Damping Effect on Thermally Induced Vibrations of Rectangular Plate

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ABSTRACT
The analysis is presented here for the design of engineering systems where materials and uniform structures are used. In this study the damping effect on thermal vibrations of an isotropic, elastic, rectangular plate of uniform thickness is considered. The frequencies corresponding to first four modes of vibration have been computed for the different combinations of boundary conditions and various values of thermal gradient and damping parameter by applying the method of Frobenius for the solution of the governing differential equation of motion.

Keywords--- Young modulus, Thermal gradient, Damping parameter, frequency parameter, Deflections and Moments.

I. INTRODUCTION
The problem of evaluating the natural frequencies of rectangular plates with variable thickness at different edge conditions have been recognized by [1-5]. Kukla [6-7] has observed free vibrations and frequencies of line system connected to rectangular plates by green function method. Oniszezuk [8] has determined the free vibrations of a system of two simply supported thin plates connected by winkler elastic layer which solved by classical navier method. Shuyu [9] has observed the flexural vibrations of thin rectangular plate by analytical method at free edges. The relation between the frequencies parameter and various type of temperature fields would be highly useful as a design tool specially in those mechanical structures where certain parts of a system have to operate under elevated temperatures its effect is far from negligible. The reason for these is that during heating up period of structures exposed to high intensity heat fluxes, the material properties undergo significant variation. Ghosh [11] observed that a parabolic distribution of temperature has a considerable effect on the natural frequencies of vibration as well as on the transverse waves which move around the circumference on the disk. Dhotarad and Ganesan [12] have analyzed the vibration of a rectangular plate subjected to a 2 thermal gradient by employing the finite element method. Fanconneau and Marangoni [13] have investigated the effect of the non-homogeneity caused by a thermal gradient on the natural frequencies of simply supported plate of uniform thickness. The concept of free vibrations is far from actual problem of vibrations due to damping factor. The damping effect can be large enough to check the vibrations to produce of any appreciable effect on frequency as well as amplitude of vibrations. This paper describe an the temperature effects has considered along x-axis only. The deflection function have consider in the form of an infinite series and then frequency parameters corresponding to the first four modes of vibrations with different combinations of boundary conditions for various values of aspect ratio, damping parameter and temperature gradient have been computed. The present investigations are helpful in designing many scientific devices where uniform structure are exposed to high intensity heat fluxes due to which the material properties undergo significant change in vibrations.

II. ANALYSIS
It is assumed that the rectangular plate material is subjected to a parabolic temperature distribution along the length i.e. in X-direction.

\[ T = T_0 \left(1 - X^2 \right) \]  

where \( T \) is the temperature excess above the reference temperature at any point \( X \) and \( T_0 \) is the temperature excess above the reference temperature at the end \( X=0 \). The temperature dependence of modulus of elasticity generally for large number of materials [10, 16, 17] is given by
\[ \bar{E}(T) = E_0 (1 - \xi T) \]  
\[ \text{(2)} \]

\[ \text{where } \bar{E}_0 \text{ is the modulus of elasticity of the material at the reference temperature and } \xi \text{ is a constant. Now considering} \]

\[ \bar{E}(X) = \bar{E}_0 [1 - \eta (1 - X^2)] \]  
\[ \text{(3)} \]

Where, \( \eta = \xi T_0 \), \( 0 \leq \eta < 1 \) is called the thermal gradient and \( \xi \) is an arbitrary constant.

The governing equation of motion is given by

\[ (D \nabla^4 w) + 2 \left( \frac{\partial D}{\partial x} \right) \frac{\partial}{\partial x} (\nabla^2 w) + 2 \left( \frac{\partial D}{\partial y} \right) \frac{\partial}{\partial y} (\nabla^2 w) + (\nabla^2 D) (\nabla^2 w) \]

\[ - (1 - \nu) \left( \frac{\partial^2 D}{\partial x^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 D}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 D}{\partial y^2} \frac{\partial^2 w}{\partial y^2} \right) + \rho h \left( \frac{\partial^2 w}{\partial t^2} \right) + k \left( \frac{\partial w}{\partial t} \right) = 0 \]  
\[ \text{(4)} \]

Where, \( D = D(x,y) = \frac{E(x,y)h^3}{12(1 - \nu^2)} \)

is flexural rigidity, \( \rho (x,y) \) is density, \( \nu \) be Poisson's ratio, \( K \) is damping constant, \( w \) is transverse deflection and

\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \text{ and } \nabla^4 = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \]

Here, the two edges parallel to x-axis are considered to be simply supported and different sets of boundary conditions have been imposed on the other pair of edges.

