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Comparison of The Markowitz and Single Index Model Based on M-V Criterion in Optimal Portfolio Formation

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ABSTRACT

This research aim to compare the result of optimal portfolio formation between Markowitz and Single Index Models based on Mean-Variance criterion. The optimal portfolio with Markowitz Model is calculated by minimizing risk and determine the specific expected return level. Optimal portofolio calculation with Single Index Model results the proportion fund of each stock, thus it obtained the expected return and risk of the portfolio. The Comparison based on M-V criterion performed by determining the same expected return level portfolio of the Single Index Model, as a constraint on the Markowitz Model to minimize the risk. This research was applied to stocks in Jakarta Islamic Index (JII). At the same expected return rate of 1.2939% per week, Markowitz Model has a risk of 0.0973% per week and the Single Index Model has a risk of 0.3318% per week. Based on the M-V Criterion, it can be concluded that the optimal portfolio formation with Markowitz Model is more dominant than the Single Index Model on Jakarta Islamic Index (JII) stocks in the period of December1st, 2015-November 30th, 2016.

***Keywords*--** Markowitz Model, M-V Criterion, optimal portfolio, Single Index Model

I. INTRODUCTION

Investment is a commitment to invest the capital in one or more assets in order to get benefit for the future. One of the trades in the capital market is a stock. Stock is the securities as proof of ownership or possession of individuals or institutions issued by Perseroan Terbatas (PT). In stock investments, investors need to consider two things: the expected return and variance. Therefore, Investors need to make diversification in invest. Diversification means that investors need to form a portfolio through the selection of a number of assets, thus the risks can be minimized without reducing the level of expected returns. In 1952, Harry Markowitz created a portfolio selection model that incorporated the principle of diversification [4]. The first step of the model is to identify an efficient portfolio. An efficient portfolio is a portfolio with a maximum yield rate at a given risk level or a portfolio with minimal risk at a given rate of return. Both ways to get an efficient portfolio will produce the same results. A set of efficient portfolios will form a line called the efficient frontier line. In a set of efficient portfolios there is an optimal portfolio based on investor preference by determining the desired return or risk level. The determination of the optimal portfolio is known as the optimal portfolio determination based on the Markowitz Model. The results of optimum Markowitz Model portfolio formation depend heavily on investor preferences on the risks or expected return that serve as constraints on the model. An optimal portfolio can also be established by finding the best combination of returns and risks. The combination can be obtained by finding the tangent point between efficient frontier and straight line drawn from the risk free return. This tangent point is the point of contact between the efficient frontier and the straight line having the largest slope. By optimizing the slope, we will get the best return and risk combination. Such optimization requires as much as n estimation of yield, n variance estimation and $n\,(n-1)\,/\,2$ covariance with total number of $2n + n\,(n-1)\,/\,2$. The calculations are not effective for large n, so William Sharpe developed a model known as the Single Index Model to simplify the calculation. The Single Index Model is based on the observation that the price of a securities fluctuates in the direction of the market price index. In general, if the stock price index rises then the stock price also rises, and vice versa, if the stock price index is fall, then generally stock prices is also fall. The Single Index Model requires only n estimated returns, n estimation of beta value, n unique risk estimation and 1 estimated market yield of macroeconomic factors, so the total calculation of this Single Index Model is $3n + 1$.

We are interested to know whether a simpler model, Single Index Model can produce a more dominant optimal portfolio compared with Markowitz Model. The comparison is based on Mean-Variance criterion. The M-V criterion compares the portfolio optimization Model based on two factors, namely the return and risk factors of both portfolios. Case studies in this research were conducted on stock of companies entering the Jakarta Islamic Index (JII).

II. METHODOLOGY

For getting stock information about companies that consistanly registered on *Jakarta Islamic Index* (JII), this research used secondary data which took from official website of Indonesian Bank at www.bi.go.id. The data was taken for four periode which are December 2014-May 2015, June 2015-November 2015, December 2015-May 2016 and June 2016-November 2016. Risk free asset return R_{br} based on Sertifikat Bank Indonesia Syariah (SBIS) can be taken from official website of Indonesian Bank at period December 1st, 2015-November 30th, 2016. Furthermore, the adjusted closed price of stock for every company was taken from www.yahoofinance.co.id at period December 1st, 2015-November 30th, 2016.

In this research comparing optimal portfolio formation with Single Index and Markowitz Model by taking the same level of expected return value. The first step is to determine the optimal portfolio with a single index Model that generates the proportion of fund, expected return and risk of portfolio. Furthermore, the determination of the optimal portfolio with the Markowitz Model is done by minimizing the variance and taking the same expected return rate as the result of Single Index Model. The last step, compare the results from both Models with M-V Criterion.

