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Analytical Study of Nonlinear Schrodinger Equation

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ABSTRACT

In this work, the generalized $\left(\frac{G'}{G}\right)-$ expansion method is used to study the nonlinear Schrödinger equation and find outthe soliton and periodic solutions of the nonlinear Schrödinger equation with a cubic nonlinearity. Many new sets of exact traveling wave solutions which are in terms of the hyperbolic, trigonometric and Jacobi elliptic functions are successfully obtained. The solutions obtained may be important to explain the phenomena under study. The method is straightforward and concise, and its application is promising.

Keywords: Hyperbolic solutions, Soliton.

I. INTRODUCTION

Nonlinear evolution equations have attracted intensive attention in the last three decades, since they occur in a variety of physical applications. Therefore, the search for exact solutions of nonlinear differential equations is an active field of research because they describe many different processes in several branches of science such as physics, biology and chemistry. Exact soliton solutions have been the natural priority, due to the elegance and their prominent position in the inverse spectral approach. Recent intensive efforts in academic, commercial, and defense establishments have reinforced such interests, since solitons now have a promising potentialin signals transmission over a large distance. The periodic case of envelope equations has received much less attention, partly because the mathematics is more involved [1,2].

In past, considerable efforts have been made to obtain exact analytical solutions of nonlinear equations with varying degree of success. Therefore, many methods have been developed to find analytical solutions of nonlinear ordinary differential equations (ODEs) and nonlinear partial differential equations (PDEs). Some of them are the truncation procedure in Painlev´e analysis, the Hirota bilinear method the tanh function method, the Jacobi elliptic function method and the Prelle–Singer method [2,3,4,5,6,7,8,9,10].

The nonlinear Schrödinger (NLS) equation, which describes the time evolution of a slowlyvarying envelope, is encountered in various branches of physics and known to be fundamental. In deriving the NLS equation as the envelope equation, we neglect higher-order terms under appropriate physical assumptions. However, due to recent developments in optical technology, higher-order corrections to the NLS equation have become necessary and important.

Therefore, here in the present work, we study the following completely integrable nonlinear Schrödingerequation [11]

$$iA_t + A_{xx} + \alpha|A|^2A = 0, \qquad (1)$$

which plays an important role in many nonlinear sciences. It arises as an asymptotic limit of aslowly varying dispersive wave envelope in a nonlinear medium and as such has significant applications such as optical soliton communications, plasma physics, etc. The domain of applications of the standard NLSE varies from optics, propagation of the electric field in optical fibers, self-focusing and collapse of Langmuir waves in plasma physics to modeling deep water waves and rogue waves in oceans. Moreover, the NLSequation admits many remarkable properties, e.g., bright and dark soliton solutions, Lax pair,Liouvilleintegrability, inverse scattering transformation, conservation laws, bilinearization,Painlev´eintegrability, multi-solitons, Backlund transformation, Darboux transformation,symmetries, etc. [12,13,14]

The goal of the present work is to demonstrate that the NLS, and envelope equations in general, display an intriguing variety of soliton and periodic solutions usingthe generalized $\left(\frac{G'}{G}\right)$-expansion method.

II. THE GENERALIZED $\left(\frac{G'}{G}\right)$-EXPANSION METHOD

Here, we briefly describe the main steps of the generalized $\left(\frac{G'}{G}\right)$-expansion method. Consider a nonlinear partial differential equation (PDE) is of the form

$$P(u,\ u_t,\ u_x,\ u_{tt},\ u_{xt},\ u_{xx}\ \ldots..) = 0, \qquad (2)$$

where u=u(x,t) is an unknown function and P is polynomial in u= u(x,t) and its partial derivatives, in which higher order derivatives and nonlinear terms are involved. In order to solve eq.(2) by this method, one has to resort the following steps:

Step 1: To find the traveling wave solution of (2), introduce the wave variable $\xi = (x - ct)$, which changes the PDE (2), to an ordinary differential equation (ODE) as

$$P(u,\ u_\xi,\ u_{\xi\xi},\ \ u_{\xi\xi\xi}, \ldots..) = 0, \qquad (3)$$

where $u_\xi,\ u_{\xi\xi}$ etc. denote derivative of u with respect to ξ. Now integrate the ODE (3) as many times as possible and set the constants of integration to be zero.

Step 2: The solution of (3) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as

$$u(\xi) = \sum_{i\to 1}^{n} a_i \left(\frac{G'}{G}\right)^i, \quad (4)$$

where $G = G(\xi)$ satisfies the following Jacobi elliptic equation,

$$\left[G'\right]^2 = e_2 G^4(\xi) + e_1 G^2(\xi) + e_0, \qquad (5)$$

where $a_i, e_2,\ e_1$ and e_0 are constants to be determined later and $a_n \neq 0$. The positive integer n can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (3).

