Volume-4, Issue-1, February-2014, ISSN No.: 2250-0758
International Journal of Engineering and Management Research
Available at: www.ijemr.net
Page Number: 285-293

Deformation and Stress Analysis under Thermal Loads of Functionally Graded Plates using Finite Element Method

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ABSTRACT

A finite element model is constructed and developed to study the behavior of a FGM plate under thermomechanical loading in. A simply supported plate subjected to mechanical and thermal loads on its top and bottom surface is considered with suitable temperature and loading function that satisfy boundary conditions are used in this paper. The continuum is divided into 3600 elements and 4410 nodes using eight node hexahedral elements. The first-order shear deformation plate model is exploited to investigate the uncoupled thermo mechanical behavior of functionally graded plates. A study is performed to illustrate the effect of thermal load and mechanical load in thickness direction of FGM plate. The volume fraction of the two constituent materials of the FGM plate is varied smoothly and continuously along the thickness of the plate. Results are computed for a FGM plate with material properties varying in the thickness direction.

***Keywords*----** functionally graded material (FGM); finite element method (FEM); thermomechanical analysis.

I. INTRODUCTION

Functionally graded materials (FGM) are composite materials, in which the volume fraction of the material is chosen as a variable function of the material position in one or more dimensions of the structure. These materials are mainly find applications in high temperature environments. These were developed to reduce such thermal stress and resist super high temperature [1].To reduce stress and resist super high temperature, they have continuous transition from metal at low temperature surfaces to ceramics at high temperature surfaces because it is used widely in high temperature working environment such as aviation and nuclear reactor and so on, it is important to analyze the temperature and thermal stress field of the body made of the materials. Yangjian et. al. [2] researched the effect of FGM layer thickness on temperature field by finite element method. In their research they concluded that with increase in FGM layer thermal stresses reduced to a safe limit. R. Ramkumar and N. Ganesan [3] studied the problem of buckling in thin walled box columns under thermal environment by using CLPT theory; they use this theory with different software packages. Chen and Tong [4] used a graded finite element approach to analyze the sensitivity in the problems of steady state and transient heat conduction in FGM .The problem of optimization of FGM can be formulated using this solution, yielding the rate of changes in the response with respect to design variables. Thermal analysis of FGM plates are reported in [5-7]. Thermo-mechanical analysis and thermal bending of FGM plates using the first-order shear deformation theory (FSDT) using finite element methods are discussed in [8-10]. Analytical methods are also used to study thermo elastic behavior [11-14]. A high-order control volume finite element method is proposed in [15] for thermal stress analysis in functionally graded materials.

II. THEORETICAL DEVELOPMENT AND FORMULATION

Functionally graded plate with non homogeneous thermal and material properties in thickness direction is taken as shown in figure

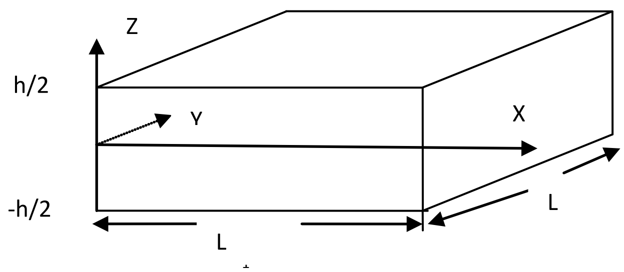

Fig.1 FGM plate

The analysis is based on the, uncoupled, quasi-static, first-order shear deformation plate theory wherein both, the change in temperature of the elastic body due to elastic deformations, and the inertia term in the equations of motion, are neglected. Material properties varied in the thickness direction according to volume fraction power law distribution such that top layer is made up of ceramic and graded to metal at the bottom surface. The volume fraction is expressed as:

