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Analysis of Thermal Stresses in a Composite Functionally Graded Material Plate by Finite Element Method

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ABSTRACT

In order to study the steady thermal stresses in a ceramic/FGM/metal composite plate a finite element model is constructed to analyze the steady temperature under convective heat transfer boundary. The mathematical model for the stresses in the plate was established, based on the functional of heat conduction. The FE basic equation of the one-dimensional heat conduction of the plate was derived. The continuum is divided into 750 elements and 751 nodes using eight node hexahedral elements. The first-order shear deformation plate model is exploited to investigate the uncoupled thermal behavior of functionally graded plates. The stress distributions of the plate were obtained and compared with those of the non graded two-layered composite plate. The results show the various degrees of the effects of thickness and composition in FGM gradient layer on thermal stresses of the plate, and the stress distributions in a graded three-layered composite plate are gentler. The model can effectively analyze the steady thermal stresses in ceramic/FGM/metal plate and the effect factors.

Key words--- FGM, finite element method, thermal stress

I. INTRODUCTION

Functionally graded materials (FGM) are microscopically inhomogeneous composite materials, in which the volume fraction of the two or more materials is varied smoothly and continuously as a continuous function of the material position along one or more dimension of the structure .These materials are mainly constructed to operate in high temperature environments. Rajesh sharma et. al.(2014) explained that FGM were developed to reduce such thermal stress and resist super high temperature. To reduce stress and resist super high temperature, FGM have continuous transition from metal at low temperature surfaces to ceramics at high temperature surfaces because it is used widely in high temperature working environment such as aviation and nuclear reactor and so on, it is important to analyze the temperature and thermal stress field of the body made of the materials. D. Saji et. al.(2008) used one dimensional heat conduction equation represent the temperature distribution across thickness of the FGM plate. Yangjian et. al.(2009) researched the effect of FGM layer thickness on temperature field by finite element method. In their research they constructed a finite element model to analyze the steady temperature field in a ceramic/metal composite FGM plate under heating boundary. The FGM is suitable for various applications, such as thermal coatings of barrier for ceramic engines, gas turbines, nuclear fusions, optical thin layers, and biomaterial electronics. R. Ramkumar and N. Ganesan (2007) studied the problem of buckling behaviors of thin walled box columns in a thermal environment by using CLPT theory; they use this theory with different software packages. Chen and Tong(2004) used a graded finite element approach to analyze the sensitivity in the problems of steady state and transient heat conduction in FGM .Therefore, FGMs have received considerable attention in the field of structural design subjected to extremely high thermal loading Y. Tanigawa(1995) and Y. Tanigawa et al.(1996).Because it is used widely in high temperature working environment such as aviation and nuclear reactor, and so on, it is important to analyze the thermal stress filed of the body made of the material. Y. Obata and N. Noda (1993), A. H. Muliana (2009) researched thermal stress of pure FGM plate by adopting perturbation and laminated analytical method, respectively. J. Huang, Y. B. Lü (2003) analyzed the thermal elastic limitation of four-layered composite plate with FGM in the middle of the plate. But these methods are too complex so as to lead to a complicated equation system, and are not convenient for engineering application. Therefore, Y. J. Xu et al.(2005), L. D. Croce (2004) studied the problem of transient thermal stress of pure FGM plate under convective heat transfer boundary by adopting simple FEM. Based on the above research work, starting from the heat conduction law; this paper will discuss the effects of FGM layer thickness and composition on thermal stresses. We hope that the numerical results obtained will be more close to actual engineering conditions and to obtain some instructive conclusions for the production and application of ZrO_2/FGM/Al composite plate.

II. DEVELOPMENT OF MODEL

Three layered infinite long composite plate made of pure Aluminum and pure ceramic (zirconia) with an interlayer of FGM is considered for analysis. The middle layer is continuous and varying in volume fraction. k(z),E(z) and α(z) denote thermal conductivity, modulus of elasticity and coefficient of thermal expansion respectively of FGM gradient layer and the layer thickness is $t_2=t_{FGM}$. The top layer is of pure ceramic i.e. have properties k_c, E_c, α_c similarly properties for bottom layer made of metal are k_m, E_m, α_m. The thickness of ceramic layer is t_1 and that of metal layer is t_3.