For harmonic damped vibrations, the deflections function \( w \) can be written as

\[ w(x,y,t) = W(x) \sin \left( \frac{m\pi y}{b} \right) e^{-\beta t} \cos(\omega t) \]  
\[ \text{(5)} \]

Using relation (4), (5) and introducing the nondimensional variables. Thus one obtains the equation of motion of a rectangular plate.

\[ \left[ E \right] \frac{\partial^4 \bar{W}}{\partial X^4} + \left[ 2 \frac{\partial E}{\partial X} \right] \frac{\partial^3 \bar{W}}{\partial X^3} + \left[ \frac{\partial^2 E}{\partial X^2} - 2S^2 \alpha^2 \bar{E} \right] \frac{\partial^2 \bar{W}}{\partial X^2} - \left[ 2S^2 \alpha^2 \frac{\partial E}{\partial X} \right] \frac{\partial \bar{W}}{\partial X} \]

\[ + \left[ \left( S^4 a^4 \bar{E} \right) \right] \left[ vS^2 \alpha^2 \frac{\partial \bar{E}}{\partial X^2} \right] - \left( \frac{3(1 - \nu^2)K}{S} \right) - \left( \frac{12(1 - \nu^2)\rho a^2 \omega^2}{H^2} \right) \right] \bar{W}(X) = 0 \]  
\[ \text{(6)} \]

where, \( \bar{W} = \frac{W}{a} \), \( X = \frac{x}{a} \), \( Y = \frac{y}{b} \), \( \bar{E} = \frac{E}{a} \), \( H = \frac{h}{a} \) and \( \bar{\rho} = \frac{\rho}{a} \)

Using equation (3), equation (6) reduces to

\[ \left[ (1 - \eta + \eta X^2) \right] \frac{\partial^4 \bar{W}}{\partial X^4} + \left[ 4\eta X \right] \frac{\partial^3 \bar{W}}{\partial X^3} + \left[ (\eta) - (1 - \eta + \eta X^2) \right] \alpha^2 \frac{\partial^2 \bar{W}}{\partial X^2} - \left[ 4\alpha^2 \eta X \right] \frac{\partial \bar{W}}{\partial X} \]

\[ + \left[ \alpha^4 (1 - \eta + \eta X^2) \right] - \left[ 2v\alpha^2 \eta \right] - \left( \Omega^2 I^2 \right) - \left( \Omega^2 I^2 \right) \right] \bar{W}(X) = 0 \]  
\[ \text{(7)} \]

Respectively, Where,
\[ \alpha = S \alpha, D_k^2 = \left[ \frac{3k^2(1-\nu^2)}{a^2\bar{p}E_0} \right], I^* = \left[ \frac{1}{H^2} \right] \text{and } \Omega^2 = \left[ \frac{12(1-\nu^2)\bar{p}a^2p^2}{E_0} \right] \]

Hence, \( D_k \) is damping parameter, \( P \) is circular frequency and \( \Omega \) is frequency parameter.

### III. Solution and Its Convergence

Let the solution for \( W \) is assured in the series from as:

\[ W(X) = \sum_{\lambda=0}^{\infty} a_\lambda X^{\lambda+c+\lambda} \text{ with } a_0 \neq 0 \]...........(8)

