.22}{0.66}$$

= 0.33

$$f\left(\frac{y}{x}\right) = \left\{\frac{\mu_x(y) - \mu_y(x)}{\max\{\mu_x(y), \mu_y(x)\}}\right\}$$

$$= \frac{(0.3,0.5,0.6,0.8,1) - (0.3,0.6,0.7,0.8,1)}{\max\{(0.3,0.5,0.6,0.8,1), (0.3,0.6,0.7,0.8,1)\}}$$

$$= \frac{(0, -0.1, -0.1,0,0)}{(0.3,0.6,0.7,0.8,1)}$$

$$= \frac{-0.04}{0.68}$$

= -0.0588

$$f\left(\frac{y}{y}\right) = \left\{\frac{\mu_y(y) - \mu_y(y)}{\max\{\mu_y(y), \mu_y(y)\}}\right\}$$

$$= \frac{(0.2,0.3,0.7,0.8,0.8) - (0.2,0.3,0.7,0.8,0.8)}{\max\{(0.2,0.3,0.7,0.8,0.8), (0.2,0.3,0.7,0.8,0.8)\}}$$

$$= \frac{(0,0,0,0,0)}{(0.2,0.3,0.7,0.8,0.8)}$$

=0

$$f\left(\frac{y}{z}\right) = \left\{\frac{\mu_z(y) - \mu_y(z)}{\max\{\mu_z(y), \mu_y(z)\}}\right\}$$

$$= \frac{(0.1,0.3,0.6,0.8,1) - (0.4,0.6,0.7,0.7,1)}{\max\{(0.1,0.3,0.6,0.8,1), (0.4,0.6,0.7,0.7,1)\}}$$

$$= \frac{(-0.3, -0.3, -0.1, 0.1, 0)}{(0.4, 0.6, 0.7, 0.8, 1)}$$

$$= \frac{-0.12}{0.7}$$

= -0.1714

$$f\left(\frac{z}{x}\right) = \left\{\frac{\mu_x(z) - \mu_z(x)}{\max\{\mu_x(z), \mu_z(x)\}}\right\}$$

$$= \frac{(0.1,0.2,0.6,0.6,0.7) - (0.4,0.5,0.7,0.8,0.9)}{\max\{(0.1,0.2,0.6,0.6,0.7), (0.4,0.5,0.7,0.8,0.9)\}}$$

$$= \frac{(-0.3, -0.3, -0.1, -0.2, -0.2)}{(0.4,0.5,0.7,0.8,0.9)}$$

$$= \frac{-0.22}{0.66}$$

= -0.33

$$f\left(\frac{z}{y}\right) = \left\{\frac{\mu_y(z) - \mu_z(y)}{\max\{\mu_y(z), \mu_z(y)\}}\right\}$$

$$= \frac{(0.4,0.6,0.7,0.7,1) - (0.1,0.3,0.6,0.8,1)}{\max\{(0.4,0.6,0.7,0.7,1), (0.1,0.3,0.6,0.8,1)\}}$$

$$= \frac{(0.3,0.3,0.1,-0.1,0)}{(0.4,0.6,0.7,0.8,1)}$$

$$= \frac{0.12}{0.7}$$

= 0.1714

$$f\left(\frac{z}{z}\right) = \left\{\frac{\mu_z(z) - \mu_z(z)}{\max\{\mu_z(z), \mu_z(z)\}}\right\}$$

$$= \frac{(0.2,0.4,0.5,0.7,0.8) - (0.2,0.4,0.5,0.7,0.8)}{\max\{(0.2,0.4,0.5,0.7,0.8), (0.2,0.4,0.5,0.7,0.8)}$$

$$= \frac{(0,0,0,0,0)}{(0.2,0.4,0.5,0.7,0.8)}$$

= 0

STEP 4

The comparison matrix

C = (C_{ij})

= AM$\left(f\left(\frac{x_i}{x_j}\right)\right)$ is given by

$$C = \begin{matrix} & x & y & z \\ x & 0 & 0.0588 & 0.33 \\ y & -0.0588 & 0 & -0.1714 \\ z & -0.33 & 0.1714 & 0 \end{matrix}$$

STEP 5

C_i = minimum of i^{th} row

x 0

y -0.1714

z -0.33

Comparison:

Pentagonal fuzzy number matrix in decision making	Normal method
'X' is the winner	'X' is the winner

STEP 6
'X' is the winner of the singing contest.
ie. Jency is the winner.

VII. CONCLUSION

Thus decision making process with the pentagonal fuzzy number have been used. It makes every problems to solve easily in the real life situations.

Hence we have succeed in finding the participant who have performed their piece exceptionally well based on the given criteria.

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