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Bayesian Analysis of Various Queue System Model: A Brief Theoretical Review

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ABSTRACT

In this paper, different queue models discussed by authors in their various research papers have been discussed in short theoretical form. Bayesian point of view as well as the updating of basic distributions in respect of their prior variations and their impact on various queue characteristics are discussed theoretically.

Keyword--- Queue system, Bayesian analysis, Robustness, Traffic intensity, arrival and service distribution, prior and posterior distribution

I. INTRODUCTION

Queuing theory analysis involves the study of a system's behavior over time. A system is said to be in transient state when its operating characteristics (behavior) vary with time. This usually occurs at the early stages of the system's operation where its behavior is still dependent on the initial condition. However, since one is mostly interested in the "long-run" behavior, most attention in queuing theory analysis has been directed to steady state results. A steady state condition is said to prevail when the behavior of the system becomes independent of time. Any situation where a flow of customers arrive for service at a service facility can be analyzed via Queuing Theory. A few situations in which queuing are important is outlined below.

(i) Supermarket: How long do customers have to wait for checkouts? What happens to the waiting time during peak hours?

(ii) Post Office: In a post office, are there specialized counters dealing with stamps, packages, financial products etc.? Are there enough counters? Should there be separate queues or a single common queue in front of counters with same specialization?

(iii) Call centers: Questions by phone regarding any products related to any company are often handled by their call centers like Insurance company, BSNL or any other mobile/internet service provider ,Railway etc. how long do customers have to wait before an operator becomes free? Are there enough operators? Are there enough dedicated telephone lines?

(iv) Toll Booths: Motorists have to pay toll in order to cross a bridge. Are there enough toll booths?

A queue is formed whenever service requests or customers arrive at a service facility and are forced to wait while the server is busy working on other request. The term customer has a wider meaning in queuing theory both animated and unanimated. In a general sense, units calling for service are defined as customers. Similarly, a service facility too could be animated or unanimated and can be defined as an arrangement which provides some benefit or assistance to customers.

According to **Gross and Harris**, “A queuing system can be described as customers arriving for service, waiting for service if it is not immediate, and if having waited for service leaving the system after being served”, while according to **Kashyap and chaudhary**, “In queuing theory, we study situations where units of some kind arrive at a service facility for receiving service of some description, some of the units have to wait for service, and depart after service”.

II. BAYESIAN ANALYSIS

Thomas Bayes (1763) introduced Bayesian inference in his famous research paper entitled, "An essay towards solving a problem in the Doctrine of Chance". Further, for basic theory and foundations one can also refer to Jeffreys (1961) and Savage (1962). Lindley (1965) and Box and Tiao (1973) have popularized and given this approach an unique important place in the field of statistics. They developed a literature based on Bayes's approach. Today a vast literature on Bayesian analysis of life testing problems in terms of some standard text is available. A few of them are Savage (1962), Bhattacharya (1967), Martz and Waller (1982), Sinha (1986) and Gelman *et al.* (1995) presented the Bayesian analysis of the system reliability using many prior distributions. Some, priors with their inherent statistical properties are also given in the study by Raiffa and Schlaifer (1961). Studies like Sharma *et al.* (1993, 1994, 1995) are also effort in the same direction. Apostolakis (1990) reviewed the literature on Bayesian theory to assess the probabilistic safety of various engineering system. But in many practical situation it may happen that the operational experiment with the complete system is limited, non-existent or very expensive to realize.

The origin of Bayesian theory may be attributed to a very primary paper by Rev. Thomas Bayes republished in 1958 due to its fundamental importance. The details of Bayesian statistical theory can be found in Raifla and Schlaifer (1961), Jeffreys (1961), Savage (1962), Lindeley (1965), Box and Tiao (1973), Berger (1985), S.K. Sinha (1998) and Bernardo and Smith (1993).

A prior distribution, which is supposed to represent, what is known about unknown parameter before the data are available, plays an important role in Bayesian analysis. The prior information concerning the parameter θ can be summarized mathematically in the form of prior distribution $g(\theta)$ on the parameter space Ω. A detailed discussion to obtain the solution to the problem concerning the choice of a prior distributions of θ is given in Raiffa and Schlaifer (1961). The natural conjugate priors satisfy the closer property implying that the prior and posterior distributions for the parameter belong to the same family. This method of choosing the priors is much popular because it leads to mathematical simplicity and tractability. This property of conjugate priors is also known as 'closer under sampling' property Weltherill (1961). Raiffa and Schlaifer (1961) have considered a method of gathering prior densities on the parameter space. A family of such densities has been called by them a 'natural conjugate family'. For example in case of an exponential density, the gamma priors form such a family.