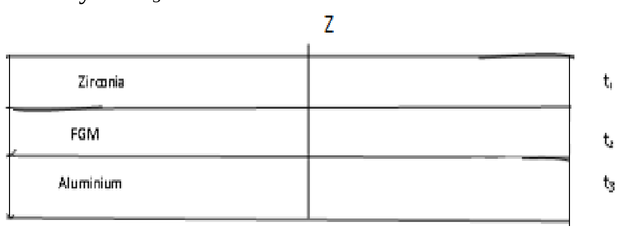

Figure 1. Zirconia/FGM/ Aluminum composite FGM plate.

The plate is stress free initially and the initial temperature of the plate is T_i. The plate is heated from lower and upper surfaces with heat transfer coefficient h_L and h_U respectively and temperature at the outer boundaries are T_L and T_U. The plate is clamped and free for bending. The plate is adiabatic in its periphery and there are no heat sources within the plate. Coordinate axis is in thickness direction and taken as Z. The material properties for same ordinate z are homogeneous and isotropic. The total thickness of plate is taken as $t=t_1+t_2+t_3$.

III. HEAT CONDUCTION EQUATIONS AND MATERIAL PROPERTIES

The steady state one dimensional heat conduction equation for composite plate is

$$\frac{d}{dz}\left\{k(z)\frac{dT}{dz}\right\}=0$$

Where $k_i(z)$ is the thermal conductivity of per layer of the three layered composite plate such as i=1, $k_1(z)=k_c$ and $k_3(z)=k_m$.

Thermal conductivity for FGM layers is k(z).there is convective heat transfer boundary condition and the conditions of continuity of the temperature in the three layered composite plate also exists.

Material properties are graded throughout the thickness direction according to volume fraction power law distribution. The volume fraction is expressed as given by Tirupathi R. et al. (2005)

$$V_c(z)=\left(\frac{2z+h}{2h}\right)^n$$

Where n is the volume fraction index.

The temperature distribution through the plate thickness for any distribution of k (z), can be written as [11]:

$$T(z)=T_c-\frac{T_c-T_m}{\int_{-h/2}^{h/2}(dz/k(z))}\int_z^{-h/2}dz/k(z)$$

where

$$k(z)=(k_c-k_m)\left(\frac{2z+h}{2h}\right)^n+k_m$$

IV. FINITE ELEMENT ANALYSIS

Consider the Fig. 2 with nodes i & j on either side, we can write

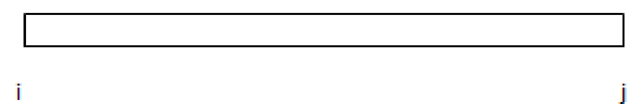

Figure 2. Linear element with nodes I and j.

$$T=N_iT_i+N_jT_j$$

where

$$N_i=\frac{y_j-y}{y_j-y_i}\ \&\ N_j=\frac{y-y_i}{y_j-y_i}$$

In local co-ordinates

$$N_i=1-y/l\ \&\ N_j=y/l$$

and temperature is dT/dy = $-\frac{1}{l}T_i+\frac{1}{l}T_j$

$=\begin{bmatrix}-\frac{1}{l} & \frac{1}{l}\end{bmatrix}\begin{Bmatrix}T_i\\T_j\end{Bmatrix}=[B][T]$ where l_e is the length of element.

The element stiffness matrix is given as [12]:

$$[K]_e=\int_\Omega[B]^T[D][B]d\Omega+\int_A h[N]^T[N]dA_s$$

$$= \int_{\Omega} [B]^T [D][B] A\, dx + \int_{A} h[N]^T [N] dA_s$$

Where Ω is the volume integral, A_s indicates surface area and h is the convective heat transfer coefficient after integration

$$[K]_e = \frac{Ak}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + hA_s\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The forcing vector can be written as

$$\{f\}_e = \int_{\Omega} G[N]^T d\Omega - \int_{A_s} q[N]^T dA_s + \int_{A_s} h[T_a][N]^T dA_s$$

