Research on the Stability of the Grade Structure of a University Title

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ABSTRACT
The evaluation of college teachers’ title is an extremely important task in personnel management, and it is also the focus of college teachers and other professional and technical personnel. The work is policy-oriented and involves a wide range of issues. Whether it is scientific, fair and reasonable is related to whether it can mobilize the enthusiasm and creativity of teachers. It is related to the construction of the teaching staff and the adjustment of the academic echelon, which is of great significance for promoting the sustainable development of higher education.

Keywords— Title Hierarchy, Stable Domain, Transfer Ratio

I. INTRODUCTION

Therefore, this paper hopes to use mathematical model to study the stability of university title hierarchy. To study the stability of the title classification structure, we first need to establish a hierarchical structure mathematical model, find the stability domain of the existing hierarchical structure by calculating the basic equations, and then judge whether the structural vector is in the stable domain, if unstable reconstruction Stable domain.

II. ESTABLISHMENT OF PROFESSIONAL TITLE HIERARCHY MODEL

Set a social system from low to high into K grades, such as university teachers have junior, intermediate, and senior positions. Time is discretized in years, that is, it is performed annually and only once. The level is recorded as \( i = 0,1,2... \) and the time is recorded as \( t = 0,1,2... \). There are two factors that cause the change of the hierarchical structure: one is the conversion between the internal levels of the system, that is, the promotion or degradation; the second is the communication inside and outside the system, that is, the transfer or exit (including transfer, retirement, death, etc.). The personnel of all levels of the system change according to a certain proportion in each period, which is a deterministic transfer problem.

Basic quantity expression:

The total number of people who quit the system in \( t \) year:

\[
W(t) = \sum_{i=1}^{k} w_i \cdot n_i(t) = n(t)w^T;
\]

The total number of people \( N(t) : N(t+1) = N(t) + R(t) - W(t) \);

Transfer equation for each level of people:

\[
n_j(t+1) = \sum_{i=1}^{k} p_{ij} \cdot n_i(t) + r_j \cdot R(t);
\]

Using vectors, matrix symbols can be expressed:

\[
n(t+1) = n(t)Q + R(t)r;
\]

Total number of people transferred in \( t \) years:

\[
R(t) = W(t) + M(t) = n(t)w^T + M(t);
\]

Arrange to get:

\[
n(t+1) = n(t)(Q + w^T r) + M(t)r .
\]

At this time to remember \( P = Q + w^T r \).

When the system internal transfer ratio matrix \( Q \) is known, the ratio \( r \) is adjusted, the initial membership level distribution \( n(0) \) and the total number of people per year \( R(t) \) or the total annual growth amount \( M(t) \), the members can be calculated. Then we can calculate the change of the membership level distribution \( n(t) \) and the basic equation of the hierarchical structure. When the percentage of the total system population increases by \( \alpha \) per year, which is \( M(t) = \alpha N(t) \), then you can use the member's hierarchical structure \( a(t) \) instead of \( n(t) \) to get:

\[
a(t+1) = (1 + \alpha)^{-1} [a(t)p + ar].
\]

If the number of people entering and leaving the system is roughly equal each year,
it can be simplified to assume that the total number of people in the system \( N(t) \) remains the same, which is \( M(t) = 0 \) (or \( a = 0 \) ). Then, there are

\[
a(t + 1) = a(t)P = a(t)(Q + w^T r).
\]

Note that the \( i, j \) element of the transfer matrix \( P \) of the hierarchical structure \( a(t) \) is \( p_{ij} + w_ir_j \), that is, the internal transfer ratio of the system \( p_{ij} \) plus the ratio \( w_ir_j \) of the internal and external communication of the system, which is discussed below on the basis of Equation \( a(t + 1) = a(t)P = a(t)(Q + w^T r) \).

