Audio Equalizer with Fractional Order Butterworth Filter

M. Eswara Rao¹, P.V.Muralidhar², S.Raghuramakrishna³
¹Department of Electronics and Communication, INDIA
²Department of Electronics and Communication, INDIA
³Department of Electronics and Communication, INDIA

ABSTRACT

An attempt is made to implement Fractional order infinite impulse response (IIR) Butterworth filter. This has given sharp cut-off frequency and better spectral response. An audio application like CD player, digital audio equalizer is one used to make sound as one desire by changing filter gains for different audio frequency bands. Other applications include adjusting the sound source to take acoustics into account removing undesired noise and boosting the desired signal in the specified pass band of Butterworth is key block in equalizer. Here, an attempt will be made to implement such band pass filters in equalizer using fractional order Butterworth filter (FBWF). The principle behind to develop FBWF is in complex w-plane (w = s^q, q is real number). In S-plane Butterworth filter and implemented in integer order, where in complex w-plane Butterworth filter will be implement in real order. Detailed mathematical calculations will be presented.

Keywords---- IIR Butterworth filter; Fractional order Butterworth filter; Audio equalizer.

I. INTRODUCTION

Filters are widely employed in signal processing and communication systems in applications such as channel equalization, noise reduction, radar, audio processing, video processing, biomedical signal processing, and analysis of economic and financial data. In an audio equalizer the input signal is filtered into a number of sub-band signals and the gain for each sub-band can be varied manually with a set of controls to change the perceived audio sensation. In a Dolby system pre-filtering and post filtering are used to minimize the effect of noise. Filters are also used to create perceptual audio-visual effects for music, films and in broadcast studios. The primary functions of filters are one of the followings: (a) to confine a signal into a prescribed frequency band as in low-pass, high-pass, and band-pass filters. (b) To decompose a signal into two or more sub-bands as in audio equalizers. (c) To modify the frequency spectrum of a signal as in audio equalizers. (D) To model the input-output relationship of a system such as music synthesizers.

A. Digital filters: A discrete-time IIR filter has a z-domain transfer function that is the ratio of two z-transform polynomials. It has a number of poles corresponding to the roots of the denominator polynomial and it may also have a number of zeros corresponding to the roots of the numerator polynomial. The main difference between IIR filters and FIR filters is that an IIR filter is more compact in that it can usually achieve a prescribed frequency response with a smaller number of coefficients than an FIR filter. A smaller number of filter coefficients imply less storage requirements and faster calculation and a higher throughput. Therefore, generally IIR filters are more efficient in memory and computational requirements than FIR filters.

B. Advantages using digital filters: The main advantages of digital over analog filters: 1. the digital filter can easily be changed without affecting the circuitry. An analog filter can only be changed by redesigning the filter circuit. 2. Digital filters are easily designed, tested and implemented on a general-purpose computer or workstation. 3. Digital filters can handle low frequency signals accurately.4. Fast DSP processors can handle complex combinations of filters in parallel or cascade (series). Fractional calculus is being widely used in modeling the dynamics of many real life phenomena due to the fact that it has higher capability of providing accurate than the integer dynamical systems[10].
II. BUTTERWORTH FILTER

A. Butterworth filter in the signal processing: The Butterworth filter is the best compromise between attenuation and phase response. It has no ripple in the pass band or the stop band, and because of this is sometimes called a maximally flat band filter [9]. The Butterworth filter achieves its flatness at the expense of a relatively wide transition region from pass band to stop band, with average transient characteristics. The normalized poles of the Butterworth filter fall on the unit circle (in the s-plane) [2]. The values of the elements of the Butterworth filter are more practical and less critical than many other filter types. The frequency response, group delay, impulse response is very better [8]. Butterworth filter is when compared with Chebyshev, Bessel and other filters. In all responses amplitude, phase and impulse responses the transient behavior gets progressively very good. So here we are used low pass Butterworth filter in the audio equalization process.