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^n$$

Where n is the volume fraction index. Temperature field is assumed to vary in thickness direction such that is dT/dx = dT/dy = 0.The equation governing the steady state heat conduction in FGM with heat source strength, by taking continuous material properties distribution along thickness, can be written as;

$$\frac{d}{dt}\left(k(z)\frac{dT}{dt}\right) = 0$$

T (z) is the temperature distribution through *z* direction. So the temperature distribution through the plate thickness for any distribution of k (z) can be written as:

$$T(\mathrm{z}) = \mathrm{T}_c - \frac{T_c - T_m}{\int_{-h/2}^{h/2}(dz/k(z))} \int_z^{-h/2} dz/k(z)$$

where

$$k(\mathrm{z}) = (k_c - k_m)\left(\frac{2z+h}{2h}\right)^n + k_m$$

And the thermal boundary conditions are imposed as

$T=T_c$ at z= +h/2

$T=T_m$ at z= -h/2

Physical material properties are given in Table 1. The temperature of the top surface (ceramic) is 300°C and the temperature of the metal rich top surface is 20°C.

Table1: Material properties

Property	Aluminum	Zirconia
Thermal conductivity	204W/mK	2.09W/mK
Thermal expansion	23×10^{-6}/°C	10×10^{-6}/°C
Poisson's ratio	0.3	0.3
Young's modulus	70Gpa	151Gpa

III. FINITE ELEMENT MODEL

The displacements and rotations at any point into a finite element e may be expressed, in terms of the *n* nodes of the element as follows

$$\begin{Bmatrix} M_x \\ M_y \\ M_z \\ \phi_x \\ \phi_y \end{Bmatrix} = \sum_{i=1}^{n} \begin{bmatrix} \varphi_i^e & 0 & 0 & 0 & 0 \\ 0 & \varphi_i^e & 0 & 0 & 0 \\ 0 & 0 & \varphi_i^e & 0 & 0 \\ 0 & 0 & 0 & \varphi_i^e & 0 \\ 0 & 0 & 0 & 0 & \varphi_i^e \end{bmatrix} \begin{Bmatrix} M_{xj} \\ M_{yj} \\ M_{zj} \\ \tau_j^1 \\ \tau_j^2 \end{Bmatrix}$$

Where φ_i^e is the lagrange interpolation function at nodes. The lagrange interpolation function for eight node hexahedron element are given in terms of natural coordinates.

$$\begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \\ \varphi_7 \\ \varphi_8 \end{Bmatrix} = \frac{1}{8} \begin{bmatrix} (1-\xi)(1-\eta)(1-\mu) \\ (1+\xi)(1-\eta)(1-\mu) \\ (1+\xi)(1+\eta)(1-\mu) \\ (1-\xi)(1+\eta)(1-\mu) \\ (1-\xi)(1-\eta)(1+\mu) \\ (1+\xi)(1-\eta)(1+\mu) \\ (1+\xi)(1+\eta)(1+\mu) \\ (1-\xi)(1+\eta)(1+\mu) \end{bmatrix}$$

Using the minimum potential energy principle we have

$$[[K_b]+[K_s]]\{M_x^0\} = \{E_T\}+\{P\}$$

where

$[K_b]$ and $[K_s]$ are the element bending and shear stiffness matrices, which are defined as

$$[K_b] = \int_A [B_b]^T [DE_b][B_b]dA$$

$$[K_s] = \int_A [B_s]^T [DE_s][B_s]dA$$

And $\{E_T\}$ and $\{P\}$ are the element thermal and mechanical load vectors respectively, defined as

$$\{P\} = \int_A [\varphi_i^e]^T \{p\}dA$$

$$\{E_{Tb}\} = \int_A [B_b]^T \{DT_b\}dA$$

$$\{E_{Ts}\} = \int_A [B_s]^T \{DT_s\}dA$$

IV. NUMERICAL RESULTS

A computer program has been developed in Abacus environment. The validation of the proposed model is examined by comparing the results which are present in the literature. An eight noded isoparemetric element with 12 degrees of freedom per node for the model has been used for the plate. Mechanical and thermal loadings are imposed on the plate under consideration.