Where \( C \) is the exponent of singularity,

Using equation (8) & (7) one obtains

\[ \sum_{\lambda=0}^{\infty} a_\lambda \left[ b_\lambda^{(3)} T_4^{(1)} + b_\lambda^{(2)} T_2^{(2)} + b_\lambda^{(1)} T_2^{(3)} \right] X^{\lambda+c+\lambda+2} = 0 \]...........(9)

respectively, where

\[ \begin{align*}
  b_\lambda^{(3)} &= (C + \lambda)(C + \lambda + 1)(C + \lambda + 2)(C + \lambda + 3)(C + \lambda + 5), \\
  b_\lambda^{(2)} &= (C + \lambda)(C + \lambda + 1), \\
  b_\lambda^{(1)} &= (C + \lambda), \\
  T_4^{(1)} &= (1 - \eta), \\
  T_2^{(2)} &= 4\eta, \\
  T_2^{(3)} &= 2[\eta - \alpha^2(1 - \eta)], \\
  T_3^{(1)} &= -2\alpha^2\eta.
\end{align*} \]

For the series expression (8) to be solution, the coefficients of the powers of \( X \) in the equation (9) must be identically zero. Hence equating to zero the coefficient of the lowest power of \( X \), the indicial roots \( C=0, 1, 2, 3 \) are obtained. For higher power of \( X \), the constants \( a_0, a_1, a_2 \) are indeterminate for \( C=0 \) and these

\[ \begin{align*}
  &\left[ (C + \lambda + 6)(C + \lambda + 5)(C + \lambda + 4)(C + \lambda + 3)T_4^{(1)} \right] a_{\lambda+6} \\
  + &\left[ (C + \lambda + 4)(C + \lambda + 3)(C + \lambda + 2)(C + \lambda + 1)T_2^{(2)} \right] a_{\lambda+4} \\
  + &\left[ (C + \lambda + 2)(C + \lambda + 1)T_2^{(3)} \right] a_{\lambda+2} \\
  + &\left[ T_3^{(1)} \right] a_{\lambda} = 0 \\
\end{align*} \]...........(10)

If the notations:

\[ a_\lambda = g_\lambda^{(3)} a_3 + g_\lambda^{(2)} a_2 + g_\lambda^{(1)} a_1 + g_\lambda^{(0)} a_0 \text{ with } \lambda = 0, 1, 2, 3 \]

are introduced one finds that

\[ g_m^{(n)} = \begin{cases} 0, & m \neq n \\ m=0,1,2,3 \\ 1, & m=n \end{cases} \]

and \( g_\lambda^{(n)} (\lambda = 4, 5, 6 \ldots \text{and } n = 0, 1, 2, 3) \) are function of \( \eta, D_k, I^* \) and \( \Omega \)

The solution for \( W \), corresponding to \( C = 0 \), is

\[ W = a_0 G_0(X, \Omega) + a_1 G_1(X, \Omega) + a_2 G_2(X, \Omega) + a_3 G_3(X, \Omega) \]...........(11)
where,

\[
G_0(X, \Omega) = 1 + \sum_{\lambda=4}^{\infty} g_{\lambda}(0) X^{\lambda}
\]

\[
G_1(X, \Omega) = X + \sum_{\lambda=4}^{\infty} g_{\lambda}(1) X^{\lambda}
\]

\[
G_2(X, \Omega) = X^2 + \sum_{\lambda=4}^{\infty} g_{\lambda}(2) X^{\lambda}
\]

\[
G_3(X, \Omega) = X^3 + \sum_{\lambda=4}^{\infty} g_{\lambda}(3) X^{\lambda}
\]

.........(12)

It is evident that no new solution will arise corresponding to the indicial roots  C=1, 2, 3. The solution corresponding to these values of  C are already contained in the solution (11). The technique used by Lamb [15] has been applied for convergence of solution (11). If the limit of  \( \lambda \to \infty \) one finds that equation (11) is uniformly convergent for

\[
0 \leq X \leq 1 \text{ when } |\mu| < 1, \text{ Where, } \mu = \lim_{\lambda \to \infty} \left( \frac{a_{\lambda+1}}{a_{\lambda}} \right)
\]

IV. BOUNDARY CONDITIONS AND FREQUENCY EQUATION

The following combinations of boundary conditions at the edge  X = 0 and  X = 1 have been considered while the other two edges  Y = 0 and  Y = 1 are simply supported in all the cases.

