International Journal of Engineering and Management Research e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume- 9, Issue- 1, (February 2019)
www.ijemr.net https://doi.org/10.31033/ijemr.9.1.10

# An Iterative Model as a Tool in Optimal Allocation of Resources in University Systems

Onanaye, Adeniyi S.
Senior Lecturer, Department of Mathematical Sciences, Industrial Mathematics Programme, Redeemer's University, Osun State, NIGERIA

Correspondence Author: onanayea@run.edu.ng

**ABSTRACT**

**In this paper, a study was carried out to aid in adequate allocation of resources in the College of Natural Sciences, TYZ University (not real name because of ethical issue). Questionnaires were administered to the high-ranking officials of one the Colleges, College of Pure and Applied Sciences, to examine how resources were allocated for three consecutive sessions(the sessions were 2009/2010, 2010/2011 and 2011/2012),then used the data gathered and analysed to generate contributory inputs for the three basic outputs (variables)formed for the purpose of the study. These variables are: $X_1$ represents the quality of graduates produced; $X_2$ stands for research papers, Seminars, Journals articles etc. published by faculties and $X_3$ denotes service delivery within the three sessions under study. Simplex Method of Linear Programming was used to solve the model formulated.**

***Keywords--*** Optimal, Mathematical Model, Linear Programming, Resources, Allocation, Management, Redeemer's University.

Subject Classification Codes: 2010: 90C90

## I. INTRODUCTION

Linear Programming is a basis with which we can manipulate and control various activities in order to achieve optimal outcome for any problem. It deals with the optimization (maximization or minimization) of a function of variables known as objective functions [1]. Optimization problems consist of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function [2]. It includes finding the best available values of some objective function given a well-defined domain. An optimization problem in general is referred to as a linear mathematical programming problem and as such, many real world and theoretical problems can be modelled into a linear mathematical program.

In the application of optimization such as in the allocation of resources, optimizer or solvers are tools that help users find the best way to allocate those resources [3]. According Huankai et'al (2013), resource allocation optimization is a typical cloud project scheduling problem: a problem that limits a cloud system's ability to execute and deliver a project as originally planned. In their own view, Connor and Shah (2014) argued that to schedule a project effectively [4], project planners must select appropriate costing and resourcing options. This selection will determine the duration of the project. In most cases, projects have multiple costing and resourcing options which lead to multiple due dates [5].

These resources may be raw materials, machine time or people time, money or anything that is in limited supply. The best or optimal solution may mean profit maximization, cost minimization or achieving the best possible quality. Resource allocation may be decided by using computer programs applied to a specific domain to automatically and dynamically distribute resources to applicants. It may be considered as a specialized case of automatic scheduling and this is especially common in electronic devices dedicated to routing and communication. For example, channel allocation in wireless communication may be decided by a base transceiver station using an appropriate algorithm.

The College of Natural Sciences is one of the colleges in the Redeemer's University. It is made up of four departments which are: Mathematical Sciences, Biological Sciences, Chemical Sciences and Physical Sciences. If the resources given to the College of Natural Sciences are well allocated, it would make the learning process in the college more efficient and also make the college to achieve better outputs. A wide range of successful applications of optimization have been developed by businesses, governments, universities, industries and any other groups. Many large companies have reported saving billions of (Naira) Dollars using optimization.

For an allocation of resources to be optimal, some conditions that must be met are that:

- It must be an efficient allocation.
- The distribution of such allocation must be equitable (i.e. fair)
- It must be simple and not complex. etc.

In using an optimizer (iterative software tools), the user must build a model that species the:

- Resources to be used (using a decision variable)
- The limit of resource usage (constraints)
- The measure to optimize (objectives).

The optimizer finds values for the decision variables that satisfy the constraints while optimizing (maximizing or minimizing) the objective [3].

Iteration is defined as the procedure that involves repetitive steps in order to achieve the desired outcome. Sometimes iteration is often referred to as a loop. In constructing an iterative model as an approach for solving optimization in allocation of resources, a good iterative model must possess the following characteristics:

i. It should be communicable,
ii. It must not be too complex to understand, it should be simple and
iii. It should be able to give feedback as a measure of its progress [3].