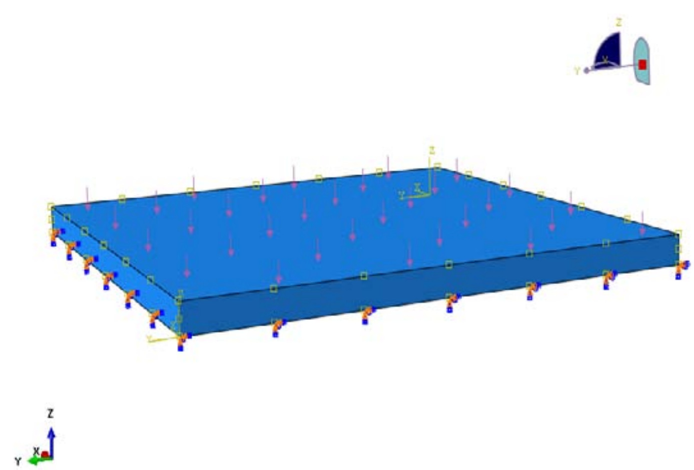

Figure 2: A simply supported FG plate under mechanical and thermal loads.

4.1 Temperature distribution in FGM Plate without heat source

The temperature distribution is a function of material parameter n as shown in Figure 3 as the value on n increases the temperature distribution becomes non linear. The temperature distribution in FG plates is smaller than those of homogeneous plates, which are made up of purely homogeneous plates. The grading parameter n has dominant effect on the thermal analysis of the FG plates, where the temperature distribution depends on the variable thermal conductivity of the FGM and the temperature of both the ceramic and metallic surfaces.

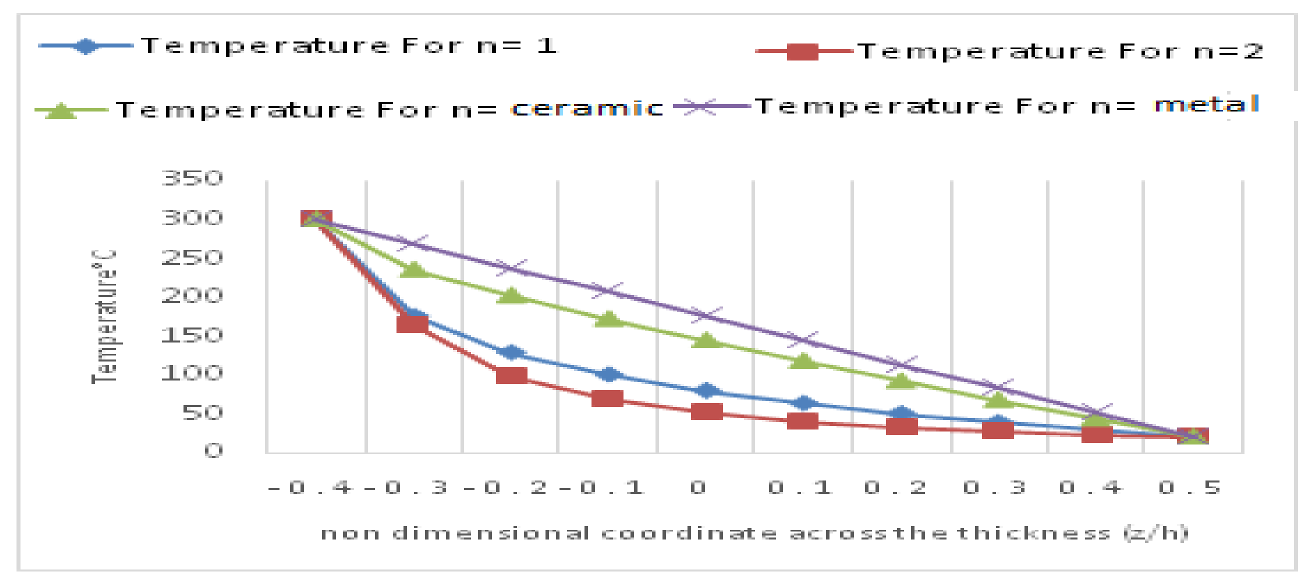

Figure 3: Temperature through the FGM plate thickness for various values of grading parameter *n* without heat source

***4.2 Temperature distribution in FGM Plate with heat source*.**

Figure 4 shows the temperature distribution for the FG plate. It is observed that the temperature distribution in FG plates is lower than corresponding to a purely ceramic or metallic plate. Figure 3 shows the temperature distribution for a steady state condition of the functionally graded material plate without heat source. However, in Figure 4 the temperature distribution for metal through the plate thickness is nearly a linear function. The reason for the large difference in the temperature distributions for isotropic materials is due to the large variation in the values of thermal conductivity for the two materials, as the temperature distribution depends on the thermal conductivity of the material. Figure 5 also shows deflections of the FGM plate due to thermo-mechanical Load.

