

# Finite Element Analysis to Predict Temperature and Velocity Distribution in Radiator Tubes

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## ABSTRACT

Automobile radiators are heat exchangers that are used to transfer thermal energy from automobile engine to the surrounding atmosphere for the purpose of cooling the engine. Over 33% of heat energy generated by the engine through combustion is loss as heat dissipated in the atmosphere. The method of solution employed in this project work to solve the governing equations is the Galerkin-integral weighted-residual method, which is achieved following the steps of transforming the governing equations into Galerkin-integral weighted residual weak form, determination of interpolations functions, determination of element properties, assemblage of elements equations into domain equations and imposition of boundary conditions and solving of the assembled domain equations.

The results showed that for temperature and velocity distributions in the radiator tubes and inlet hose to radiator as the number of elements is increased the more the finite element solution approximates the analytical solutions. Temperature values are observed to decrease, with increase in length, from 150°C to 80°C in the radiator tubes for finite element analysis, analytical, and ANSYS software used; and the finite element solutions exactly approximate analytical solutions at the nodes and agree with the ANSYS result. For velocity distribution in the radiator tube diameter, at the tube walls the no-slip boundary conditions are satisfied with velocity increasing from the wall at velocity of 0 to the midsection at velocity of 50.195m/s; while for the inlet hose diameter, velocity increases from wall at velocity 0 to the maximum at the midsection velocity 669.269m/s. Finally, the finite element analysis method can be used to determine how temperature will be distributed during radiator design stage in order to improve on its efficiency.

**Keywords--** Radiator Tube, Finite Element, Temperature Distribution, Velocity Distribution

mixture is combusted in the combustion chamber. Temperatures of metal components such valves, cylinder, piston and cylinder head around the combustion chamber can exceed 538°C (Pathade et al., 2017). Therefore, automobile radiators, which are heat exchangers that are used to transfer thermal energy from automobile engine to the surrounding atmosphere for the purpose of cooling the engine. Over 33% of heat energy generated by the engine through combustion is loss as heat dissipated in the atmosphere (Sathyan, 2016). According to Gangireddy and Kishore (2017), to increase the surface area available for a radiator with its surroundings, multiple fins are usually attached in contact with the radiator tubes through which pumped liquid flow through. Air or other exterior fluid in contact with the fins carries off heat. More so that radiators used for vehicle engine cooling are either down-flow in which the direction of coolant flow is vertical or cross-flow with horizontal flow. In their review, Patel and Dinesan (2014) in their parametric study used CFD to compare heat transfer and pressure drop of radiators with different parameters for optimum performance. The results showed that CFD results have high correlation level with the actual experimental results. In the work of Priyadharshini (2016), Pro-E 3-D modeling and finite element analysis were used in analyzing a radiator. The result showed Alalloy temperature of variation above 145°C and heat flux value of 0.142W/m<sup>2</sup>; for increasing length of fins, heat transfer value will be in the range of 740 watts while for copper alloy temperature was in the range of 145°C with heat transfer range above 1000 watts. And 150°C for brass with heat transfer in the range above 635watts.

Ng et al. (2005) noted that for the purposes of increasing the boiling point of the coolant and preventing corrosion in the cooling systems, coolants are generally a mixture of water, ethylene glycol (anti-freezing agent), and possibly various corrosion inhibitors. It is noted that the use of glycol mixture generally reduces the heat transfer performance compared with pure water. Oliet et al. (2007)

## I. INTRODUCTION

Automobile engines are internal combustion engines that generate huge amount of heat when air-fuel

observed that the heat transfer and the performance of a radiator are parameters that strongly affected by air and coolant mass flow rate; that as air and coolant flow increases cooling capacity also increases; and that when air inlet temperature increases, heat transfer and cooling capacity decreases. According to Romanov and Khozeniuk (2016), thermal load on an engine can be reduced by controlling the convective component of heat flows using the changing flow pattern of the coolant in the cooling system.

The dynamics of flow in a radiator are known to be governed by sets of partial differential equations (PDEs) along with boundary conditions which are usually formulated to simplify the PDEs. By similarity transformations, the PDEs are transformed into ordinary differential equations (ODEs) which can then be solved numerically when the boundary conditions are applied (Reddy, 1998).

One major problem of radiator is low rate of heat transfer which usually results in low efficiency challenges. This low heat flow rate depends on the distribution of temperature and the manner of velocity distributions in the radiator. These have prompted a lot of numerical models being developed to predict the cooling air flow, either in 2D or 3D using off-the-shelf commercial Computational Fluid Dynamics (CFD) software like Fluent, Vectris, StarCD and

StarCCM+. In all the literature reviewed mathematical model for temperature and velocity distributions were not developed using finite element analysis (FEA) method (Pang et al., 2012; Pathade et al., 2017; Gangireddy and Kishore, 2017; Ng et al., 2005). Therefore in this study, finite element analysis method is used to develop mathematical models that will predict temperature and velocity distributions in a car radiator.