B. Fractional order Butterworth filter: When a Butterworth filter is to be designed, the optimum order which is capable of meeting these specifications may be a fraction which is conventionally raised to the nearest integer value due to unavailability of filter order being a fraction. This is just similar to adding a bit more factor of safety in the design. This can easily be met with the concept of fractional order filters with tunable parameters. With further enhancement of the realization techniques for fractional order filters, it will be possible to satisfy the exact requirements according to the given specifications [1]. The fractional Butterworth filter has been formulated in the w-plane. An analytical formulation of the filter is done in the w-plane and certain conditions for the location of the stable and the unstable poles are derived. The essence of classical Butterworth filter is kept unchanged and it is found that even in case of the fractional order formulation, the poles are placed along the circumference of a circle whose radius is equal to the cut-off frequency [3]. Thus, by the proposed formulation all the stable poles have been taken into account. The corresponding transfer functions are obtained in w-plane and are then mapped back into the s-plane using the relation \( w = s^q \). The corresponding response curves are obtained to confirm the maximally flat Butterworth nature. Since this is one of the first approaches towards filter design in the w-plane, some generic issues related to fractional order filter design. A filter having an order \( P/Q \) implies that \( P \) numbers of poles are distributed over \( Q \) Riemann sheets. However, this result is a serious issue related to the order of the filter. Considering whether increase in the number of poles and number of Riemann sheets for capturing delicate frequency domain information is actually required, since this would lead to higher design complexity. For a 0.9 order filter, the proposition will require 9 poles distributed over 10 Riemann sheets. This resulting complexity would make it difficult to generalize filters in fractional domain and the advantage of greater flexibility will be outweighed by fabrication costs [4]. Filter design refers to FIR or IIR structures which are designed to satisfy some frequency domain specifications like cut-off frequencies for the pass-band or stop band, maximum allowable ripples in magnitude response etc. Fractional order systems are inherently infinite dimensional, they naturally model the exact magnitude roll-off [6]. The order of realization can be chosen by the user after the exact order of the filter is calculated from the frequency domain specifications [5]. The mathematical formulation for the realization of fractional Butterworth filter is introduced and design fractional order Butterworth filter, by using fractional order, we are taken as cascading of integer and fraction. After we are converted into band pass filter. Those band pass filters are used in the audio equalizer. A fractional Butterworth like filter has been proposed using the concept of w-plane stability. The poles are located on the w-plane which consists of all the poles lying on different higher Riemann sheets [2]. Then the stability criterion is taken under consideration and all the unstable poles are rejected and a fractional order BW like filter can be obtained. The concept is somewhat similar to the conventional notion of finding integer BW filter poles lying on a circle and then discarding the right-half plane poles in order to avoid instability. The fractional Butterworth filter is developed on the lines of the integer order Butterworth filter with the considerations of the stability of fractional linear systems. The pole locations of a FO Butterworth low-pass filter of any real order \( P/Q \), \( \{P, Q \in \mathbb{Z} \} \) are given by \( W_k = \pm j \Omega_k e^{j(2\pi - 1)\pi} \), where \( k=1, 2, 3..., p \).

\( \Omega \) = Cutoff frequency

Condition for the stability of the root is given by \( \arg|W_k| > \frac{q \pi}{2} \), \( q=1/Q \), \( q \) is the commensurate order. \( P \) is number of poles distributed over \( Q \) Riemann sheets [5].

III. DIGITAL AUDIO EQUALIZER

A. Equalization: Equalization is the increase or decrease of signal strength for a portion or a band of frequencies. The audio we record that is the sound made by instruments or voice is complex. By this we mean that is composed of energy at different audio frequencies [13]. If we take bass control (a simple equalizer) and turn the knob clock wise, we will get an increase in strength of the signal or the signal component that has lower frequencies. Thus equalization affects the tone because it changes the level relationship of the fundamental and harmonic frequencies. Generally every musical sound consisting of combination of the actual sound called fundamental tone. Number of tones with higher frequency called over tones. Digital
audio equalizer contains number of frequency bands and it is used for audio equalization purpose. Digital audio equalizer divides incoming audio spectrum into different frequency bands such as treble, mid range, bass, etc. it equalizes the incoming audio signal, means increase or decrease of signal strength for a portion of audio frequencies. Then we can produce many sound effects by changing the gains of the equalizer [13].

**B. Working of audio equalizer:** Digital audio equalizer is a system that allows the spectral characteristics of a signal to be changed. In speech, audio applications, signal processing functions and related operations may suppress or boost certain frequencies of a signal. It will change in tonal properties of the output signal as compared to input. Additionally, input and output devices that is microphones, speakers, etc. May emphasize or de-emphasize certain frequencies in a signal due to their mechanical characteristics and limitations. An audio equalizer enables compensation for these changes by providing the user the ability to modify the spectral characteristics of signal [13].