(1) Clamped at both the edges  X=0 and  X=1 (C-SS-C-SS) (2). Clamped at  X=0 and simply supported at  X=1 (C-SS-SS-SS) and (3) Simply supported at both the edges  X=0 and  X=1 (SS-SS-SS-SS). The boundary conditions for different edge conditions are:

For clamped edge: \( w = 0 \) and \( \frac{\partial w}{\partial x} = 0 \) ..........(13)

For simply supported edge: \( w=0 \) and \( M_8 =0 \) ..........(14)

[C-SS-C-SS] – PLATES :

Using equation (11) and applying boundary conditions (13) one gets after eliminating  a_0, a_1, a_2 and a_3 then governing frequency equation is:

\[
\begin{bmatrix}
G_0(0,\Omega) & G_1(0,\Omega) & G_2(0,\Omega) & G_3(0,\Omega) \\
G_0'(0,\Omega) & G_1'(0,\Omega) & G_2'(0,\Omega) & G_3'(0,\Omega) \\
G_0(1,\Omega) & G_1(1,\Omega) & G_2(1,\Omega) & G_3(1,\Omega) \\
G_0'(1,\Omega) & G_1'(1,\Omega) & G_2'(1,\Omega) & G_3'(1,\Omega)
\end{bmatrix}
\]

...........(15A)

where, from (12)

\[
\begin{bmatrix}
G_0(0,\Omega)=1, G_1(0,\Omega)= G_2(0,\Omega)=G_3(0,\Omega)=0 \\
G_0'(0,\Omega)=1, G_1'(0,\Omega)= G_2'(0,\Omega)=G_3'(0,\Omega)=0
\end{bmatrix}
\]

..........(15B)

Therefore equation (15A) reduces to

\[
\begin{bmatrix}
G_2(1,\Omega) & G_3(1,\Omega) \\
G_2'(1,\Omega) & G_3'(1,\Omega)
\end{bmatrix}
\]

= 0

..........(16)
where a prime denotes the derivative with respect to x.

[C-SS-SS-SS] - PLATES:

Again using equation (11) and applying boundary conditions (13) and (14), the frequency equation for this plate after eliminating $a_0$, $a_1$, $a_2$ and $a_3$ and further using (15B) is

$$
\begin{bmatrix}
G_2(1,\Omega) & G_3(1,\Omega) \\
G_2'(1,\Omega) & G_3'(1,\Omega)
\end{bmatrix}
= 0
$$

..........(17)

[SS-SS-SS-SS] - PLATES:

Similarly using equation (11) and (13), the frequency equation for this plate after eliminating $a_0$, $a_1$, $a_2$ and $a_3$ and further using (12) at this edges, is

$$
\begin{bmatrix}
G_1(1,\Omega) & G_3(1,\Omega) \\
G_1'(1,\Omega) & G_3'(1,\Omega)
\end{bmatrix}
= 0
$$

..........(18)