## II. MODEL GEOMETRY

In this study 1-dimensional flow at inlet and outlet hoses of the radiator as well as through the radiator tubes from top to bottom, down-flow radiator are considered (Fig. 1).

The fluid is assumed to enter the tube at inlet ( $x, y_i$ ) with uniform inlet velocity ( $u_i = 0$ ), where it comes in contact with the inlet tube surface with viscous reaction between fluid and the tube thus assumed to set up a one dimensional (1-D), steady, uniform inward flow. The viscous force between the tube surface and the adjacent fluid layer tends to slow down the fluid velocity, which results in developing velocity gradient in the flow field. As the fluid is set in motion, pressure differential (drop) is also set up in the direction of flow.

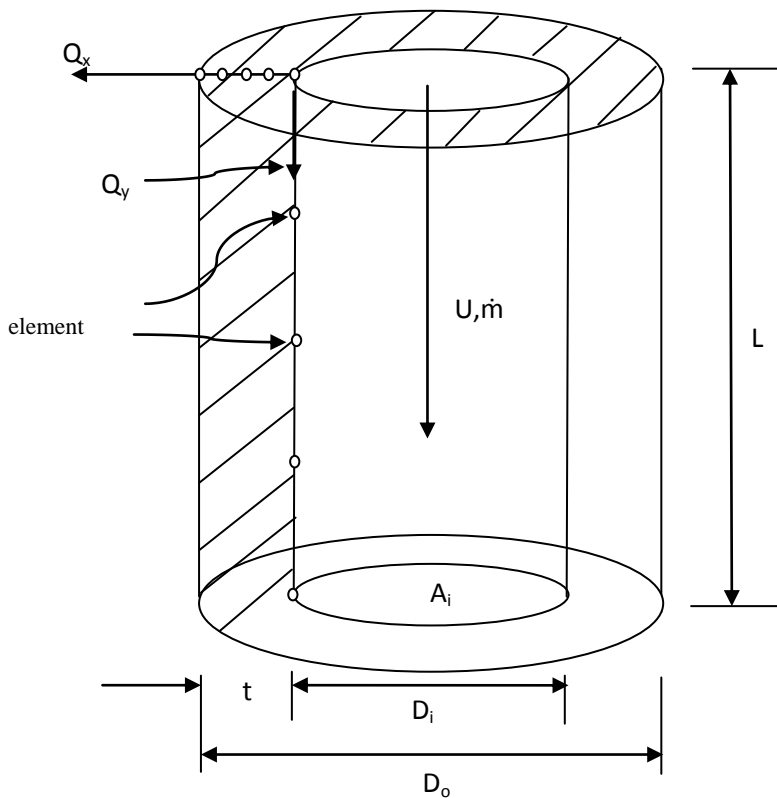


Fig. 1: Radiator tube geometry

Fig. 2 shows the flow characteristic of an ideal fully-developed viscous flow profile in the flexible hose tubes and radiator tubes. As the fluid enters at inlet  $x_i$  or  $y_i$

as the case may be, it sustains a uniform flow at this point but becomes laminar and fully-developed significant distance from the inlet with parabolic flow profile.

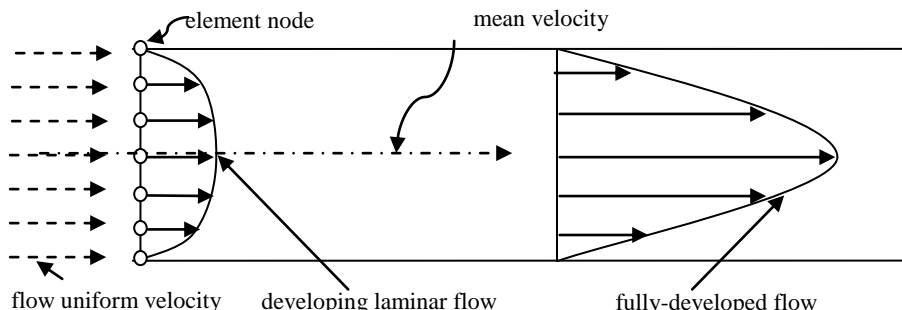


Fig. 2: radiator tube and flexible tube domain flow

2.1 Domain Discretization

In this study, uniform and linear elements were chosen for the domain discretization. The radiator tube domain  $\Omega_1$  ( $-1 \leq x \leq 1$ ) is subdivided into N number of linear elements mesh along the y axis; while the flexible

tube domain  $\Omega_2$  ( $-1 \leq x \leq 1$ ) is also subdivided into N number of linear elements mesh along the y axis. The radiator tube flow field and wall are subdivided into  $N = 4$ , linear and uniform elements mesh as in Fig. 3(a) and (b).