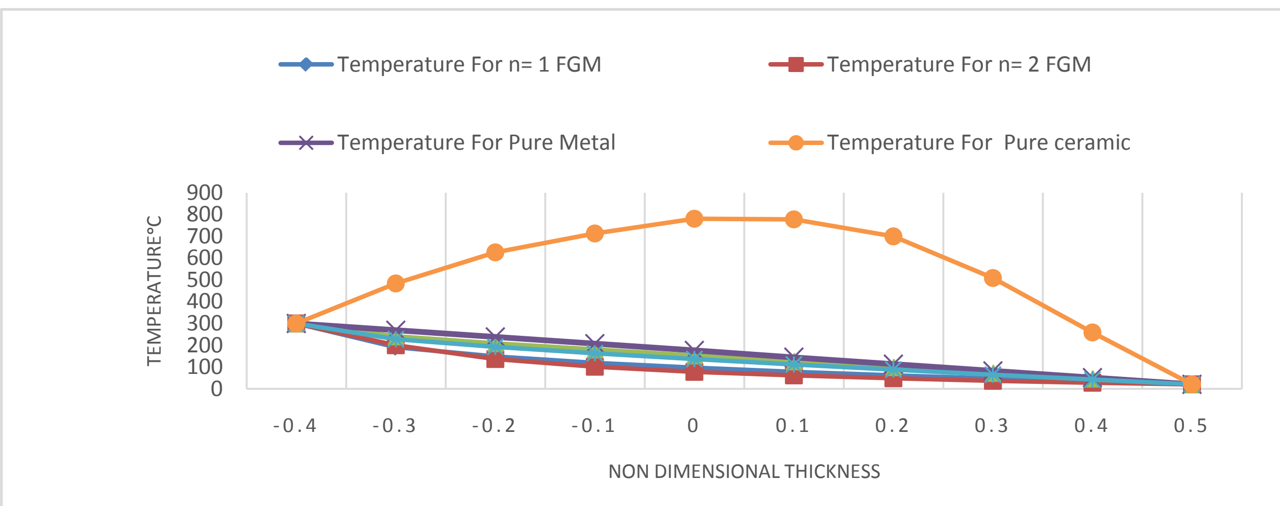

Figure 4: Temperature distribution through the thickness of FGM plate for different values of n.