V. RESULTS AND DISCUSSION

Numerical results for an isotropic, elastic, non-homogeneous, rectangular plate of uniform thickness have been computed from the equations (16), (17) and (18) when the temperature fields varies as parabolically, using latest computer technology for various combinations of thermal gradients $\eta$, damping parameters $D_{4}'$ and length to breadth ratio (a/b). In all the cases considered, the Poisson's ratio (0.3) and thickness (0.1) of the plate has been assumed to remain constant. Terms of the series up to an accuracy of $10^{-8}$ in their absolute values have been retained. The variation of frequency parameters $\Omega$ corresponding to the first four modes of vibration have been computed for (C-SS-C-SS), (C-SS-SS-SS) and (SS-SS-SS-SS) plates for different values of $\eta$ and $D_{4}'$. However the results of first two modes have been shown in the figures (1 to 4). Also the results for frequency parameters $\Omega$ with $\eta=0=D_{4}'$ and a/b = 0.25 have been compared with Soni [14]. The results, plotted in figures from 1 to 4 depicts the effect of damping parameter $D_{4}'$ on the frequency parameter $\Omega$ of rectangular plates of uniform thickness corresponding to the first four modes of vibration for various combinations of parabolic thermal gradient $\eta$ with (C-SS-C-SS), (C-SS-SS-SS) and (SS-SS-SS-SS) boundary condition. For (C-SS-C-SS), (C-SS-SS-SS) and (SS-SS-SS-SS) plates, it is observed that the frequency parameters $\Omega$ decreases with the increasing value of damping parameters $D_{4}'$ for heated aswell as unheated ($\eta=0$) plates. One can also noted that the for higher values of $D_{4}'$ the fall in frequency parameters $\Omega$ is very sharp specially for (SS-SS-SS-SS) boundary conditions when the $\eta$ is higher. The frequencies for (C-SS-C-SS) plate is higher than the corresponding to (C-SS-SS-SS) and (SS-SS-SS-SS) plates for all the four modes. The variation of $\Omega$ with thermal gradient $\eta$ for different values of damping parameter $D_{4}'$ and length to breadth ratio (a/b), is plotted in figures from 5 to 8 for all the three edge conditions for first four modes of vibrations. It is noted that the $\Omega$ decreases with the increasing value of thermal gradient $\eta$ for all the three edge conditions for first four modes of vibrations considered here. Hence it is clearly noted that the for first four modes of non homogeneous rectangular plate, as the value of $\eta$ and $D_{4}'$ increases the frequency parameter $\Omega$ also decrease for all the three cases of boundary conditions considered here. Further noted that the effect of parabolically distributed temperature has more effect on the frequency parameter $\Omega$ than the linearly distributed temperature field.

REFERENCES

FIG 2: VARIATION OF FREQUENCY PARAMETER ‘Ω’ WITH DAMPING PARAMETER ‘DK’ FOR AN RECTANGULAR PLATE CORRESPONDING TO SECOND MODE OF VIBRATION UNDER THE PARABOLIC THERMAL GRADIENT ‘η’. [LEGEND ; H=0.1,a/b=0.25,m=1.0, V=0.3]

FIG 3: VARIATION OF FREQUENCY PARAMETER ‘Ω’ WITH DAMPING PARAMETER ‘DK’ FOR AN RECTANGULAR PLATE CORRESPONDING TO THIRD MODE OF VIBRATION UNDER THE PARABOLIC THERMAL GRADIENT ‘η’. [LEGEND ; H=0.1,a/b=0.25,m=1.0, V=0.3]
FIG 4: VARIATION OF FREQUENCY PARAMETER ‘Ω’ WITH DAMPING PARAMETER ‘ΔK’ FOR AN RECTANGULAR PLATE CORRESPONDING TO FOURTH MODE OF VIBRATION UNDER THE PARABOLIC THERMAL GRADIENT ‘η’.
[LEGEND: H=0.1, a/b=0.25, m=1.0, V=0.3]

FIG 5: VARIATION OF FREQUENCY PARAMETER ‘Ω’ WITH PARABOLIC THERMAL GRADIENT ‘η’ OF A DAMPED RECTANGULAR PLATE CORRESPONDING TO FIRST MODE OF VIBRATION.
[LEGEND: H=0.1, a/b=0.25, m=1.0, V=0.3]
FIG 6: VARIATION OF FREQUENCY PARAMETER ‘Ω’ WITH PARABOLIC THERMAL GRADIENT ‘η’ OF A DAMPED RECTANGULAR PLATE CORRESPONDING TO SECOND MODE OF VIBRATION. [LEGEND; H=0.1, a/b=0.25, m=1.0, V=0.3]

FIG 7: VARIATION OF FREQUENCY PARAMETER ‘Ω’ WITH PARABOLIC THERMAL GRADIENT ‘η’ OF A DAMPED RECTANGULAR PLATE CORRESPONDING TO THIRD MODE OF VIBRATION. [LEGEND; H=0.1, a/b=0.25, m=1.0, V=0.3]
FIG 8: VARIATION OF FREQUENCY PARAMETER ‘Ω’ WITH PARABOLIC THERMAL GRADIENT ‘η’ OF A DAMPED RECTANGULAR PLATE CORRESPONDING TO FOURTH MODE OF VIBRATION.

[LEGEND: H=0.1, a/b=0.25, m=1.0, V=0.3]