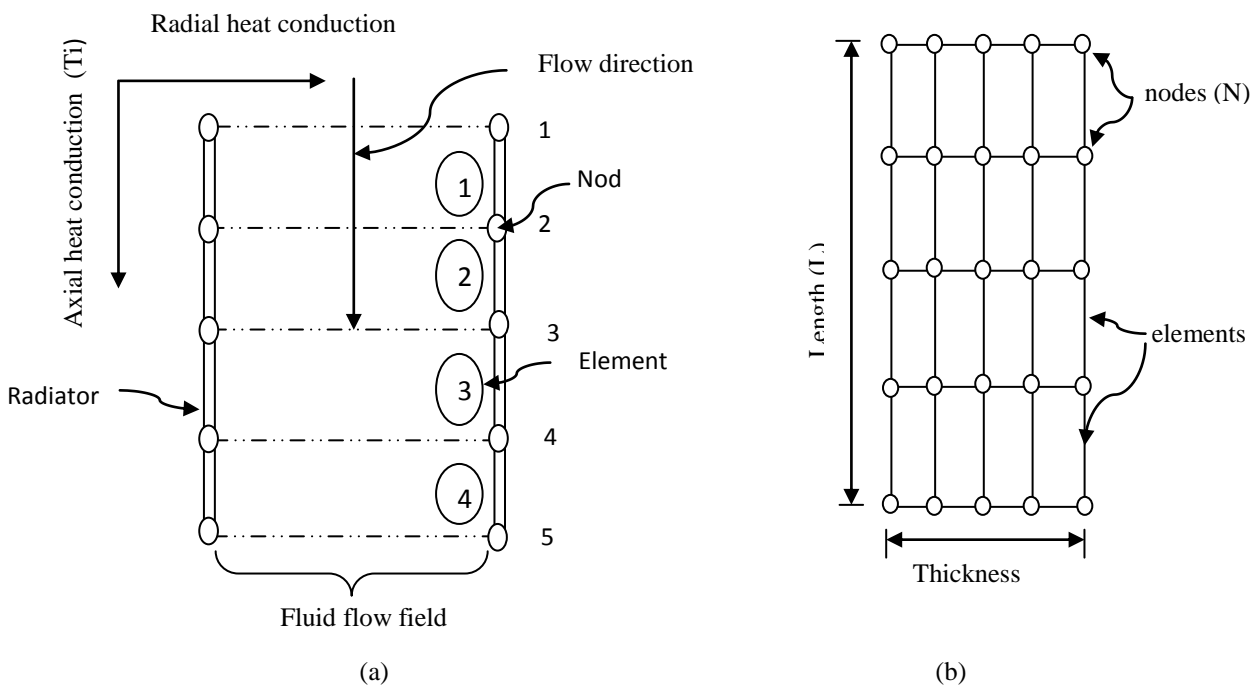


Fig. 3: (a) Radiator tube flow field (b) Radiator thickness meshes

III. MATHEMATICAL FORMULATIONS

For this study, flow of fluid in the flexible tube and radiator tube cores are governed by the continuity, Navier-Stokes and energy partial differential equations.

3.1 Governing Equations

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

x-momentum equation:

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x \quad (2)$$

y-momentum equation:

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \rho g_y \quad (3)$$

z-momentum equation:

$$\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \rho g_z \quad (4)$$

Energy equation

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k \left[ \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + \phi \quad (5)$$

### 3.2 Relevant Assumptions

The following assumptions are applied to simplify continuity, Navier-Stokes and equations:

- (i) One dimensional in the hose and radiator (the flow in the radiator is in y-direction with the flow in x-axis direction assumed insignificantly negligible),
- (ii) Incompressible (i.e. density is assumed constant),
- (iii) Steady,
- (iv) Laminar and fully-developed,
- (v) Newtonian and isotropic fluid flow,
- (vi) Body forces (gravitational and inertia ) are negligible,
- (vii) No-slip condition exists at tube inner surfaces,
- (viii) Elements are linear and uniformly spaced between nodes,

- (ix) Heat conduction is steady with no heat generation,
- (x) Heat conduction is one dimensional, along the y-axis only,
- (xi) Thermal conductivity is constant (isotropic material),
- (xii) Element length is uniform over domain region,
- (xiii) Heat transfer by radiation negligible

By modeling the radiator this way is to enable us to know how temperature varies in the direction of heat transfer and to develop a simple approximate mathematical model to predict the temperature distribution in one dimension in the radiator.