**IV. PROPOSED METHOD**

In order to find filter transfer function first find the order of the filter. The order of a digital filter was defined earlier as the number of previous inputs which have to be stored in order to generate a given output [12]. This definition is appropriate for non-recursive (FIR) filters, which use only the current and previous inputs to compute the current output In the case of recursive filters. The order of a recursive filter is the largest number of previous input or output values required to compute the current output. Filter required specifications are \((\Omega_s)\)Stop band frequency=6552, \((\Omega_p)\)pass band frequency=3173, \((\alpha_s)\) pass band attenuation=3 db, \((\alpha_p)\) stop band attenuation=15 db

\[
N = \log \left( \frac{10^{0.1\alpha_p}}{10^{0.1\alpha_s} - 1} \right) \log \left( \frac{\Omega s}{\Omega p} \right)
\]

Here (N) order of the filter=2.3

By using Butterworth polynomials we are design fractional order Butterworth filter. Cut-off frequency \((\Omega_c)\) can be calculated from the specification of the stop band.

\[
\Omega_c = \frac{\Omega}{\left(10^{0.1\alpha s} - 1\right)^{1/\beta}}
\]

The value of N cannot always take integer values. In most cases it is more likely to take fractional values. So in the traditional integer order case, we had to over satisfy the design needs [5]. For example, if a filter of order 1.5 was required in order to exactly meet the given specifications, a second order filter had to be designed. This can be overcome by fractional filter of an order exactly equal to 1.5. As mentioned before in order to design a filter of order 1.5, a 0.5 order filter can be cascaded with a first order filter. Here in the fractional order Butterworth filter we are taken as

\[
N+ = (P/Q)=2+0.36.
\]

**A. Integer order Butterworth based band pass filters:**

For order \(N=2\) Transfer function [7],

\[
H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}
\]

Low pass filter need to be converted into band pass filter for audio equalization [9]

Using \(S \rightarrow \frac{S^2 + \omega_k^2}{\omega_0}\)

The warping effect can be eliminated by prewarping analog filter. This can be done by finding prewarping analog frequencies formula [8]

\[
\Omega = \frac{2}{T} \tan \left( \frac{\omega}{2} \right)
\]

Here \(\omega = \omega_{ah} - \omega_{ol}; \omega_{ah} = \text{upper cut off frequency in rad/sec,} \omega_{ol} = \text{lower cut off frequency in rad/sec}\)

\[
\omega_h = \frac{2}{T} \tan \left( \frac{\omega_{ah}}{2} \right), \omega_s = \frac{2}{T} \tan \left( \frac{\omega_{ol}}{2} \right)
\]

Then 1st band pass filter

\[
\omega_{ah} = 471 \text{rad/sec}, \omega_{ol} = 786 \text{rad/sec}
\]

\[
H(s) = \frac{1}{s + 445.41 + 839645 s + 1.65 \times 10^5 s + 1.3705 \times 10^6}
\]

**B. Bilinear transformation:** The technique of digitizing an analogue design is the most popular IIR filter design technique, since there is a large amount of theory on standard analogue filters available. The bilinear transform is a mathematical transformation from the s-domain to the Z-domain which preserves the frequency characteristics and is defined by

\[
s = \frac{2}{T} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right)
\]

Where T=sampling period

Under this mapping [8], the entire j\(\omega\) axis in the -plane is mapped on to the unit circle in the Z-plane; the left-half S-plane is mapped inside the unit circle and the right-half s-plane is mapped outside the unit circle. The bilinear transformation gives a non-linear relationship
between analogue frequency $\omega_a$ and digital frequency $\omega_d$. Since the frequency response of a digital filter is evaluated by setting $Z = e^{j\omega t}$, the mapping is almost linear; for most of the frequency scale; however, the mapping is highly non-linear\[11]\.