III. RESEARCH MODELS

In 1952, Harry Markowitz created the Markowitz Model to form an optimal portfolio. Then William Sharpe create the Single Index Model to simplify the calculation of the Markowitz Model

MARKOWITZ MODEL

The calculation of the optimal portfolio with Markowitz Model is done by minimizing risk portfolio and determine the specific expected return level portfolio to obtain the proportion of each stock [3]:

$$\underset{w_i, w_j}{\text{inimize}} \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \text{Cov}(R_i, R_j) \qquad (3.1)$$

constrains

1. $\sum_{i=1}^{n} w_i = 1$
2. $\sum_{i=1}^{n} w_i E(R_i) = R^*$
3. $w_i \geq 0$ untuk $1 \leq i \leq n$

Where $\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \text{Cov}(r_i, r_j)$ is variance or risk portfolio [1], $\sum_{i=1}^{n} w_i E(R_i)$ is expected return portofolio [1] and w_i are the proportion on stock i. R^* is the specific expected return portofolio that determined by investor.

Since the objective function is non-negative, it can be multiplied by any non–negative constant without changing the solution. So the objective function can be written as follows:

$$\underset{w_i, w_j}{\text{inimize}} \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \text{Cov}(R_i, R_j) \qquad (3.2)$$

This problem is optimization problem where the solution can be solved by using Lagrangian equation.

$$= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \text{Cov}(r_i, r_j) - \lambda_1\left(\sum_{i=1}^{n} w_i - 1\right) - \lambda_2\left(\sum_{i=1}^{n} w_i E(R_i) - R^*\right) \qquad (3.3)$$

where λ_1 and λ_2 are Lagrangian multiplier.

Searching for the first derivative of the objective function L to the proportion of each stock and Lagrangian multiplier, then set derivative to zero as follows:

$$\frac{dL}{dw_i} = \sum_{j=1}^{n} w_j \text{Cov}(R_1, R_j) - \lambda_1 - \lambda_2 E(R_i) = 0, \qquad (3.4)$$

$i = 1, 2, \ldots, n$

$$= \sum_{i=1}^{n} w_i - 1 = 0 \qquad (3.5)$$

$$= \sum_{i=1}^{n} w_i E(R_i) - R^* = 0. \qquad (3.6)$$

Suppose:

$\bar{1} = [1 \quad 1 \quad \cdots \quad 1_n]^T$,

$\mu = [E(R_1) \quad E(R_2) \quad \cdots \quad E(R_n)]^T$ and

$$\Omega = \begin{bmatrix} \text{Cov}(R_1,R_1) & \text{Cov}(R_1,R_2) & \cdots & \text{Cov}(R_1,R_n) \\ \text{Cov}(R_2,R_1) & \text{Cov}(R_2,r_2) & \cdots & \text{Cov}(R_2,R_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}(R_n,R_1) & \text{Cov}(R_n,R_2) & \cdots & \text{Cov}(R_n,R_n) \end{bmatrix}$$

$$= \begin{bmatrix} \text{Cov}(R_1,R_1) & \text{Cov}(R_1,R_2) & \cdots & \text{Cov}(R_1,R_n) \\ \text{Cov}(R_1,R_2) & \text{Cov}(R_2,r_2) & \cdots & \text{Cov}(R_2,R_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}(R_1,R_n) & \text{Cov}(R_2,R_n) & \cdots & \text{Cov}(R_n,R_n) \end{bmatrix}$$

From equation (3.4), then obtained:

$\sum_{j=1}^{n} w_j \text{Cov}(R_1, R_j) - \lambda_1 - \lambda_2 E(R_i) = 0$

$\sum_{j=1}^{n} w_j \text{Cov}(R_1, R_j) = \lambda_1 - \lambda_2 E(R_i)$

$\Leftrightarrow$

$$\begin{bmatrix} \text{Cov}(R_1,R_1) & \text{Cov}(R_1,R_2) & \cdots & \text{Cov}(R_1,R_n) \\ \text{Cov}(R_2,R_1) & \text{Cov}(R_2,r_2) & \cdots & \text{Cov}(R_2,R_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}(R_n,R_1) & \text{Cov}(R_n,R_2) & \cdots & \text{Cov}(R_n,R_n) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

$$= \lambda_1 \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1_n \end{bmatrix} + \lambda_2 \begin{bmatrix} E(R_1) \\ E(R_2) \\ \vdots \\ E(R_n) \end{bmatrix}$$

$\Leftrightarrow \Omega \text{w}^* = \lambda_1 \bar{1} + \lambda_2 \mu$, with

$$\text{w}^* = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = [w_1 \quad w_2 \quad \cdots \quad w_n]^T.$$

Because Ω is covariance then Ω is definit positive. Because Ω is definit positive, then Ω is invertible.