Joiner in 2009 developed a mathematical model to determine the optimal structure (dollars, space) for allocating resource packages when recruiting new faculty, based on expected financial returns from those faculty using the University of Arizona College of Medicine as an illustrative case study (the model was applied there from 2005 to 2008), according to her, the model is a simple and flexible approach that can be adopted by other medical schools irrespective of the magnitude of the resources allocated [6]. Tarek in 1999 proposed improvement to resource allocation and levelling heuristics using the Genetic Algorithms (GA) to search for near-optimum solution, considering both aspects simultaneously. According to his work, the improved heuristics, random priorities were introduced into selected tasks and their impact on the schedule is monitored [7].

According to Zhu and Cipriano (2002), in their work on using mathematical optimization approach for resource allocation in large scale data centres using Hewlett Packard laboratory, Palo Alto as a case study centre. According to them, they addressed the resource allocation problem (RAP) for large scale data centres using mathematical optimization techniques given a physical topology of resources in a large data centre, and an application with certain architecture and requirements, so as to determine which resources in the physical topology should be assigned to the application architecture such that application requirements and bandwidth constraints in the network are satisfied, while communication delay between assigned servers is also minimized [8].Okonta and Chikwendu in2008used an iterative model for optimum allocation of government resources to the less privilege in Ethiope West Local Government Area of Delta State of Nigeria. In their methodology, four principal projects which are Education, Electricity, Water supply and Health care were put into key considerations. Budgeted amount and the actual expenditure between the year 2001 and 2006 were key parameters used by them making use of the Simplex Method of the linear programming problems to generate their iterative model [1].

Guptar and Hira in 1985defined operation research (OR) a study that encompasses a wide range of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency, such as simulation, mathematical optimization, queuing theory and other stochastic-process models, Markov decision processes, econometric methods, neural networks, expert systems, decision analysis, and the analytic hierarchy process [8]. Operation research gives executives the power to make more effective decisions and build more productive systems based on more complete data, consideration of all available options, careful predictions of outcomes and estimates of risk and the latest decision tools and techniques. Guptar and Hira again in 1985 described linear programming (LP or linear optimization) as a mathematical method for determining a way to achieve the best outcome (such as maximum profit or minimum cost) in a given mathematical model for some list of requirements represented as linear relationships. It is the process of taking various linear inequalities relating to some situation, and finding the "best" value obtainable under those conditions. More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints [8]. According to Robert (2007), a model is a miniature representation of something, a pattern of something to be made, an example for imitation or emulation, a description or analogy use to help visualize something [9]. Mathematically a model is a description of a system using mathematical concepts and languages, the process of developing a mathematical model is called mathematical modelling. Robert in 2007 defined Simplex method is an iterative procedure for solving Linear Programming Problems (LPP) with a finite number of steps [9]. This method provides an algorithm which consist of moving from one vertex of the region of feasible solution to another in such a manner that the value of the objective function at the succeeding vertex is less or more as the case may be than the previous vertex. The procedure is repeated and since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or indicates the existence of unbounded solution. According to Okonta and Chikwendu in 2008, said that sensitivity analysis deals with finding out the amount by which we can change the input data for the output of our linear programming model to remain comparatively unchanged [1]. This helps us to determine sensitivity of the data we supply for the problem. If a small change in the input produces a large change in the optimal solution for some model, and a corresponding small change in the input for some other model doesn’t affect its optimal solution as much, we can conclude that the second problem is less sensitive to the changes in the input data. A typical example of LP Model can be expressed as follows:

e-ISSN: 2250-0758 | p-ISSN: 2394-6962
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https://doi.org/10.31033/ijemr.9.1.10

Maximize Z: $\sum_{j=1}^{n} C_j X_j$

Subject to: $\sum a_{ij} X_j \leq b_i$ (1)

$X_j \geq 0$, $i = 1, 2, \ldots, m$

where: $X_j$ are the output variables from the system been modelled,

$a_{ij}$ are the input coefficients of $X_j$ as contributions to the objective function, Z .

$b_i$ are the quantities of expectations in each of the processes,

$C_j$ arethe marginal values of resources (inputs) available.

Now, in case of minimization models, the inequalities in (1) above do changed to greater than or equal to (≥).

## II. PROBLEM STATEMENT

Allocation of resources (resources such as capital, time, land, personnel, facilities etc) in an organization is not a small job. Correct allocation of such resources adequately in such a way that every department/unit is sufficiently satisfied cannot be over emphasized. Therefore, we want to make room for the best allocation of resources to the College of Natural Sciences in Redeemers University, Nigeria so that the college can carry out her duties more efficiently. In order to do this, we developed an iterative model for optimal allocation of those resources to the different departments in the college of Natural Sciences.