Stabilize control with the ratio of the transfer, assuming that the ideal hierarchical structure of people in the actual problem is \( a^* \), because the hierarchical structure \( a(t) \) changes according to the law of \( a(t + 1) = a(t)P = a(t)(Q + w^T r) \), people naturally hope that once \( a^* \) is reached, it can be selected by The appropriate ratio is adjusted so that a remains the same. As you will see below, not any level structure can be controlled by the ratio control. Our purpose is to give an internal transfer ratio matrix \( Q = \{p_{ij}\} \), and study which hierarchical structure can be kept constant with the appropriate ratio of the transfer ratio, which is called the stable control of the scale-in structure. According to the equation, for a certain hierarchical structure \( a \), if there is a ratio \( r \) to \( a = (Q + w^T r) \) then \( a \) is called a stable structure. Note that \( r \) here must satisfy \( r_i \geq 0, \sum_{i=1}^{k} r_i = 1 \).

It is not difficult to obtain \( r = \frac{a - aQ}{aw^T} \). It can be seen that the norm of the stable structure is determined by \( a \geq aQ \), which is called the stable domain of the hierarchical structure.

### III. THE SOLUTION OF PROFESSIONAL TITLE RANK STRUCTURE

This paper mainly studies the three professional titles in a university: primary, intermediate, and advanced, which are represented by \( i = 1, 2, 3 \) respectively. The initial total number of people is known to be \( N_0 = 1854 \), the initial \( a_0 = (140, \text{774}, 940) \), the initial structure \( a_0 = (0.51, 0.41, 0.08) \), annual growth rate \( \beta = 0.03 \). The proportion of junior high-level upgrades is 20% per year, and the transfer ratio is 10%; the proportion of intermediate-level promotion is 15%, the transfer rate is 5%; and the high-level transfer rate is 5% (mainly retirement).

We get the annual transfer ratio matrix

\[
Q = \begin{pmatrix} 0.7 & 0.2 & 0 \\ 0 & 0.8 & 0.15 \\ 0 & 0 & 0.95 \end{pmatrix},
\]

solving the stability domain of the hierarchical structure, solution equation

\[
\begin{align*}
 a_1 & \geq 0.7a_1 \\
 a_2 & \geq 0.2a_1 + 0.8a_2 \\
 a_3 & \geq 0.1a_2 + 0.95a_3
\end{align*}
\]

Any hierarchical structure can be regarded as a point in the three-dimensional space, and is located in the plane of the first quadrant, that is \( a = (a_1, a_2, a_3) \). This is an equilateral triangle with vertices of \((1, 0, 0), (0, 1, 0), (0, 0, 1)\), and lay it on the plane, as shown in the figure, as A, called a feasible domain. In the stable domain found in A, draw two lines \( a_2 = a_1 \) and \( a_3 = 2a_2 \), that intersect at point \( s_1 \). The definition of the inequality is defined by the triangle with vertex \( s_1, s_2 = (0.1/3, 1/3) \) and \( s_3 \) (as shown in the figure), and B is the stable domain.

\[
\begin{align*}
 0 & \leq a_1 \\
 0 & \leq a_2 \\
 0 & \leq a_3
\end{align*}
\]

And can calculate the coordinates of \( s_i \) as \( (0.25, 0.25, 0.5) \). Now set the ratio of \( r = (0.8, 0.2, 0) \), you can get \( N_1 = 1910, n_i = (143, 659, 1009) \). Now ask for \( a_1 \) and determine if \( a_1 \) satisfies the stability domain \( (a_1 \geq a_0Q) \). Get \( a_{0Q} = (0.56, 0.34, 0.55) \) by MATLAB operation.
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\[
P = \begin{pmatrix} 0.78 & 0.22 & 0 \\ 0.04 & 0.81 & 0.15 \\ 0.04 & 0.01 & 0.95 \end{pmatrix}
\]

\[
Q = \begin{pmatrix} 0.7 & 0.2 & 0 \\ 0 & 0.9 & 0.05 \\ 0 & 0 & 0.95 \end{pmatrix}
\]

\[a(1) = (0.12, 0.35, 0.53), \text{ because } a_3 < 2a_2 \]

\[
a_2 \geq a_1
\]