$\Omega^{-1}\Omega \text{w}^* = \Omega^{-1}(\lambda_1 \bar{1} + \lambda_2\, \mu)$

$\Leftrightarrow I\text{w}^* = \Omega^{-1}(\lambda_1 \bar{1} + \lambda_2\, \mu)$, I is identity matrix

$$\Leftrightarrow \text{w}^* = \Omega^{-1}(\lambda_1 \bar{1} + \lambda_2\, \mu). \qquad (3.7)$$

Calculate λ_1 and λ_2.

From equation (3.7), then obtained:

$\text{w}^* = \Omega^{-1}(\lambda_1 \bar{1} + \lambda_2\, \mu)$

$\Leftrightarrow \text{w}^* = \Omega^{-1}\lambda_1 \bar{1} + \Omega^{-1}\lambda_2\, \mu$

$\Leftrightarrow \bar{1}^T \text{w}^* = \bar{1}^T \Omega^{-1}\lambda_1 \bar{1} + \bar{1}^T \Omega^{-1}\lambda_2\, \mu$, λ_1 and λ_2 are constant

$\Leftrightarrow \bar{1}^T \text{w}^* = \lambda_1 \bar{1}^T \Omega^{-1} \bar{1} + \lambda_2 \bar{1}^T \Omega^{-1} \mu$

Based from the first constrain, can be written as follows:

$$\sum_{i=1}^{n} w_i = w_1 + w_2 + \cdots w_n = [1 \quad 1 \quad \cdots \quad 1] \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} =$$

$\bar{1}^T \text{w}^* = 1$, then obtained:

$$1 = \bar{1}^T \text{w}^* = \lambda_1 \bar{1}^T \Omega^{-1} \bar{1} + \lambda_2 \bar{1}^T \Omega^{-1} \mu. \qquad (3.8)$$

From equation (3.7), then obtained:

$w^* = \Omega^{-1}(\lambda_1 \bar{1} + \lambda_2 \mu)$
$\Leftrightarrow\ w^* = \Omega^{-1}\lambda_1 \bar{1} + \Omega^{-1}\lambda_2 \mu$
$\Leftrightarrow \mu^T w^* = \mu^T\Omega^{-1}\lambda_1 \bar{1} + \mu^T\Omega^{-1}\lambda_2 \mu$, λ_1 and λ_2 are constant
$\Leftrightarrow \mu^T w^* = \lambda_1 \mu^T \Omega^{-1} \bar{1} + \lambda_2 \mu^T \Omega^{-1}\mu.$
Based from the second constrain, can be written as follows:

$$\sum_{i=1}^{n} w_i E(R_i) = w_1 E(R_1) + w_2 E(R_2) + \cdots + w_n E(R_n)$$
$$= [E(R_1) \quad E(R_2) \quad \cdots \quad E(R_n)] \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \mu^T w^* = R^*,$$

then obtained:
$$R^* = \mu^T w^* = \lambda_1 \mu^T \Omega^{-1} \bar{1} + \lambda_2 \mu^T \Omega^{-1}\mu \quad (3.9)$$
Suppose:
$a = \bar{1}^T \Omega^{-1} \bar{1}$
$b = \bar{1}^T \Omega^{-1} \mu$
$c = \mu^T \Omega^{-1} \mu$
Based (3.8) and (3.9), then obtained the system of equation:
$$1 = \lambda_1 a + \lambda_2 b \quad (3.10)$$
$$= \lambda_1 b + \lambda_2 c \quad (3.11)$$
Based from the system of equation, then obtained:
$\lambda_1 = \frac{c - bR^*}{ac - b^2}$ dan $\lambda_2 = \frac{aR^* - b}{ac - b^2}$
or
$$= \frac{(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)R^*}{(\bar{1}^T\Omega^{-1}\bar{1})(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)^2} \quad (3.12)$$
$$= \frac{(\bar{1}^T\Omega^{-1}\bar{1})R^* - (\bar{1}^T\Omega^{-1}\mu)}{(\bar{1}^T\Omega^{-1}\bar{1})(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)^2}. \quad (3.13)$$