In case, when the decision maker does not have any prior knowledge about the parameter, non-informative quasi density may be used. The role of non-informative prior quasi densities in Bayesian analysis is available in Bhattacharya (1967). Jeffreys (1961) proposed a general rule for obtaining the prior distribution of θ. According to this rule, the unknown parameter θ, which is assumed to be a random variable follows such a distribution which is proportional to the square root of the fisher information on θ.

III. LOSS FUNCTION

Suppose θ be an unknown parameter of some distribution $f(x|\theta)$ and we estimate θ by some statistics $\hat{\theta}$. Let $L(\hat{\theta}, \theta)$ represent the loss incurred when the true value of the parameter is θ and we are estimating θ by the statistic $\hat{\theta}$. The loss function denoted by $L(\hat{\theta}, \theta)$ is defined to be real valued function satisfying:

(i) $L(\hat{\theta}, \theta) \geq 0$ for all possible estimates $\hat{\theta}$ and all $\theta \varepsilon \Theta$

(ii) $L(\hat{\theta}, \theta) = 0$ for $\hat{\theta} = \theta$.

We now consider the following loss functions:

(i) Quadratic Loss Function

A function defined as $L(\hat{\theta}, \theta) = k(\hat{\theta} - \theta)^2$ is called quadratic loss function. Such a loss function is widely used in most estimation problems, if k is a function of θ, the loss function is called the weighted quadratic loss function. If k = 1, we have $L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2$, known as squared error loss function (SELF). Under the SELF, the Bayes estimator is the posterior mean. The squared error loss function is a symmetric function of $\hat{\theta}$ and θ. The reason for the popularity of SELF is due to makes the calculations relatively straightforward and simple.

(ii) Linex Loss Function

A symmetric loss function assumes that positive and negative errors are equally serious. However, in some estimation problems such as assumption may be inappropriate, Canfield (1970) points out that the use of symmetric loss function may be inappropriate in the estimation of reliability function. Over estimation of reliability function or average lifetime is usually much more serious than under estimation of reliability function or mean time to failure (MTTF). Also, an under-estimation of the failure rate results in more serious consequences then the over estimation of the failure rate. This lead to the statistician to think about asymmetrical loss function which has been proposed in statistical literature Ferguson (1967), Zellner and Geisel (1968), Aitcheson and Dunsmore (1975) and Berger (1980) have considered the linear asymmetric loss function. Varian (1975) introduced the following convex loss function known as Linex (linear - exponential) loss function:

$$L(\Delta) = be^{a\Delta} - c\Delta - b; \qquad a,\ c \neq 0, b > 0 \ldots(1)$$

where $\Delta = \hat{\theta} - \theta$ .it is clear that L (0) = 0 and the minimum occurs when c = a .b, therefore,

$$L(\Delta) = b\left[e^{a\Delta} - a\Delta - 1\right],\ d \neq 0, b > 0 \ldots(2)$$

where a and b are the parameters of the loss function may be defined as shape and scale respectively. This loss function has been considered by Zellner (1986), Rajo (1987) and Base and Ebrahimi (1991) considered the L (Δ) as

$$L(\Delta) = be^{a\Delta} - c\Delta - b;\ a, c \neq 0,\ b > O \ldots(3)$$

where $\Delta = \frac{\hat{\theta}}{\theta} - 1$

And studied the Bayesian estimation under the Linex loss function for exponential life time distribution. This loss function is suitable for the situations where observations of θ is more costly than its underestimation. This loss function L (Δ) have the following nice properties:

(i) For a = 1, the function is quite asymmetric about zero with overestimation being more costly than underestimation.

(ii) For a < 0, L (Δ) rises exponentially when Δ < 0 (underestimation) and almost linearly when Δ > 0 (overestimation).