### *2.1 Aim and Objectives*

The aim of this paper is to aid in adequate and correct allocation of resources in the college of Natural Sciences in the Redeemers University and by extension to other colleges/departments/units in the University and any other organisations both in public and private.

To achieve the above aim, we carried out the following objectives:

- Based on existing model of resource allocation method, a new model was designed to improve the existing one in resources allocation.
- We recommend areas that should have more input of resources so that the College would achieve better outputs.

## III. METHODOLOGY

Data generated by questionnaire were used to formulate the iteration model used for the study. The mainstream resources allocated and available at the College of Natural Sciences of the University include academic and non-academic staff strength; library facilities and journals; lecture halls; laboratories; transportation; utilities, furniture, office and residential accommodations; internet and intercom services and as such. The questionnaire were administered to the high ranking officials such as the Dean of the College, Head of Departments (HOD’s) and the College Officer based on a three-session academic school calendar (i.e. 2009/2010, 2010/2011 and 2011/2012 sessions were used for this study). The generated information from the questionnaire was defined as the primary data while the journal articles, personal observations and interviews were defined as the secondary data for the study. The model generated from the available information was solved by using an iterative tool called Simplex Method (SM) of the Linear Programming model.

### *3.1 Formulated Mathematical Model*

Linear programming problems (LPP) of the Simplex method involves the optimization of a linear function, called the objective function which is subject to some linear constraints, which may either be equalities or inequalities in the unknowns.

The objective function is of the form:

Maximize Z: $\sum_{j=1}^{n} C_j X_j$ (2)

Subject to: $\sum a_{ij} X_j \leq b_i$

$X_j \geq 0$, $i = 1, 2, \ldots, m$

where $X_j$ is the output based on the iteration model derived from the three sessions academic University calendar; $a_{ij}$ is the input allocated resources based on the information from the questionnaire.

$b_i$ is the quantity of the resources allocated from sessions 2009/2010 – 2011/2012

$C_j$ is the marginal value of resources available being derived by ranking in order of needs.

$a_j$ and $b_i$ were obtained as the objective and subjective allocated resources respectively.

Thus, the linear function to be maximized is mathematically given as:

$$\text{Max } Z = C_1X_1 + C_2X_2 + C_3X_3$$

Subjects to the constraints:

$$\begin{aligned} A_{11}X_1 + A_{12}X_2 + A_{13}X_3 &\le b_1 \\ A_{21}X_1 + A_{22}X_2 + A_{23}X_3 &\le b_2 \\ A_{31}X_1 + A_{32}X_2 + A_{33}X_3 &\le b_3 \end{aligned} \quad (3)$$

where:

$X_1$ = Quality of graduate produced from the college

$X_2$ = Research papers, journals and seminars e.t.c

$X_3$ = Service delivery e.t.c

$X_j$ = Output

- For the quality of graduates produced from the college; the following were considered as input: the staff strength based on qualifications, productivities and years of experience including non-academic supporting staff; access to data base of high quality Journals for different areas of disciplines and access to internet; laboratories/equipment/consumables and hostel accommodations.
- For quality of researches done, journals articles published and seminars presentations; the following were considered as inputs: access to data base of high quality Journals for different areas of disciplines, access to internet, laboratories/equipment/consumables, research funds, and conducive office accommodations provided within each session
- For service delivery, the following were considered as inputs: transportation, residential accommodations, stationeries, computer systems, internet facility, and other utilities provided within each session.

$C_j$ = Marginal value of resources which is been derived based on the ranking of resources allocated

$b_i$ = calculated final points based on the inputs resources from the primary data.

***3.2 Theorems***

***Theorem 1***

The set of all feasible solutions to the linear programming problem (LLP) is a convex set.

(Source: Okonta and Chikwendu, 2008)

***Theorem 2***

If for any basic feasible solutions $X_0 = (X_{10}, X_{20}, ..., X_{m0}$, the conditions $Z_j - C_j \ge 0$ $(ie.\, C_j - Z_j \le 0)$ hold for $j = 1, 2, ..., n$, then a maximum feasible solution is has been obtained.