Substituting (3.12) and (3.13) to (3.7), then obtained:

$$w^* = \Omega^{-1}\left[\frac{(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)R^*}{(\bar{1}^T\Omega^{-1}\bar{1})(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)^2}\right]\bar{1} + \Omega^{-1}\left[\frac{(\bar{1}^T\Omega^{-1}\bar{1})R^* - (\bar{1}^T\Omega^{-1}\mu)}{(\bar{1}^T\Omega^{-1}\bar{1})(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)^2}\right]\mu.$$

Minimum variance portfolios

Based equation (3.7) then obtained minimum variance portfolios, as follows:
$\sigma_P{}^2 = w^{*T}\Omega w^*,$
$= [\Omega^{-1}(\lambda_1\bar{1} + \lambda_2\mu)]^T\Omega[\Omega^{-1}(\lambda_1\bar{1} + \lambda_2\mu)]$
$= [\lambda_1\Omega^{-1}\bar{1} + \lambda_2\Omega^{-1}\mu]^T\Omega[\lambda_1\Omega^{-1}\bar{1} + \lambda_2\Omega^{-1}\mu].$
Because $(A+B)^T = A^T + B^T$ dan $(AB)^T = B^T A^T$, then
$\sigma_P{}^2 = [(\lambda_1\Omega^{-1}\bar{1})^T + (\lambda_2\Omega^{-1}\mu)^T]\Omega\,[\lambda_1\Omega^{-1}\bar{1} + \lambda_2\Omega^{-1}\mu]$
$= [\lambda_1\bar{1}^T(\Omega^{-1})^T + \lambda_2\mu^T(\Omega^{-1})^T]\Omega[\lambda_1\Omega^{-1}\bar{1} + \lambda_2\Omega^{-1}\mu]$
because Ω is symetry, so that $\Omega^T = \Omega$ and $(\Omega^{-1})^T = \Omega^{-1}$.
$\sigma_P{}^2 = [\lambda_1\bar{1}^T\Omega^{-1} + \lambda_2\mu^T\Omega^{-1}]\Omega[\lambda_1\Omega^{-1}\bar{1} + \lambda_2\Omega^{-1}\mu]$
$= [\lambda_1\bar{1}^T\Omega^{-1}\Omega + \lambda_2\mu^T\Omega^{-1}\Omega][\lambda_1\Omega^{-1}\bar{1} + \lambda_2\Omega^{-1}\mu]$
$= [\lambda_1\bar{1}^T I + \lambda_2\mu^T I][\lambda_1\Omega^{-1}\bar{1} + \lambda_2\Omega^{-1}\mu],$
I is identity matrix
$= [\lambda_1\bar{1}^T + \lambda_2\mu^T][\lambda_1\Omega^{-1}\bar{1} + \lambda_2\Omega^{-1}\mu]$
$= \lambda_1\lambda_1\bar{1}^T\Omega^{-1}\bar{1} + \lambda_1\lambda_2\mu^T\Omega^{-1}\bar{1} + \lambda_1\lambda_1\bar{1}^T\Omega^{-1}\mu + \lambda_2\lambda_2\mu^T\Omega^{-1}\mu$
$= \lambda_1(\lambda_1\bar{1}^T\Omega^{-1}\bar{1} + \lambda_2\mu^T\Omega^{-1}\bar{1}) + \lambda_2(\lambda_1\bar{1}^T\Omega^{-1}\mu + \lambda_2\mu^T\Omega^{-1}\mu)$
$= \lambda_1(\lambda_1 a + \lambda_2 b) + \lambda_2(\lambda_1 b + \lambda_2 c)$
$= \lambda_1 + \lambda_2 R^*$

$$= \left(\frac{(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)R^*}{(\bar{1}^T\Omega^{-1}\bar{1})(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)^2}\right) + \left(\frac{(\bar{1}^T\Omega^{-1}\bar{1})R^* - (\bar{1}^T\Omega^{-1}\mu)}{(\bar{1}^T\Omega^{-1}\bar{1})(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)^2}\right) R^*$$
$$= \frac{(\bar{1}^T\Omega^{-1}\bar{1})(R^*)^2 - 2(\bar{1}^T\Omega^{-1}\mu)R^* + \mu^T\Omega^{-1}\mu}{(\bar{1}^T\Omega^{-1}\bar{1})(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)^2} \quad (3.14)$$

Based on the result, it can be concluded that the variance (risk) based on the Markowitz Model is still dependent on the expected return portfolio R*