(iii) For small value of |a|, L(Δ) is almost symmetric and not far from a squared error loss function. Indeed, on expanding

$$e^{\alpha\Delta} \approx 1 + \alpha\Delta + \frac{\alpha^2\Delta^2}{z}$$

or L(Δ) is a squared error loss function. Thus for small values of |a|, optimal estimates are not far different from those obtained with a squared error loss function.

(iii) Precautionary Loss

Norstrom (1996) introduced an alternative asymmetric precautionary loss function and also present a general class of precautionary loss function with quadratic loss function such as a special case. These loss functions approach infinitely near the origin to prevent underestimation and thus giving a conservative estimators, especially when, low failure rates are being estimated. These estimators are very useful when underestimation may lead to serious consequences. A very useful and simple asymmetric loss function is

$$L(\hat{\theta}, \theta) = \frac{(\hat{\theta} - \theta)}{\hat{\theta}} \ldots(4)$$

(iv) Entropy Loss

In many practical situations, it appears to be more realistic to express the loss in terms of the ratio $\frac{\hat{\theta}}{\theta}$. In this case, Calabria and Pulcini (1994) point out that a useful asymmetric loss function is the entropy loss

$$L(\delta) \propto \left[\delta^p - p\log_e(\delta) - 1\right] \ldots(5)$$

where $\delta = \frac{\hat{\theta}}{\theta}$

And who minimum occurs at $\hat{\theta} = \theta$ when a positive error ($\hat{\theta} - \theta$) causes more serious consequences than a negative error and *vice-versa*. For small |p| value, the function is almost symmetric when both $\hat{\theta}$ and θ are measured in a logarithmic scale and approximately

$$L(\delta) \propto p^2 \left[\log_e(\hat{\theta}) - \log_e(\theta)\right]^2 \ldots(6)$$

Also, the loss function L (δ) has been used in [Dey *et al.* (1987)] and [Dey and Liu (1992)], in the original from having p = 1. Thus, L (δ) can be written as

$$L(\delta) = b\left[\delta - \log_e(\delta) - 1\right];\ b > 0 \ldots(7)$$

where $\delta = \frac{\hat{\theta}}{\theta}$.

IV. REVIEW OF LITERATURE

Queuing theory is concerned with the statistical description of the behavior of queues with findings. For example, the probability distribution of the number in the queue from which the mean and variance of queue length can be found. In queuing theory, the investigators must measure the existing system to make an objective assessment of its characteristics and must determine how changes may be made to the system and what effect of various kinds of changes in the system's characteristics would be there. Probability mass function (p.m.f.) obtained in a steady state situation is the basis of analyzing various queue systems in respect of their characteristics. Traffic intensity (ρ) defined as the ratio of the arrival rate to service rate is an important parameter of the p.m.f. Saaty (1961), Ackoff and Sasieni (1968) and Taha (1976) studied various queue characteristics which have been defined using the parameter ρ. D.G. Kendal (1953) introduced a useful notation for multiple - server queuing models which describe the three characteristics namely, arrival distribution, departure distribution and number of parallel service channels. Later, A. Lee (1966) added the fourth and fifth characteristics to the notation; that is, the service discipline and the maximum number in the system. In Taha (1976), the Kendall-Lee notation is augmented by the sixth characteristics describing the calling source. The

complete notation has thus appears in the following symbolic form:
(a / b / c) : (d / e / f)
where

a ≡ arrival (or inter arrival) distribution
b ≡ departure (or service time) distribution
c ≡ number of parallel service channels in the service
d ≡ service discipline
e ≡ maximum number allowed in the system (in service + waiting)
f ≡ calling source.

The following conventional codes are usually used to replace the symbols a, b and d.
Symbols a and b replaced by
M ≡ Poisson (Markovian) arrival or departure distribution (or equivalently exponential inter-arrival or service time distribution).
D ≡ Deterministic inter-arrival or service times.
E_k ≡ Erlangian or gamma inter-arrival or service time distribution with parameter k.
GI ≡ General independent distribution of arrivals (or inter-arrival times).
G ≡ General distribution of departures (or service times).
Symbol d:
FCFS ≡ first come, first served
LCFS ≡ last come, first served
SIRO ≡ service in random order
GD ≡ general service discipline

The symbol c is replaced by any positive number representing the number of parallel servers. The symbol e and f represent a finite or infinite number in the system and calling a source, respectively.