Where G is the internal heat generation per unit volume, q is the boundary surface heat flux and T_a is the atmospheric temperature. In our case heat generation within the element is zero i.e. G =0 .The no heat flux boundary condition is denoted by q=0, so finite element equation for a each layer of FGM composite plate with two nodes becomes

$$[K]_e = \frac{Ak}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + hA_s\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ hT_a A \end{Bmatrix}$$

V. THERMAL STRESS EQUATIONS

The initial thermal strain due to change in temperature is $\varepsilon_0 = \alpha \nabla T$, where α is the coefficient of thermal expansion from stress-strain relation we know that from Tirupathi R. et al. (2005):

$$\sigma_T = E(\varepsilon - \varepsilon_0)$$

The strain energy per unit volume, u_0 is equal to

$$u_0 = \frac{1}{2}\sigma(\varepsilon - \varepsilon_0)$$

The total strain energy is U in the structure is obtained by integrating u_0 , over the volume of the structure:

$$U = \int \frac{1}{2}\sigma(\varepsilon - \varepsilon_0)^T E(\varepsilon - \varepsilon_0) A.dx$$

For linear element the equation becomes

$$U = \sum \frac{1}{2} A_e \frac{l_e}{2} \int \frac{1}{2}\sigma(\varepsilon - \varepsilon_0)^T E(\varepsilon - \varepsilon_0) d\xi$$

Noting that $\varepsilon = Bq$

$$U = \sum_e \frac{1}{2} q^T \left(E_e A_e \frac{l_e}{2} \int_{-1}^{1} B^T B d\xi\right) q - \sum_e q^T \left(E_e A_e \frac{l_e}{2} \varepsilon_0 \int_{-1}^{1} B^T d\xi\right) + \sum_e \frac{1}{2} E_e A_e \frac{l_e}{2} \varepsilon_0^2$$

Examining the strain energy expression, we see that the first term on the right side yields the element stiffness matrix. The 2nd term yields the desired of temperature change i.e.

$$\theta^e = E_e A_e \frac{l_e}{2} \varepsilon_0 \int_{-1}^{1} B^T d\xi$$

using $B = [-1 \quad 1]/(x_2 - x_1)$ and $\varepsilon_0 = \alpha \nabla T$, thus

$$\theta^e = \frac{E_e A_e l_e \alpha \nabla T}{x_2 - x_1}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

After solving the finite element equations KQ=F for displacements Q, where F is global force vector the stress in each element can be obtained as

$$\sigma = E(Bq - \alpha \nabla T)$$

$$\sigma = \frac{E}{x_2 - x_1}[-1 \quad 1] q - E\alpha \nabla T$$

VI. RESULT AND DISCUSSIONS

The total thickness of plate is 10 mm and t_1=t_3. The finite element mesh of zirconia/FGM/Al composite plate is divided into 750 elements and 751 nodes. The nodes are at the interfaces of three layered plate. In this section we present several numerical simulations in order to assess the thermal stresses in ZrO_2/FGM/Al composite plate due to change in FGM layer thickness and composition.

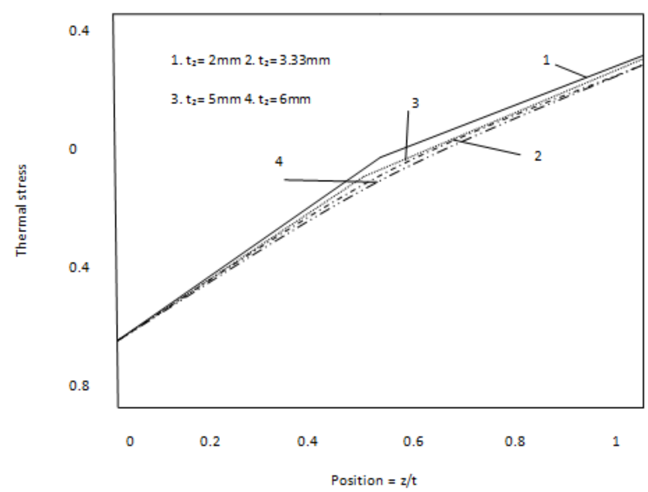

Figure 3. Variation of thermal stress with change in FGM layer thickness

6.1 Variation of thermal stress with change in FGM layer thickness

It can be shown from figure 3 that the thermal stresses changes from compressive to tensile as we goes from metal layer to ceramic layer without considering FGM layer. Tensile stresses appearing on the ceramic surface is larger, as we know ceramics are weak in tension, this is unfavorable to the strength of ceramics. In metal and ceramic layers thermal stress curves are almost linear and slope of each curve is almost same. As thickness of FGM layer is increases from t_2 = 2 mm to t_2 = 6 mm the curves becomes more smooth and tends to gentle. Whole stress distribution in FGM layer is more reasonable.