Global minimum variance portfolio

Calculated the expected return portfolio as follows:
$$\frac{d\sigma^2{}_p}{dR^*} = \frac{2(\bar{1}^T\Omega^{-1}\bar{1})R^* - 2(\bar{1}^T\Omega^{-1}\bar{1})}{(\bar{1}^T\Omega^{-1}\bar{1})(\mu^T\Omega^{-1}\mu) - (\bar{1}^T\Omega^{-1}\mu)^2} = \frac{2aR^* - 2b}{ac - b^2} = 0$$
$\Leftrightarrow 2aR^* - 2b = 0$
$\Leftrightarrow R^* = \frac{b}{a}$
with variance portfolio, as follows:
$$\sigma_P{}^2 = \lambda_1 + \lambda_2 R^* = \frac{c - bR^*}{ac - b^2} + \frac{aR^* - b}{ac - b^2}\left(\frac{b}{a}\right) = \frac{1}{a}$$
This figure show the global minimum variance portfolio, so the optimal portfolio can be chosen from efficient frontier with $E(R_P) > \frac{b}{a}$.

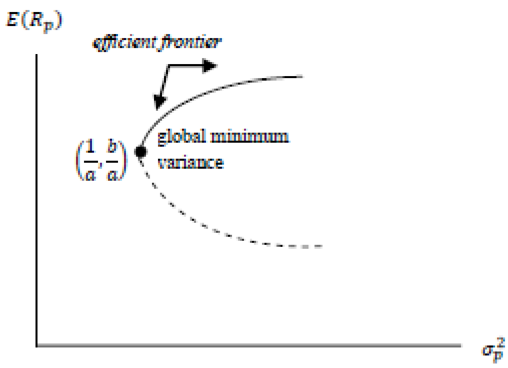

Figure 1: The global minimum variance portfolio

SINGLE INDEX MODEL

The Single Index Model is based on the observation that the price of a securities fluctuates in the direction of the market price index. On this basis, returns from securities and returns from common market indices can be written as follows [2]:
$$R_i = a_i + \beta_i R_m \qquad i = 1,2,\ldots,n \qquad (3.15)$$
Where R_i are return on stock i, a_i are random variable showing of the return of each stock which is independent to market performance, β_i are coefficient that measures the change of R_i as a result of R_m [1], R_m is rate of return of the market index.

Then a_i composed by $\alpha_i + e_i$, where α_i are intercept of a straight line or Alpha coefficient and e_i are error term with the mean of zero $(E(e_i) = 0)$. So the Single Index Model can be written as follows [5]:
$$R_i = \alpha_i + \beta_i R_m + e_i \qquad i = 1,2,\ldots,n \qquad (3.16)$$
There are assumptions on the Single Index Model which is $\text{Cov}(e_i, e_j) = 0$ and $\text{Cov}(e_i, R_m) = 0$, then the variance and covariance of Single Index Model as follows[3]:

$\sigma_i{}^2 = \beta_i{}^2 \sigma_m{}^2 + \sigma_{ei}{}^2$ and $\sigma_{ij} = \beta_i\, \beta_j\, E[R_M - E(R_M)]^2$ (3.17)

The establishment of an optimal portfolio based on the Single Index Model is used slope optimization which will result in the proportion of funds to be allocated to each company. Slope can be calculated by the following formula [3]:

$$\theta = \frac{E(R_p)-R_{br}}{\sigma_P} \quad (3.18)$$

Where $E(R_p)$ is expected return of portfolio P, R_{br} is risk free asset and σ_P is risk (deviation standard) of portofolio P.

Maximizing the slope can be calculated by searching for the first derivative of the objective function θ to the proportion of each stock $\frac{d\theta}{dw_k} = 0$, so as to obtain the following system of equations [5]:

$$\psi(w_1\sigma_1{}^2 + w_2\sigma_{12} + \cdots + w_n\sigma_{1n}) = [E(R_1) - R_{br}]$$
$$\psi(w_1\sigma_{21} + w_2\sigma_2{}^2 + \cdots + w_n\sigma_{2n}) = [E(R_2) - R_{br}]$$
$$\vdots$$
$$\psi(w_1\sigma_{n1} + w_2\sigma_{n2} + \cdots + w_n\sigma_{nn}) = [E(R_n) - R_{br}]$$

Or in general can be written as follows:

$$\psi\left(\sum_{j=1}^{n} w_j\, \sigma_{kj}\right) = [E(R_k) - R_{br}]. \quad (3.19)$$

Where $\psi = \left[\frac{\left(\sum_i^n w_i[E(R_i)-R_{br}]\right)}{\left(\sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij}\right)}\right]$