To illustrate, the use of this notation, consider (M/M/c):(FCFS/N/∞). This denotes Poisson arrival (exponential inter-arrival), Poisson departure (exponential service time), c parallel servers, "first come, first served" discipline, maximum allowable number N in the system, and infinite calling source.

The Bayesian approach has been pioneered by Armero and Bayarri (1994-a,1994-b,1997,2000). They have worked with a likelihood function constructed by observing some independent inter arrival and service times. Statistical analysis of bulk arrival queues has been studied by Armero and Conesa (2000). Bayesian analysis in a set of individual M/M/1 queue with correlated arrivals and service rates has been studied by Sohn (1996). In this continuation Sharma and Kumar (1999) addressed the issue of statistical inference both from frequents as well as from Bayesian point of view. A unique feature of their work was the construction of critical regions for testing hypothesis on performance measures using randomized testing procedures. Oflate, Maurya (2004) has confined his considerable attention to analyze a more generalized queue (M/G/∞): (∞/GD) regarding statistical inference on its use full characteristics.

Since in Bayesian methodology we considers parameter to be random in place of constant. The analyst's prior knowledge or belief about random behavior of the parameter is expressed via prior distribution. This when combined with likelihood produces the posterior distribution of the parameter given data. Inference on the parameter(s) and for other characteristics is carried out through this posterior distribution.

Following the above studies, the authors have provide a brief theoretical review of their contribution in queuing theory area. To discuss a queuing system, we must specify the arrival and services patterns. The assumptions states that the chance of the next arrivals occurrence is independent of the time that has passed since the last arrival. More precisely, if h is sufficiently small amount time and λ is the mean rate of arrivals, then the probability of an arrival in the time t to (t + h) is λh independent of time t. The distribution of arrivals generated by this assumption is called Poisson, because it may be shown that the probability of r arrivals in any finite interval of time, t, is $\frac{e^{-\lambda T}(\lambda T)^r}{r!}$. This is the Poisson distribution with parameter λt. The probability of an interval exceeding t between two consecutive arrivals is the same as the probability of no arrivals in the interval t immediately following the first arrival. Thus, under the assumption, the time between arrivals has an exponential distribution. The assumption of Poisson arrivals or equivalent assumption that an arrival is equally likely to occur at any point of time is more frequently justified than might appear at first sight. For example- the iron-ore ships are scheduled to arrive at their destinations on given dates, but it has been observed that fluctuations in weather and tides cause schedules to be missed in such a way that actual arrivals follow a Poisson distribution.

The study Goel, Gupta and Kumar(2009) deals with the posterior analysis of various characteristics of a power supply queue system using Squared Error Loss Function (SELF) and LINEX (linear exponential) i.e. an asymmetric loss function. Bayes point estimation for queue characteristics are obtained by assuming prior distribution for the parameters involved in arrival and service time distributions. It is further assumed that the information on arrival and service for a fixed time interval is known. For analyzing the behavior of posterior or Bayes estimates of queue characteristics of a power supply system model with SELF and LINEX loss functions, keeping some of the parameters fixed and varying others. The variations in queue characteristics have been summarized in Table 1 and 2. In this research the effect of varying r [or number of arrivals recorded in time interval (0,T)] and k [or number of services recorded in time interval (0, T)] ,the trends of all queue characteristics are shown in both of the Tables respectively. Analyzing the behavior of Bayes estimates of queue characteristics from Table -1 and 2, one can easily reach a tread-off between r, k and involved parameters and the intended system by analyzing trends with different sets of parameters.

Since in above paper (2009), It is further assumed that the information on arrival and service for a fixed time interval is known. In this continuation, here it should be recognized that prior do have an impact on the basic distribution and therefore updating these basic distributions in its parameters is another important aspect for analyzing the queue characteristics of the model in changed scenario. The variations in the parameter can be neutralized by averaging as we do in the case of compound distribution of the concern variable (Johnson and Kotz, 1969). Therefore, Goel, Gupta and Kumar(2009) in their study deals with the development of the methodology for updating the basic distributions in respect of prior variations in arrival and service time distributions in a different point of view. These updated distribution have been used to develop the queue characteristics of the power supply model when predictive basic queue model used in the analysis, and thus authors explain the robust character of various queue characteristics in a power supply system model when $E(\rho)[\geq$ or$<] \rho$.