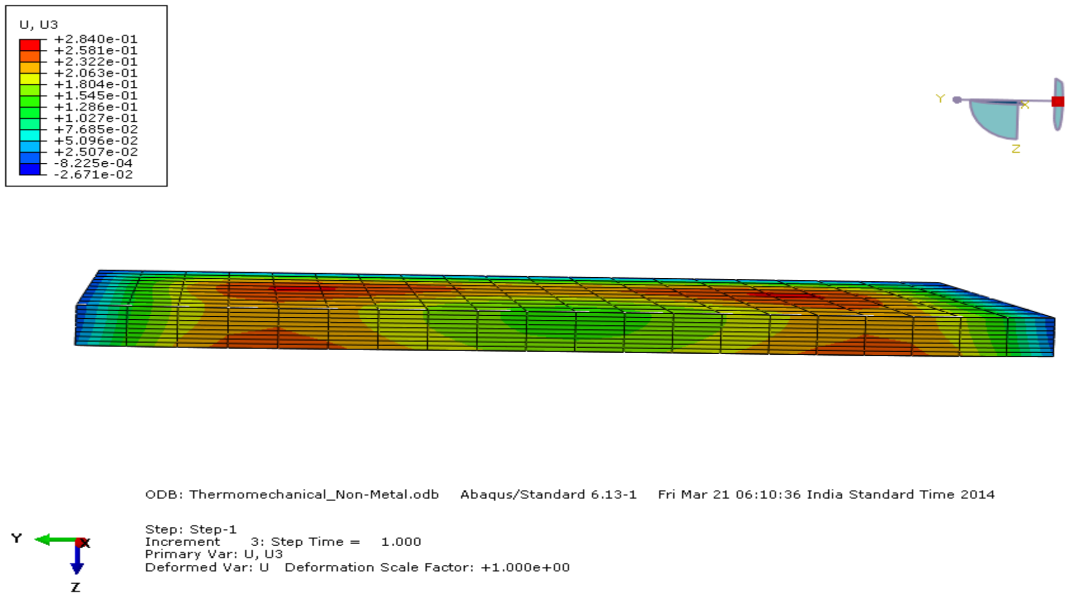

Figure 5: Deflection of Body due to Thermo-Mechanical Load.

***4.3 Deformation of FGM plate under Mechanical Loading*.**

To investigate the thermo elastic behavior of FG plates, with Aluminum at the bottom and Zirconia at the top of the plate is chosen, several numerical simulations are carried out, for different values of the grading parameter *n*. The following non dimensional parameters are used throughout the numerical simulations.

Non dimensional load intensity $\bar{P} = L^4 p/E_m h^4$;

Central deflection $\bar{w} = w/h$,

Thickness $\bar{h} = h/L$

Where p is load intensity and E_m is the young's modulus of the Aluminum bottom face.

It can be noticed from Figure 6 that the central deflection of a decrease by increasing the volume fraction index (*n*), due to increased material rigidity. As the material changes from metal to ceramic from bottom to top face of the FG plate the rigidity of the material increases because ceramic are more rigid than metals. Table 2 shows the numerical results of a set of numerical simulations with different values of (*n*).The results of Figure 6 are consistent with that of presented in [16].

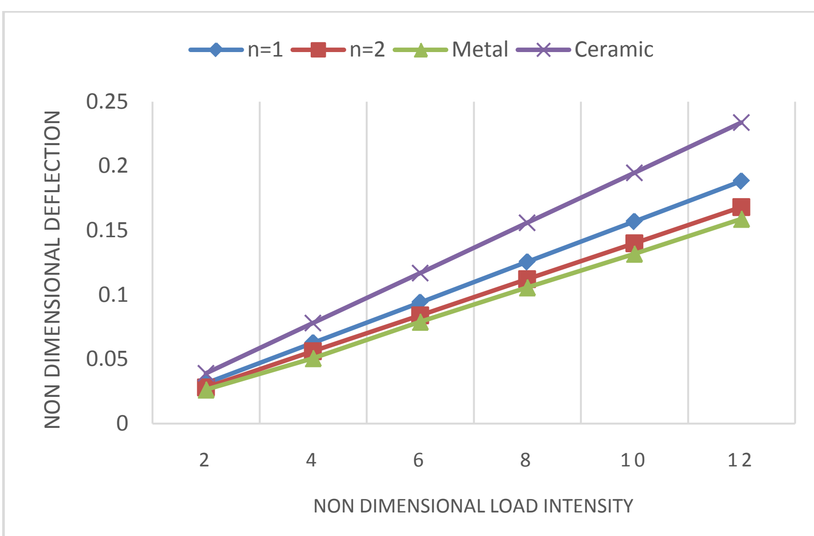

Figure 6: Non dimensional central deflection of FG plate versus non dimensional load intensity.

4.4 Deformation of FGM plate under Thermo-mechanical loading.

Figure 7 shows the central deflection due to a series of mechanical loads for different values of grading parameter *n*. In addition to the uniformly distributed load on the top surface, the plate is subjected to a thermal load where the ceramic rich top surface is held at 300°C and metal rich bottom surface is held at 20°C. It can be observed from figure that materials having properties in between ceramic and metal constituents shows intermediate values of deflection. This was expected as the metallic plate is one with minimum stiffness and ceramic plate with maximum stiffness. The results are in good agreement with those presented by Croce and venini [16] shown in Table 1.