Let $\psi w_i = Z_i$ then in general the optimization equation as follows:

$$Z_i\sigma_i{}^2 \sum_{\substack{j=1\\ j\neq i}}^{n} Z_i\, \sigma_{ij} = E(R_i) - R_{br} \quad (3.20)$$

Substituting the variance and covariance equations of the Single Index Model (3.17) to equation (3.20), thus obtaining:

$$Z_i(\beta_i^2\, \sigma_m^2 + \sigma_{ei}^2) + \sum_{\substack{j=1\\ j\neq i}}^{n}\left(Z_j\, \beta_i\, \beta_j\, \sigma_m^2\right) = E(R_i) - R_{br}$$

$$Z_i\, \beta_i^2\, \sigma_m^2 + Z_i\, \sigma_{ei}^2 + \sum_{\substack{j=1\\ j\neq i}}^{n}\left(Z_j\, \beta_i\, \beta_j\, \sigma_m^2\right) = E(R_i) - R_{br}$$

$$Z_i\, \sigma_{ei}^2 + \sum_{j=1}^{n}\left(Z_j\, \beta_i\, \beta_j\, \sigma_m^2\right) = E(R_i) - R_{br}$$

$$Z_i\, \sigma_{ei}^2 + \beta_i\, \sigma_m^2 \sum_{j=1}^{n}\left(Z_j\, \beta_j\right) = E(R_i) - R_{br}$$

$$Z_i = \frac{E(R_i)-R_{br}}{\sigma_{ei}^2} - \frac{\beta_i\, \sigma_m^2}{\sigma_{ei}^2}\sum_{j=1}^{n}\left(Z_j\, \beta_j\right).$$

By multiplying the value $\frac{E(R_i)-R_{br}}{\sigma_{ei}^2}$ with $\frac{\beta_i}{\beta_i}$, obtained:

$$Z_i = \left(\frac{E(R_i)-R_{br}}{\sigma_{ei}^2}\right)\frac{\beta_i}{\beta_i} - \frac{\beta_i\sigma_m^2}{\sigma_{ei}^2}\left(\frac{\sum_{j=1}^{n}\frac{E(R_j)-R_{br}}{\sigma_{ej}^2}\beta_j}{\left(1+\sigma_m^2\sum_{j=1}^{n}\frac{\beta_j}{\sigma_{ej}^2}\right)}\right)$$

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left(\frac{E(R_i)-R_{br}}{\beta_i} - \sigma_m^2\left(\frac{\sum_{j=1}^{n}\frac{E(R_j)-R_{br}}{\sigma_{ej}^2}\beta_j}{\left(1+\sigma_m^2\sum_{j=1}^{n}\frac{\beta_j}{\sigma_{ej}^2}\right)}\right)\right)$$

$$= \frac{\beta_i}{\sigma_{ei}^2}\left(\frac{E(R_i)-R_f}{\beta_i} - \frac{\sigma_m^2\sum_{j=1}^{n}\frac{E(R_j)-R_f}{\sigma_{ej}^2}\beta_j}{\left(1+\sigma_m^2\sum_{j=1}^{n}\frac{\beta_j}{\sigma_{ej}^2}\right)}\right)$$

$$= \frac{\beta_i}{\sigma_{ei}^2}\left(ERB_i - C^*\right) \quad (3.21)$$

Where $ERB_i = \frac{E(R_i)-R_{br}}{\beta_i}$ and $C^* = \frac{\sigma_m^2\sum_{j=1}^{n}\frac{E(R_j)-R_{br}}{\sigma_{ej}^2}\beta_j}{\left(1+\sigma_m^2\sum_{j=1}^{n}\frac{\beta_j}{\sigma_{ej}^2}\right)}$ with $C^* = \max\{C_i\}_{i=1}^{n}$.

Excess return is defined as the difference between the expected return value and the risk-free asset return. Excess return to Beta (ERB) is a measure of the excess return relative to a unit of risk that can not be diversified and measured by Beta. Furthermore, if C^* is the $\max\{C_i\}_{i=1}^{n}$, it can be concluded that the stock with $ERB_i \geq C^*$is the stock to be included in the optimal portfolio. This condition is done so that Z_i is positive so no short sale occurs. So C^* is called cut off point.