Step 3: Substituting eq.(4) into eq.(3) and using eq. (5), we obtain polynomial in $G^j(\xi), G'(\xi)G^j(\xi)$ (j=$\pm$1, $\pm$2,....). Equating each coefficient of the resulted polynomials to zero yields a set of algebraic equations for together and then equating each coefficient of the resulting polynomial to zero gives the values of $a_i, e_2,\ e_1$ and e_0.

Step 4: Since the general solutions of eq.(5) are well known for us [15], then substituting a_i and the general solutions of eq.(5) into eq.(4), we obtain many new traveling wave solutions in terms of Jacobi elliptic function of nonlinear differential equation (2).

After the brief description of method, we now solve nonlinear Schrödinger equation with cubic nonlinearity using above method.

III. EXACT TRAVELLING WAVE SOLUTIONS OF eq.(1)

Using the transformation A(x,t)=w(x) $e^{i\alpha t}$, eq.(1) change into following ODE

$$w'' - \alpha w + \alpha w^3 = 0, (6)$$

where α is considered as a real parameter. Suppose the solution of the ODE (6) is expressed by a polynomial in $\left(\frac{G'}{G}\right)$- as given in eq.(4) and G=G(x) satisfies the Jacobi elliptic equation (5).

Now on substituting eq.(4) into eq.(6) and with the help of eq.(5), we get the set of algebraic equations which on solving with the help of Mathematica, we get

$$a_1 = \pm\sqrt{\frac{-2}{\alpha}}, a_0 = 0, \alpha = -2e_1. \qquad (7)$$

With the help of the results obtained in eq.(7), the solution of the NLSE is written as

$$A(x,t) = \pm\sqrt{\frac{-2}{\alpha}}\left(\frac{G'}{G}\right)e^{i\alpha t}. \qquad (8)$$

Now using the general solutions of eq. (5) we have thefollowing set of exact solutions of NLSE with cubic nonlinearity.

Family 1: If $e_0 = 1, e_1 = -(1+m^2), e_2 = m^2$, then the solution becomes

$$A(x,t) = \pm\sqrt{\frac{-2}{\alpha}}\ cs(x)\,dn(x)e^{2i(1+m^2)t}. \qquad (9)$$

or

$$A(x,t) = \mp\sqrt{\frac{-2}{\alpha}}\ (1-m^2)sd(x)nc(x)e^{2i(1+m^2)t}. \qquad (10)$$

Family2: If $e_0 = 1-m^2, e_1 = 2m^2-1, e_2 = -m^2$, the following solution obtained

$$A(x,t) = \mp\sqrt{\frac{-2}{\alpha}}\ sn(x)dc(x)e^{-2i(2m^2-1)t}. \qquad (12)$$

Family 3: If $e_0 = m^2-1, e_1 = 2-m^2, e_2 = -1$, then eq.(8)becomes

$$A(x,t) = \mp\sqrt{\frac{-2}{\alpha}}m^2 sn(x)\,cd(x)e^{-2i(2-m^2)t}. \qquad (13)$$

Family4: If $e_0 = 0, e_1 = 1, e_2 = -1$, then we obtain

$$A(x,t) = \mp\sqrt{\frac{-2}{a}}\ tanh(x)e^{-2it}. \quad (14)$$

Family5: If $e_0 = 0, e_1 = 1, e_2 = 1$, then we get

$$A(x,t) = \mp\sqrt{\frac{-2}{a}}\ coth(x)e^{-2it}. \quad (15)$$

Family 6: If $e_0 = 0, e_1 = 1, e_2 = -1$, then we obtain

$$A(x,t) = \pm\sqrt{\frac{-2}{a}}\ tan(x)\,e^{2it}. \quad (16)$$

Similarly, we can also write down the other sets of exact solutions which are omitted for convenience. The solutions which are interms of Jacobi elliptic functions can also reduced in terms of hyperbolic, trigonometric functions.

IV. CONCLUSION

In this work, within theframework of generalized $\left(\frac{G'}{G}\right)$-expansion method, we obtained the exact solutions of the nonlinear Schrödinger wave equation with a cubic nonlinearity which have wide applicationsin physics specially in plasma physics, nonlinear optics etc.Thesolutions obtained of these equations are successfully describesthe phenomena under study. It is also note that the solutionsobtained by above method are the general traveling wave solutionswhich can give soliton and periodic solutions underdifferent parametric restrictions. It is interesting to note thatfrom the general results, one can easily recover solutions whichare obtained from others methods. This direct and concise methodcan further be used to explore more applications.

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