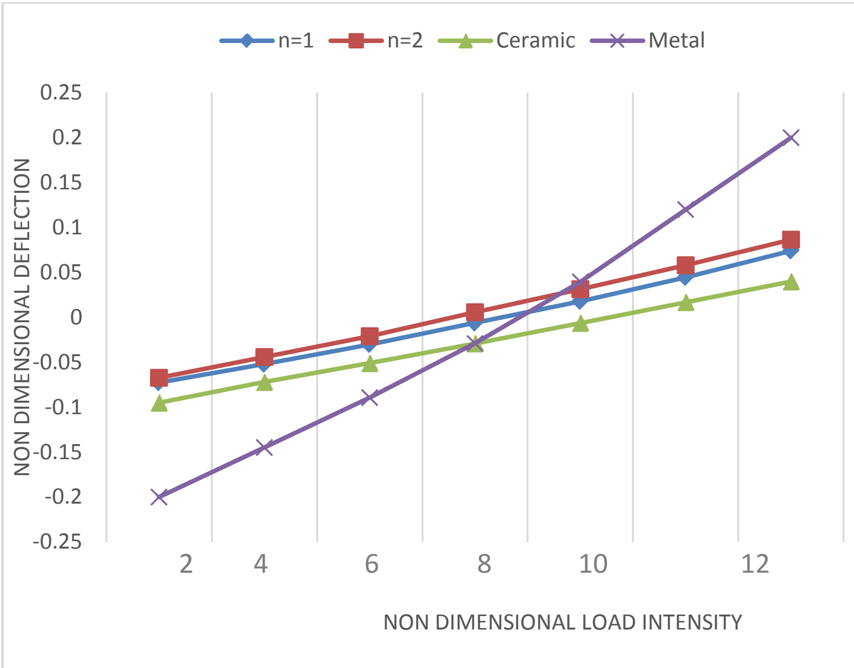

Figure: 7 Non dimensional deflection of FG plate in steady state without heat source versus non dimensional load intensity.

Table2: Non dimensional deflection versus non dimensional load intensity.

	Metal		n=0.5		n=1		n=2		Ceramic	
Load Intensity	Present	Established [16]	Present	Established[16]	Present	Established [16]	Present	Established [16]	Present	Established [16]
2	0.091	0.089	0.060	0.061	0.056	0.056	0.0532	0.0267	0.0210	0.0210
4	0.183	0.0180	0.121	0.124	0.113	0. 114	0.010	0.0534	0.0418	0.0420
6	0.275	0.269	0.1841	0.185	0.169	0.169	0.015	0.0801	0.0626	0.0629
8	0.366	0.361	0.242	0.248	0.226	0.228	0.212	0.1067	0.0834	0.0839
10	0.458	0.452	0.303	0.308	0.282	0.285	0.265	0.1334	0.1042	0.1049
12	0.550	0.54	0.363	0.365	0.339	0.343	0.318	0.313	0.1251	0.1258

4.5 Axial stresses in FGM plate under Thermo-mechanical Loading.

Figure 8 shows the variation of axial stresses at the centre of FG plate along the thickness direction for different values of grading factor n. The maximum tensile stresses along the thickness of FG plates are located at the bottom edge and increases with the modular ratio. The results of Figure 6 are in good agreement with established results. Axial stresses are also shown at the top and bottom of FG plate in Figure 9.