(Source: modified version of Okonta and Chikwendu, 2008)

## IV. SOLUTION TO THE PROBLEM FORMULATED

After analysing the input based for the proposed outputs, we were able to formulate the objective function to be considered and solved as shown below:

$$\text{Max } Z = 95.5x_1 + 75x_2 + 88x_3$$

subject to the constraints:

$$\begin{aligned} 19x_1 + 14x_2 + 14x_3 &\le 78 \\ 18x_1 + 15x_2 + 19x_3 &\le 84.2 \\ 18x_1 + 15_2 + 17x_3 &\le 83.1 \end{aligned} \quad (4)$$

**Hints**: From the above optimization model, it should be noted that:

e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume- 9, Issue- 1, (February 2019)
https://doi.org/10.31033/ijemr.9.1.10

- $b_i$ = calculated final points based on the inputs resources from the primary data (i.e questionnaire),
- 95.5, 75 and 88 were marginal values (i.e $C_1, C_2, and\ C_3$) of high ranking from the total or summation of inputs that contribute $x_1, x_2, x_3$ and as outputs respectively.
- The individual constraints analysed above were based on highest ranking from the respective catchment areas (Departments, College).
- In other to remove the inequalities signs in (4) above, we introduce slacks (dummy variables), $S_1, S_2, and\ S_3$ which now made us to re-write (4) as follows:

$$Max\,Z = 95.5X_1 + 75X_2 + 88X_3 + 0S_1 + 0S_2 + 0S_3$$

Subject to the constraints

$$19X_1 + 14X_2 + 14X_3 + S_1 + 0S_1 + 0S_1 = 78$$
$$18X_1 + 15X_2 + 19X_3 + 0S_1 + S_2 + 0S_1 = 84.2 \quad (5)$$
$$18X_1 + 15X_2 + 17X_3 + 0S_1 + 0S_1 + S_3 = 83.1$$

For non-negativity condition, it implies that:

$$X_1, X_2, X_3 \geq 0$$

We now construct the initial tableau for the simplex method as follows:

**Table 1 Initial Simplex Table**

| **Column $c_i$** | | | **95.5** | **75** | **88** | **0** | **0** | **0** | |
|---|---|---|---|---|---|---|---|---|---|
| **row** | **$c_i$** | **Solution** | ***$X_1$*** | ***$X_2$*** | ***$X_3$*** | ***$S_1$*** | ***$S_2$*** | ***$S_3$*** | ***$P_0$*** |
| **1** | **0** | ***$S_1$*** | **19** | **14** | **14** | **1** | **0** | **0** | **78** |
| **2** | **0** | ***$S_2$*** | **18** | **13** | **19** | **0** | **1** | **0** | **84.2** |
| **3** | **0** | ***$S_3$*** | **18** | **15** | **17** | **0** | **0** | **1** | **83.1** |
| **$z_j$** | | | **0** | **0** | **0** | **0** | **0** | **0** | **0** |
| **$c_i$-$z_j$** | | | **95.5** | **75** | **88** | **0** | **0** | **0** | |

Since the entries in $c_i$-$z_j$ row in the above table contains elements that are positive, [that is for us to have optimal solution all none of the entries in the $c_i$-$z_j$ row must positive ($c_i$-$z_j$< 0 or =0)] it shows that table is not for optimal solution. We therefore introduced $X_1$ into the solution column because it has the highest value of coefficient in the objective function in (4) above. We also determined the slack to be removed for $X_1$ by dividing all entries in $P_0$ by all the entries in $X_1$

$$\frac{78}{19} = 4.105 \ldots\ldots\ldots\ldots\ldots\ldots(S_1)$$

$$\frac{84.2}{18} = 4.678 \ldots\ldots\ldots\ldots\ldots..(S_2)$$

$$\frac{83.1}{18} = 4.617 \ldots\ldots\ldots\ldots\ldots\ldots(S_3)$$

We remove the slack $S_1$ in the solution column because of its lowest ratio.
Based on this, we then reconstruct the simplex tableau as follows:

**Table 2 Simplex Tableau after the First Iteration**

| Column $C_i$ | | | 95.5 | 75 | 88 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|---|
| row | $C_i$ | Solution | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | $S_3$ | $P_0$ |
| 1 | 95.5 | $X_1$ | 1 | 0.737 | 0.737 | 0.052631579 | 0 | 0 | 4.1052632 |
| 2 | 0 | $X_3$ | 0 | -5 | 109 | -18 | 19 | 0 | 195.8 |
| 3 | 0 | $X_2$ | 0 | 33 | 71 | -18 | 0 | 19 | 174.9 |
| $Z_j$ | | | 95.5 | 70.37 | 73.37 | 5.026315789 | 0 | 0 | 392.05263 |
| $C_i$ - $Z_j$ | | | 0 | 4.632 | 17.63 | -5.02631579 | 0 | 0 | |