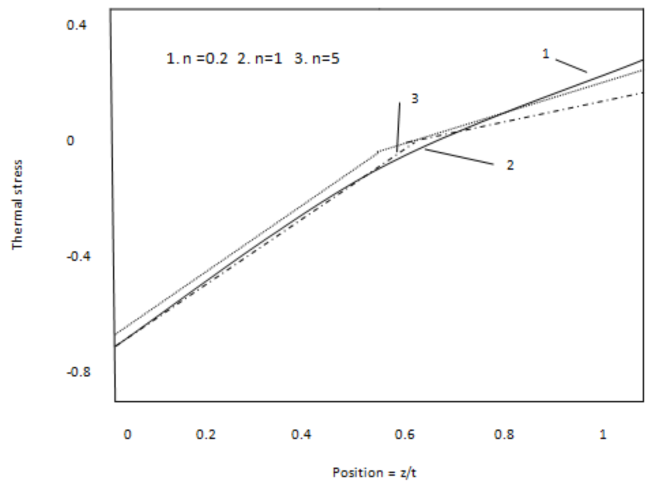

Figure 4. Variation of thermal stress with change in FGM layer composition

6.2 Variation of Thermal stresses with change in FGM layer composition

From figure 4 it is shown that for grading parameter n= 0.2(curve1), the thermal compressive stress on the metal surfaces reaches the biggest and thermal tensile stress on ceramic surface reaches smallest. When n= 5(curve 3) thermal compressive stress on the metal surface reaches the smallest and thermal tensile stress on ceramic surface reaches largest. When n= 1(curve2) thermal stress curves tend to smooth and gentle without any steep turning point.

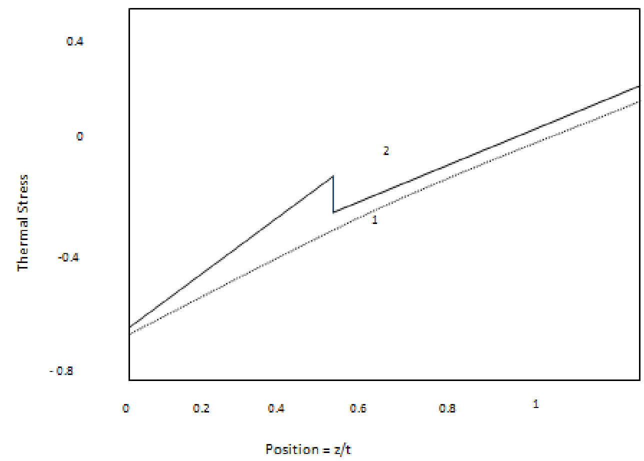

Figure 5. Variation of thermal stress with change in FGM layer composition

6.3 Variation of Thermal stresses with different composite plate

The thermal stress curves as shown in figure 5 in FGM gradient layer is gentle and smooth but thermal stress distribution at the interface of ceramic /metal two layered plate at curve 2 becomes linear and makes sharp bend at the interface of ceramic and metal surfaces because of big difference of material properties of metal and ceramic materials.

VII. CONCLUSION

The thermal stresses in FGM composite plate with convective boundary condition are investigated using one dimensional finite element method in this paper. Numerical simulations for thermal stresses on ZrO_2/FGM/Al composite plate are performed and it is observed that FGM layer has little effect on the thermal stress of elongation clamped and bending free plate. The grading parameter has significant effect on steady thermal stress field distributions. Comparison of thermal stress at the interface of ceramic/metal two layered composite plate, the thermal stress of ZrO_2/FGM/Al composite plate is very gentle.

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