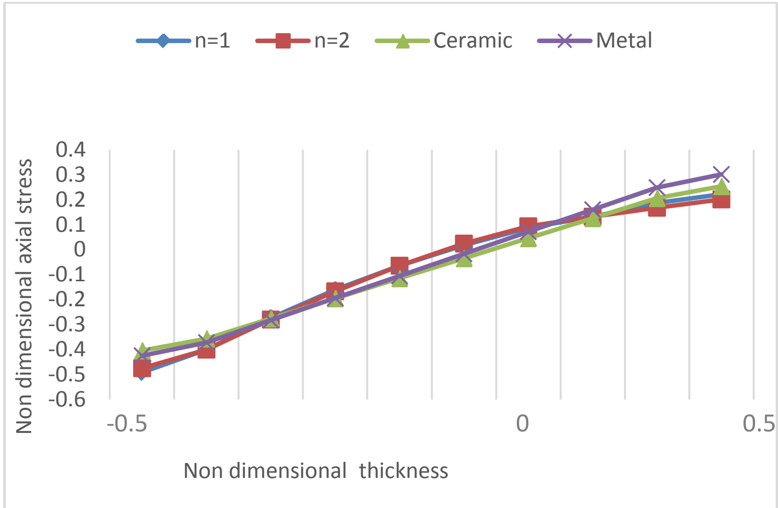

Figure 8: non dimensional axial stress distribution through the thickness of FGM plate.

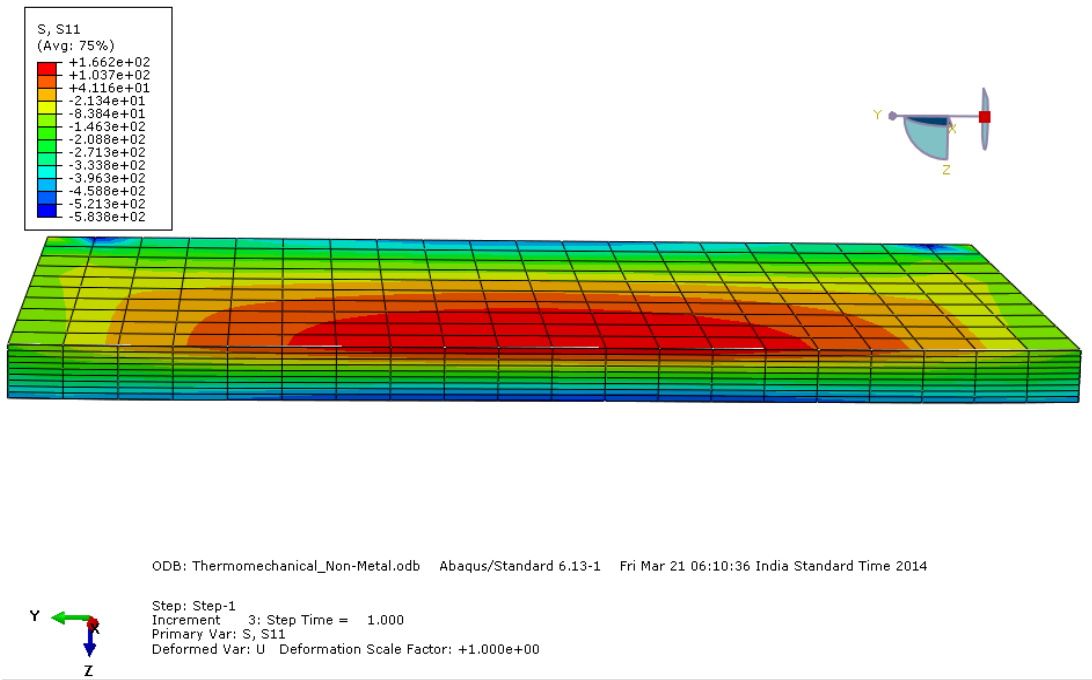

Figure 9: Axial stresses due to thermo mechanical load.

While increasing the value of material parameter *n*, the FG plate becomes more sensitive for the amount of heat delivered from the heat source. Figure 10 shows deflection due to sequence of mechanical loads for different values of n. Figure 11 shows the non dimensional deflection of the FG steady state plate with change in heat source strength. Figures 7 and 10 shows that FGM are more sensitive to change in heat source intensity with comparison to homogeneous materials, which have negligible effect of heat source intensity on their deformation Figure 11 also confirms that isotropic material gives no response for heat strength change. But FGMs provide a changeable deflection by changing the amount of heat source strength.