Again the entries in $c_i$-$z_j$ row in the above table still contains some elements that are positive, it shows that table is not yet for optimal solution. We repeated the same procedures as mentioned in the first iteration until optimal solution was reached after the fifth iterations and the final Simplex tableaus shown below:

**Table 3 Final Simplex Tableau**

| Column $C_i$ | | | 95.5 | 75 | 88 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|---|
| row | $C_i$ | Solution | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | $S_3$ | $P_0$ |
| 1 | 95.5 | $X_1$ | 1 | 0 | 0 | 0.307692309 | 0.134615384 | -0.40384615 | 1.775 |
| 2 | 88 | $X_3$ | 0 | 0 | 1 | -0.17307692 | 0.158653846 | 0.024038462 | 1.85625 |
| 3 | 75 | $X_2$ | 0 | 1 | 0 | -0.17307692 | -0.34134615 | 0.524038462 | 1.30625 |
| $Z_j$ | | | 95.5 | 75 | 88 | 1.173077232 | 1.216346371 | 2.850961562 | 430.83125 |
| $C_i$ - $Z_j$ | | | 0 | 0 | 0 | -1.17307723 | -1.21634637 | -2.85096156 | |

Since all entries in $c_i$-$z_j \leq 0$ , then the iterations in the table had produced the optimal solution.

***Interpretation of Result***

Considering the final or optimal tableau above, the optimal values for decision variables are $x_1$=1.775, $x_2$ =1.30625, and $x_3$ =1.85625 with value of objective function as Z=430.83125. From the above analysis we can then say that $x_3$ has the highest value of quality of output which connotes that services delivery has greater input of resources followed by $x_1$ and then $x_2$ in that order respectively. For the best or optimal allocation of resources, the values of $x_1, x_2$ and $x_3$ are meant to be at equilibrium or almost equilibrium. This implies that the management of the College had reasonable resources that were evenly distributed among the four Departments and the College Office. However, we strongly recommend that the University, through the office of the Dean of College of Natural Sciences, should deploy more resources for better outputs in future and that this study could be extended to the entire University to test the effectiveness of allocation of resources vis-a-vis her outputs.

## V. CONCLUSION

In conclusion we were able to analyse the existing model, identifying near optimal allocation of the available resources by the management of the College, and we also recommended hat more resource should be deployed in the College by the University management for better outputs in future and further research on the subject matter to cover the entire University.

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## APPENDIX

***1) Proof of Theorems***

- **Proof of Theorem One:** In general, let $[x_i]_{i=1}^{k}$ be a family of feasible solutions and let $a_i \in [0,1]$ for all $i = 1, 2, \ldots, k$ such that $a_1 + a_2 + \ldots + a_3 = 1$ where $x = \sum a_1 x_1$, then it implies that $Ax = a_i A x_i \underset{\geq}{\overset{\leq}{-}} a_i \delta = b$ where $x \leq 0$.

- **Proof of Theorem Two:** let

$$P_0 = \sum_{i=1}^{n} y_i P_i \quad \text{(i)}$$

and

$$Z_0 = \sum_{i=1}^{n} y_i C_i \quad \text{(ii)}$$

where Z is the corresponding value of the objective function. Therefore, by hypothesis if $Z_j - C_j \geq 0$

$(ie.\, C_j - Z_j \leq 0) \Rightarrow Z_j \geq C_j$ then $Z_0 = \sum y_0 Z_j \geq Z$.

Using equations (i) and (ii) to obtain the equation below, it implies that:

$$y_{10} \sum_{i=1}^{n} X_{11} P_i + y_{20} \sum_{i=1}^{n} X_{12} P_i + \ldots) + y \sum_{i=1}^{n} X P_i = P_0 \quad \text{(iii)}$$

Given that $P_0 = x_{10} P_1 + x_{20} P_2 + \ldots + xP$

Since, $P_1, P_2, \ldots, P_m$ are linearly independent, we can equate the co-efficient of equation (iii) which becomes:

$$x_{10} c_1 + x_{20} c_2 + \ldots + x_0 c_0 = Z$$