The total fund to be invested by one unit or $\sum_{i=1}^{n} w_i = 1$, then the proportion of funds to be invested to each company as follows:

$w_i = \frac{w_i}{\sum_{i=1}^{n} w_i}$, multiplying the numerator and denominator by ψ, then obtained:

$$w_i = \frac{\psi w_i}{\psi\sum_{i=1}^{n} w_i} = \frac{\psi w_i}{\sum_{i=1}^{n}\psi w_i} = \frac{Z_i}{\sum_{i=1}^{n} Z_i} \quad (3.22)$$

M-V CRITERION

A portfolio is said to dominate portfolio B if the expected return of portfolio A is greater than that of portfolio B, and the standard deviation of portfolio A is less than the standard deviation of portfolio B. The equation can be written as follows [1]:

$$E(R_A) \geq E(R_B) \text{ and } \sigma_A \leq \sigma_B \quad (3.23)$$

IV. APPLICATION MODEL

In this research comparing optimal portfolio formation with Single Index Model and Markowitz by taking the same level of expected return value. The first step is to determine the optimal portfolio with a single index Model that generates the proportion of fund, expected return and risk of portfolio. Furthermore the determination of the optimal portfolio with the Markowitz Model is done by minimizing the variance and taking the same expected return rate as the result of Single Index Model. The last step, compare the results from both Models with M-V Criterion.

DATA

Companies that entered into the Jakarta Islamic Index (JII) amounted to 30 companies. In this study only companies consistently listed in the Jakarta Islamic Index (JII) for four periods are December 2014-May 2015, June 2015-November 2015, December 2015-May 2016 and June 2016-November 2016 periods. Are as follows:

Astra Agro Lestari Tbk. (AALI)	PP London Sumatra Indonesia Tbk. (LSIP)
Adaro Energi Tbk. (ADRO)	Perusahaan Gas Negara (Persero) Tbk. (PGAS)
AKR Corporindo Tbk. (AKRA)	PP Persero (PTPP)
Astra International Tbk. (ASII)	Siloam International Hospitals Tbk. (SILO)

Alam Sutera Realty Tbk. (ASRI)	Semen Indonesia (Persero) Tbk. (SMGR)
Bumi Serpong Damai Tbk. (BSDE)	Summarecon Agung Tbk. (SMRA)
Indofood CBP Sukses Makmur Tbk. (ICBP)	Sawit Sumberma Sarana Tbk. (SSMS)
Vale Indonesia Tbk. (INCO)	Telekomunikasi Indonesia (Persero) Tbk.(TLKM)
Indofood Sukses Makmur Tbk. (INDF)	United Tractors Tbk. (UNTR)
Indocement Tunggal Perkasa Tbk. (INTP)	Unilever Indonesia Tbk. (UNVR)
Kalbe Farma Tbk. (KLBF)	Wijaya Karya (Persero) Tbk. (WIKA).
Lippo Karawaci Tbk. (LPKR).	

Furthermore, stocks with an expected return value less than equal to risk free or $E(R_i) \leq R_{br} = 0.001238$ (R_{br} taken from Bank Indonesia Syariah Certificate for the period December 1, 2015-30 November 2016) will not be included in the optimal portfolio forming candidate. This is because if investors invest funds owned to the Bank, will be more profitable and there are no risks in the future than if investors invest funds into the stock market. Companies that will be incorporated into candidates for the establishment of an optimal portfolio ie, AALI, ADRO, AKRA, ASII, ASRI, ICBP, INCO, INDF, KLBF, LSIP, PTPP, SILO, TLKM, UNTR and UNVR.

Single Index Model in the Formation of JII's Optimal Stock Portfolio

1. Calculation to find the expected market return $E(R_m)$ and market variance ${\sigma_m}^2$. This calculation is derived from data of Jakarta Composite Index (IHSG) with the same period with period of taking stock data, that is period 1 December 2015-30 November 2016.

$E(R_m)$ weekly : 0.273019%

${\sigma_m}^2$ weekly : 0.025036%

$E(R_m)$ annually : 15.231912 %

${\sigma_m}^2$ annually : 1.301845%

2. The results of calculations $a_i, \beta_i$and ERB_i shares can be seen in the table as follows:

	a_i	β_i	ERB_i
AALI	0.000303	0.37235	0.00022
ADRO	0.019996	1.66135	0.01402
AKRA	-9.82E-05	0.83005	0.00112
ASII	0.002337	0.76459	0.00417
ASRI	0.00031	0.70424	0.00141
ICBP	0.015107	0.81861	0.01967
INCO	0.016093	1.01005	0.01744
INDF	0.007023	0.77154	0.01023
KLBF	0.001048	0.98381	0.00254
LSIP	0.006061	0.6205	0.01050
PTPP	0.001704	0.62633	0.00347
SILO	0.002582	0.01149	0.11971
TLKM	0.003066	0.87219	0.00483
UNTR	0.005598	0.77297	0.00837
UNVR	0.000783	0.81524	0.00217