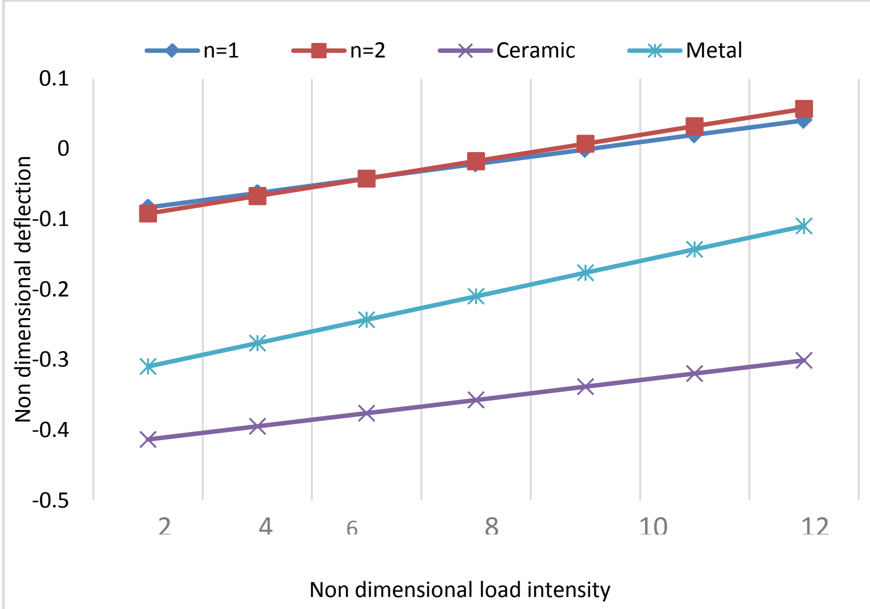

Figure 10: Non dimensional deflection of FGM plate with heat source versus load intensity

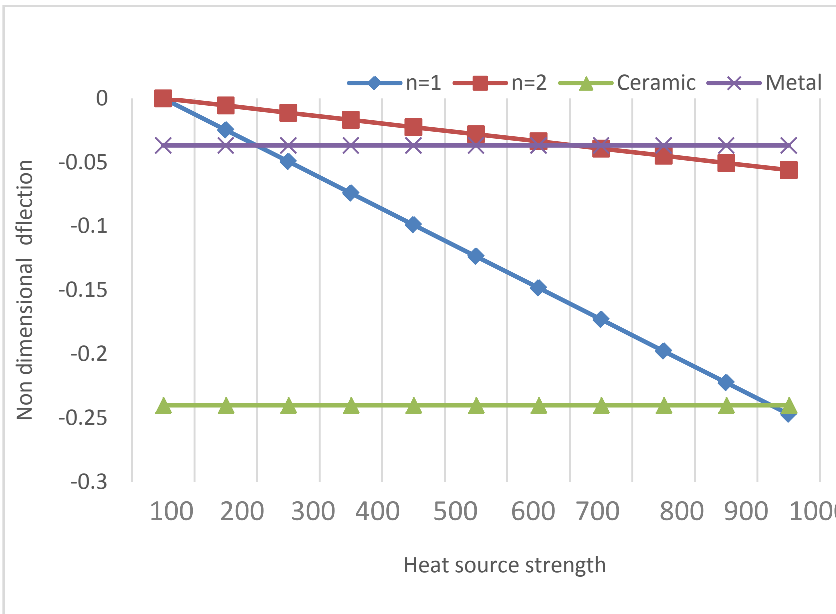

Figure 11: Non dimensional deflection versus the heat source strength.

V. CONCLUDING REMARKS

The behavior of FGM plate under thermal and mechanical loads is investigated and following conclusions are made:

1. The thermal load has dominant effect than mechanical load.
2. Functionally graded materials show a high ability to withstand thermal stresses, which reflect its capacity to operate at high temperatures.
3. The FGMs provide a high resistance to the thermal loading comparing to that of the isotropic materials.
4. The stresses are varied smoothly with comparison to conventional laminated plates without any sort of singularities due to the continuity of the material properties.

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