1\textsuperscript{st} band pass filter transfer function for order 2 is

$$H_{11}(Z) = \frac{Z^4 - 2Z^2 + 1}{3354.65Z^4 - 13222.59Z^2 + 19574.90Z^2 - 12899.76Z + 3192.87}$$

(1)

2\textsuperscript{nd} band pass filter transfer function for order 2 is

$$\omega_{0a} = 943 \text{ rad/sec}, \quad \omega_{0b} = 1575 \text{ rad/sec}$$

By using bilinear transform

$$H_{21}(Z) = \frac{Z^4 - 2Z^2 + 1}{860.1527Z^4 - 3325.07Z^2 + 4850.67Z^2 - 3164.71Z + 779.2365}$$

(2)

3\textsuperscript{rd} band pass filter transfer function for order 2 is

$$\omega_{0a} = 1892 \text{ rad/sec}, \quad \omega_{0b} = 3174 \text{ rad/sec}$$

$$H_{31}(Z) = \frac{Z^4 - 2Z^2 + 1}{225.7342Z^4 - 827.2496Z^2 + 1166.6153Z^2 - 749.3078Z + 185.2914}$$

(3)

**C. Integer order Butterworth based band pass filter**

**Frequency response:** The frequency response of a filter describes how the filter alters the magnitude and phase of the input signal frequencies. The frequency response of a filter can be obtained by taking the Fourier transform of the impulse response of the filter, or by simple substitution of the frequency variable $e^{j\omega}$ for the $z$ variable. The frequency response of a filter is a complex variable and can be described in terms of the filter magnitude response of the filter.

---

**D. Audio equalizer response with integer order Butterworth filters:** Designed integer order Butterworth filters are applied to Audio equalizer then we can observed required response.
Figure (4): Equalized audio spectrum for integer order

E. Fractional order Butterworth based band pass filter design and response:

Fractional order Butterworth filter we are taken as N+ (P/Q) = 2 + 0.36.

1st filter transfer function for order 2 is

\[ H_1(Z) = \frac{2365.7797 + 12.316467 + 7.485007 + 0.332515276 + 79.2365}{12.316467 + 7.485007 + 0.332515276 + 79.2365} \]

2nd filter transfer function for order 0.3 is

\[ H_2(Z) = \frac{z^2 - 1}{5.546 z^2 - 7.7084 z + 2.2336} \]

Transfer function obtained by cascading (6) & (7)

\[ H_{12}(Z) = [1.0, -3.0, -1] \]

Cascade those two equations (4) & (5) then

\[ H_{12}(Z) = \frac{z^2 - 1}{3.6 z^2 - 3.7468 z + 0.28798} \]

3rd filter transfer function for order 2 is

\[ \Omega_{\omega_{\text{m}}} = 1892 \text{ rad/sec}, \quad \Omega_{\omega_{\text{m}}} = 3174 \text{ rad/sec} \]

\[ H_{12}(Z) = \frac{z^2 - 2 z + 1}{225.734 z^2 - 827.2496 z + 1166.6153 z^2 - 749.3078 z + 185.29145} \]

Fractional order 2nd band pass filter

\[ \Omega_{\omega_{\text{m}}} = 943 \text{ rad/sec}, \quad \Omega_{\omega_{\text{m}}} = 1575 \text{ rad/sec} \]

\[ H_{12}(Z) = [1.0, -3.0, -1] \]

Cascade those two equations (9) & (10) then

\[ H_{12}(Z) = \frac{z^2 - 1}{3.6 z^2 - 3.7468 z + 0.28798} \]

Figure (5): Filter 1 results for N = 2.3
Figure (6): Filter2 results for N=2.3

Figure (7): Filter3 results for N=2.3

**F. Audio equalizer response with fractional order Butterworth filters:** Designed fractional order Butterworth filters are applied to Audio equalizer then we can observe required response.

Figure (8): Equalized audio spectrum for fractional order

Figure (9): Combined audio equalized spectrum
V. CONCLUSION

An audio equalizer is equalizing the different band of frequencies in the certain range value. Those band pass filters designed by using fractional order Butterworth filter. A new kind of fractional Butterworth like filter has been designed using the consideration of poles lying on a circle in the transformed w-plane and also using concept of stability in complex w-plane. The obtained FBWFs are analyzed from the nature of poles, as well as in the time and frequency domain. The new fractional filter design technique is capable of meeting the design specifications in an exact manner rather than under or over satisfying it. This has been possible by making the filter’s order exactly as per the design requirements without rounding-off to the nearest upper or lower integer. Future scope of research can be directed towards extending the present concept for discrete domain filtering techniques which has higher flexibility of realization using digital signal processors. Also, the concept can be extended for other class of FO IIR filters.

REFERENCE

[12] DIGITAL FILTER DESIGN FOR AUDIO PROCESSING Elenberg ,Stephanie Ng, Anthony