Various queue's system have been analyzed in respect of their characteristics using the corresponding probability mass function (p.m.f.). Traffic intensity (ρ) defined as the ratio of the arrival rate to the survival rate is an important parameter of this p.m.f. In reliability theory this ratio is also known as availability ratio. Ackoff and Sasiei (1968) and Taha (1976) studied various queue characteristics which have been defined using the parameter ρ. In all such analysis, ρ is assumed as constant. With the advancement in science and technology over a period, the parameters involved in queue's characteristics can not be considered as constant. The situation become alarming when one is going for queue's characteristics of the model of the same nature accomplishing the same task in varying conditions. Obviously to overcome this situation, it seems logical to assume variations in traffic intensity, represented by known suitable prior distribution. In this regard, on the repeated analysis of various queue systems, we have a strong base for collecting prior information showing variations in ρ. In all of above define studies, the investigators aim has been to update the prior knowledge about the parameter using experimental data. Here it should be recognized that prior do have an impact on basic queue system and therefore updating the basic queue distribution in its parameter is another important aspect for analyzing the queue characteristics of the model in a changed scenario. Since the variation in the parameter can be neutralized by averaging as we do in the case of compound distribution of the concern variable [Johnson-Kotz (1969)]. Therefore, in a situation when ρ is treated as a random variable, it seems statistically logical to infer about the parameter of the prior distribution using this compound distribution which also involve these parameters.

In the light of the above discussion, Gupta,P. K. and Goel Jaideep (2008, 2015) deals with the development of the methodology for updating the basic distribution in respect of prior and posterior variations in ρ. Consequently, the distribution of basic queue system (5) can be viewed as an updated basic and predictive basic distribution of the queue system. Thus, in the process, one gets three queue distributions as given in (1), (3) and (5) respectively. These distributions are useful for analyzing the queue's characteristics of the model in the following three specific situations.

(1) When the basic queue system model is used in the analysis. Here ρ is considered as constant.

(2) When the updated basic queue system model is used in the analysis. Here ρ is regarded as random variable with its prior variations.

(3) When the predictive basic queue system model is used in the analysis. Here the parametric variations in ρ are represented by the posterior prior distribution, which also incorporates the experimental data $\underset{\sim}{X} = (x_1, x_2, \ldots. x_k)$ or sample information.

These updated distributions have been used to study the robust character of various queue characteristics of a power supply system model in (2008) and for (M / M / 1): (∞: FCFS) queue system model in (2015).

With these three queue system defined in (1), (2) and (3) respectively, the queue's characteristics in the corresponding situations have been derived. For analyzing the robust character of all queue characteristics when □ is considered as a random variable, a comparison made between all these three specific situations. For introducing statistical validity in such comparison, the parametric values in the three specific situations, i.e., (ρ,u,v) are so chosen so that E(X) for the distributions in (1),(2) and (3) are equal.

In power supply system , we observe that the trends in all the queue characteristics L_s, L_q, Q_m, V_s, C.V. and η values clearly reveal that updated and predictive values of these characteristics are tend to be uniformly higher when E(□□□□□□. However, even for □ E(□□□□□□□, the updated values for all the queue characteristics are uniformly lower, while for the predictive values, these estimates tend to be higher up to a certain point, but thereafter these predictive values tends to be uniformly smaller for E(□□□□□□. The point at which the trend is reversed, is not difficult to estimate. Further, it is notable that the estimate tend to be more precise and consistent (as CVs tends to be uniformly smaller) in the case of predictive distribution as that compared with the compound distribution. The results seem obvious in view of the fact that queue characteristics of a power supply are observed to be non-robust and as such, this model should be very cautiously used whenever we suspect variations in □□□□ .

Also, in (M/M/1):(∞:FCFS) queue system model, the values for the queue characteristics tend to be uniformly higher when the prior variations are suspected. The trends in all the three tables clearly highlighted that modified/updated queue's characteristics values are uniformly higher when $\rho \leq E(\rho)$ i.e., (its expected mean variations). On the other hand, when $\rho > E(\rho)$, this updated estimates tend to be higher up to a certain point but thereafter updated queue's characteristics values are uniformly lower . The point at which the trends are reversed is not difficult to estimate. On comparing the variations in co-efficient of variations with respect to ρ and E(ρ) in, we observe that the estimates tend to be more and more consistent as either ρ or its expected mean variations increases. Cost benefit analysis in all the three situations are also explain for various values of ρ and its prior variations in profit functions. In market analysis, by analyzing the above trends, in respect of profit analysis, one can easily make an economic trade-off in an (M/M/1):(∞:FCFS) queue system model.