3. Calculate σ_{ei} and calculate *Cut off Point* C^* by previously sorting Excess Return to Beta (ERB_i).The results of these calculations can be seen in the following table:

Pers.	ERB_i	σ_{ei}	C_i
SILO	**0.11971**	0.001566	2.5E-06
ICBP	**0.01967**	0.025472	0.00013
INCO	**0.01744**	0.006812	0.00075
ADRO	**0.01402**	0.005051	0.00229
LSIP	**0.01050**	0.002172	0.00259
INDF	**0.01023**	0.00129	0.00325
UNTR	**0.00837**	0.002326	0.00348
TLKM	**0.00483**	0.000758	0.00368
ASII	**0.00417**	0.001325	**0.00371**
PTPP	0.00347	0.001125	0.00370
KLBF	0.00254	0.001431	0.00361
UNVR	0.00217	0.001041	0.00350
ASRI	0.00141	0.001628	0.00343
AKRA	0.00112	0.001709	0.00333
AALI	0.00022	0.002852	0.00332

Based on the table, it can be seen the value $C^* = 0.003714$, so that shares that have value $ERB_i \geq C^*$ is the stock to be included in the formation of optimal portfolio. Of the 15 JII companies, nine companies are included in the optimal portfolio, namely SILO, ICBP, INCO, ADRO, LSIP, INDF, UNTR, TLKM and ASII.

4. The proportion is calculated by the formula searched in equation (3.22). The calculation results in the proportion of funds to be allocated to nine company as follow: SILO of 5.4149%, ICBP of 3.2643%, INCO of 12.9512%, ADRO of 21.5780%, LSIP of 12.3442%, INDF of 24.7948%, UNTR of 9.8473%, TLKM of 8.1414% and ASII of 1.6640%.

The expected return of portfolio and the risk of portfolio (weekly) based on the Single Index Model as follows:

$E(R_p)$: 1.293944% ${\sigma_p}^2$: 0.331829%

Markowitz Model in the Formation of JII's Optimal Stock Portfolio

Based on the description of the data that has been done in the previous sections, the stock data entered into the candidate for optimal portfolio formation is the shares that entered in JII for four periods and the stock having the expected return value which is above the risk free asset. Suppose that with the same expected return rate with Single Index Model that is equal to 1.293944% per week and used the program assistance LINGO 11.0 to solve the optimization problem (3.1), output as follows:

Variable	Value
W1	0.000000
W10	0.3849560E-01
W11	0.4372707E-01
W12	0.5074074E-01
W13	0.1767587
W14	0.000000
W15	0.000000
W2	0.2328353
W3	0.000000
W4	0.5054194E-02
W5	0.000000
W6	0.3324011E-01
W7	0.1294837
W8	0.2896645
W9	0.000000

Figure 2:Output LINGO 11.0

LINGO 11.0 was obtained nine companies are included in the optimal portfolio, with INCO proportion of 12.1295%, %, INDF of 28.9665% ICBP of 3.3240%, ASII of 0.5054%, ADRO of 23.2835%, TLKM of 17,6759%, SILO of 5.0741%, PTPP of 4.3727% and LSIP 3.8496%. The expected return of portfolio $E(R_p)$ and the portfolio variance $\sigma_p{}^2$ (weekly) based on the Single Index Model as follows:

$E(R_p)$: 1.293944% $\sigma_p{}^2$: 0.0973%

Comparison Based M-V Criterion

The comparison of optimal portfolio formation between Markowitz and Single Index model using M-V Criterion on Jakarta Islamic Index (JII) stocks in the period of December1st, 2015-November 30th,2016 can be seen in the following table, as follows:

	Markowitz	Single Index
Expected Return	1.2939%	1.2939%
Risk (St. Dev)	0.0973%	0.3318%

Based on the M-V Criterion, it can be concluded that the optimal portfolio formation with Markowitz Model is more dominant than the Single Index Model.

V. CONCLUSION

Based on Jakarta Islamic Index (JII) data in the period of December1st, 2015-November 30th, 2016 Markowitz Model has a risk of 0.0973% per week and the Single Index Model has a risk of 0.3318% per week at the same expected return rate of 1.2939% per week. Based on the M-V Criterion, it can be concluded that the optimal portfolio formation with Markowitz Model is more dominant than the Single Index Model.

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