satisfied \(a_3 \geq a_2\). From this result, we can see \(a_1 \not\in B\), so the structure is unstable. Below we analyze the reasons for the instability of the structure, we guess that it may be due to the instability of the title structure of colleges and universities. That is to say, the number of senior positions is relatively small. In order to achieve stability at a given transfer ratio, the structure can be stabilized by adjusting the number of transfers and the transfer rate. \(Q\) will be adjusted to

\[
\begin{align*}
Q & = \begin{pmatrix} 0.7 & 0.2 & 0 \\ 0 & 0.9 & 0.05 \\ 0 & 0 & 0.95 \end{pmatrix} \\
& \text{Repeat the above calculation process, the same reason can be obtained:}
\end{align*}
\]

\[
\begin{align*}
a_1 & \geq 0.7a_1 \\
a_2 & \geq 0.2a_1 + 0.9a_2 \\
a_3 & \geq 0.05a_2 + 0.95a_3 \\
\Rightarrow & \begin{cases} a_2 \geq 2a_1 \\ a_3 \geq a_2 \end{cases}
\end{align*}
\]

We can get a new stable domain, we can see that the current \(a(1) \in B\) then \(a(1)\) satisfies the stable domain. It shows that the adjusted grade structure is stable.

**IV. RESULTS ANALYSIS**

Now use MATLAB to perform multiple iterations and test the structure for the next 10 years as shown in the following table.

<table>
<thead>
<tr>
<th>(t)</th>
<th>(a(t))</th>
<th>Whether in the stable region</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(a(0) = (0.08, 0.42, 0.51))</td>
<td>Yes</td>
</tr>
<tr>
<td>1</td>
<td>(a(1) = (0.10, 0.40, 0.50))</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>(a(2) = (0.11, 0.39, 0.50))</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>(a(3) = (0.12, 0.39, 0.49))</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>(a(4) = (0.13, 0.38, 0.49))</td>
<td>Yes</td>
</tr>
<tr>
<td>5</td>
<td>(a(5) = (0.14, 0.38, 0.48))</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>(a(6) = (0.14, 0.38, 0.48))</td>
<td>Yes</td>
</tr>
<tr>
<td>7</td>
<td>(a(7) = (0.14, 0.38, 0.48))</td>
<td>Yes</td>
</tr>
</tbody>
</table>
The results show that the structure still satisfies the stability domain within 10 years, that is, the structure is stable, indicating that the mathematical model using the hierarchical structure is better.

V. CONCLUSION

The evaluation of college teachers’ title is an important task that affects the future development of colleges and universities. This work is highly policy-oriented, wide-ranging, scientific, fair and reasonable. It is related to whether it can mobilize the enthusiasm and creativity of teachers, and it is related to teachers. The construction of the team and the adjustment of the academic echelon are of great significance for promoting the sustainable development of higher education. This paper mainly studies the stability of the grade structure of college titles, how to adjust the number of relevant personnel at each level, in order to develop a reasonable level distribution system. The faculty of the university is strong, and the primary, middle and senior teachers are excellent, but the school position is a dynamic flow process. I analyzed the current level of flow in a university. In this case, the three levels of teachers are not in the stable domain. After a few years, the teacher level may be unevenly distributed. Therefore, I adjusted the flow of a certain university. The proportion has been tested and it has been ensured that the structure of teachers in a university within 10 years is within a reasonable stability domain, which is conducive to maintaining the stability of the teaching staff and promoting the smooth progress of teaching and scientific research. That is to say, the teacher’s title is only upgraded, there is no downgrade, and only one level can be upgraded in one cycle. The number of retired, transferred, and transferred people in the teacher system is relatively stable, that is, the total number of teachers is relatively unchanged, which makes the teaching assistant Up to 20% per year as a lecturer, 10% of the proportion maintains the original position; 90% of the lecturer maintains the original position, 5% of the proportion is promoted; and 95% of the professors retain the original position. And to ensure that 80% of the fresh teacher team enters the teaching assistant position, and 20% enter the lecturer position. This paper explores only a relatively basic problem. With the continuous introduction of policies related to the title level of colleges and universities, this model needs to be further improved, and the relevant reference coefficients are modified to make the model closer to the actual situation.

REFERENCES