Gupta et.al(2013) considered the (M / M / 1): (∞ / FIFO) queue system model. In this model while recording arrival and service information for a large interval of time, it is reasonable to assume random variations in the parameter involve in the arrival and service time distributions. Here it should be recognized that prior do have an impact on the basic distribution and therefore updating these basic distributions in its parameter is another important aspect for analyzing the queue characteristics of the model in changed scenario. Again the variations in the parameter can be neutralized by averaging as we do in the case of compound distribution of the concern variable (Johnson and Kotz, 1969). This study deals with the development of the methodology for updating the basic distributions in respect of prior variations in arrival and service time distributions. Further, since the traffic intensity is the ratio of mean service time to mean inter arrival time, thus the traffic intensity (ρ) can be modified as the ratio of the updated mean service time to updated mean inter-arrival time. This updated or modified traffic intensity will used to develop all queue characteristics in changed scenario. For analyzing the effect of random variations in arrival and service time distributions, the authors compare the queue characteristics as given in basic form with those developed in updated form. Choosing suitability of the parameters, the corresponding value of the $\lambda = E(\lambda) = \frac{n}{m}$ and $\mu = E(\mu) = \frac{v}{u}$ may be used for analyzing the effect of random variations on various queue characteristics. The estimates of the various queue characteristics with variations in λ [or E(λ)], (while μ is fixed) as well as for fixed λ, and with variations in μ [or E(μ)] are shown in paper and explain the impact of variations on queue characteristics**.** The trends with variations in λ [or E(λ)] clearly reveals that these updated values are tend do be smaller up to a certain point, but thereafter, these values tend to be uniformly higher when $E(\lambda) \geq \lambda$. On the other hand, for $E(\lambda) < \lambda$, the updated values of queue characteristics are tend to be uniformly smaller. By the same way with variations in μ [or E(μ)] the reverse tendency of characteristics are found, i.e. these updated values are tend do be smaller up to a certain point, but thereafter, these values tend to be uniformly higher when $E(\mu) \leq \mu$. On the other hand, for $E(\lambda) > \lambda$, the updated values of queue characteristics are tend to be uniformly smaller. On comparing the variations in CV_s^* in respect of variations in λ and E(λ) , authors find that the estimates tend to be more and more consistent as either λ or E(λ) increases. Similarly, on comparing the variations in CV_s^* in respect of μ and E(μ) the reverse trend are found i.e. the estimates tend to be less and less consistent as either μ or E(μ) increases. Finally, the conclusion is that, the queue characteristics of a (M / M / 1): (∞: FIFO) queue system model are observed to be non-robust and, as such, this model should be very cautiously used whenever we suspect variations in arrival and service rates or in ρ.

In this continuation, Gupta, P.K. and Goel, Jaideep (2014) derived the inferential characteristics such as the maximum likelihood estimator (MLE), uniformly minimum variance unbiased estimator (UMVUE) and Bayes estimator of ρ and various other queue characteristics in General Earlang queuing system model. Furthermore, testing of various estimators of performance measures has been also presented at the end of the paper to demonstrate the estimation technique applied as to showing its practical significance.

V. CONCLUSION

For analyzing the behavior of these MLE’s, UMVUE’s and Bayes estimators of various queues characteristics, keeping some of the parameters fixed and varying others. With variations in the mean of traffic intensity ρ , the variations in MLE’s, UMVUE’s and Bayes estimators are studied and the trends so observed clearly reveals that MLE’s and UMVUE’s of all the queue’s characteristics are constant while Bayes estimates of all the characteristics tends to be decreases as mean of the traffic intensity decrease. On the other hand with variations in the number of the system, the trends in various characteristics are studied and the trends so observed clearly highlighted that MLE’s, UMVUE’s and

Bayes estimators of all the characteristics tend to be decreased as the number of system increases. Following this pattern, the behavior of all these characteristics with respect to variations in other parameters can be studied, measured and interpreted.

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