### 3.3 Initial inlet and Boundary Conditions

The following initial inlet and essential boundary conditions are specified at the tube-fluid and radiator-fluid boundary interfaces as well as at inlet and outlet as follows:

#### Boundary conditions at flexible hose tube inlet

$$v(y = 1) = v(y = -1) = 0; V_{ave} = 0.20 \frac{m}{s}; P_A = 18 \frac{N}{m^2}$$

#### Boundary conditions at flexible hose tube outlet

$$V(x, y_i) = 0.018 \frac{m}{s}; P_A = 23 \frac{N}{m^2}$$

**Boundary conditions at radiator tube inlet and outlet**

$$u(x = 1) = u(x = -1) = 0;$$

$$T_o = 80^\circ C, T_L = 150^\circ C;$$

$$T_o = 30^\circ C, T_s = 150^\circ C$$

By applying the assumptions and boundary conditions to equations (1), (2), (3), (4) and (5) reduce them to the followings governing equations (6), (7) and (8):

x-momentum equation applicable to tubes:

$$0 = -\frac{1}{\rho} \frac{dp}{dx} + \frac{\mu}{\rho} \left( \frac{d^2 u}{dy^2} \right) \quad (6)$$

y-momentum equation applicable to radiator:

$$0 = -\frac{dp}{dy} + \mu \left( \frac{d^2 v}{dx^2} \right) \quad (7)$$

Energy equation

$$0 = Aq + k \left[ \frac{d^2 T}{dy^2} \right] \quad (8)$$

**IV. METHOD OF SOLUTION**

The method of solution employed to solve the close-formed governing equations (6), (7) and (8) is Galerkin-integral weighted-residual method.

$$\frac{d}{dy} \left( kA \frac{dT}{dy} \right) + qA = 0 \quad (9)$$

Subject to the following boundary conditions:

$$T(y = 0) = 150^\circ C; T(y = L) = 80^\circ C; V_x = 0.20 \frac{m}{s}; p_y = 18 \frac{N}{m^2} \quad (10)$$

Finite element models for 3,4 and 6 elements mesh are (see Appendix A):

**For 3 elements mesh:**

$$T_1 = 150^\circ C; T_2 = 127^\circ C; T_3 = 103^\circ C; T_4 = 80^\circ C$$

**4.1 Heat transfer in Wall of Length L**

For steady, 1-dimensional heat conduction along wall, length L (y-direction), the above equation reduces to:

**For 4 elements mesh:**

$$T_1 = 150^\circ C; T_2 = 134^\circ C; T_3 = 117^\circ C; T_4 = 99^\circ C; T_5 = 80^\circ C$$

**For 5 elements mesh:**

$$T_1 = 150^\circ C; T_2 = 138^\circ C; T_3 = 126^\circ C; T_4 = 112^\circ C; T_5 = 96^\circ C; T_6 = 80^\circ C$$

**4.2 Heat Transfer in Wall of Thickness S**

For steady, one dimensional heat conduction across the wall thickness S, x-direction, the governing equation (3.13) becomes:

$$\frac{d}{dx} \left( kA \frac{dT}{dx} \right) + qA = 0 \quad (12)$$

Subject to the following boundary conditions:

$$T(x=0) = 150^\circ C; T(x=S) = 30^\circ C; V_y = 0.018 \frac{m}{s}; p_x = 23 \frac{N}{m^2}$$

Finite element models for 3,4 and 5 elements mesh are (see Appendix A):

**For 3 elements mesh:**

$$T_1 = 150^\circ C; T_2 = 110^\circ C; T_3 = 70^\circ C; T_4 = 30^\circ C$$

**For 4 elements mesh:**

$$T_1 = 150^\circ C; T_2 = 121^\circ C; T_3 = 92^\circ C; T_4 = 61^\circ C; T_5 = 30^\circ C$$

**For 5 elements mesh:**

$$T_1 = 150^\circ C; T_2 = 128^\circ C; T_3 = 106^\circ C; T_4 = 82^\circ C; T_5 = 56^\circ C; T_6 = 30^\circ C$$

**4.3 Governing Equation for Flow in Radiator Tube**

The governing equation for inlet tube flow in one-dimension is:

$$\frac{d^2 v}{dx^2} - \frac{P_y}{\mu} = 0 \quad (13)$$

subject to the following initial inlet and boundary conditions:

$$v(y=-1) = 0; v(y=+1) = 0; V_x = 0.20 \frac{m}{s}; p_y = 18 \frac{N}{m^2}$$

Finite element models for 3 and 4 elements mesh are (see Appendix A):

**For 3 elements:**

$$v_1 = 0; v_2 = 2 \left( \frac{P_y h^2}{2\mu} \right); v_3 = 2 \left( \frac{P_y h^2}{2\mu} \right); v_4 = 0$$

**For 4 elements:**

$$v_1 = 0; v_2 = 3 \left( \frac{P_y h^2}{2\mu} \right); v_3 = 4 \left( \frac{P_y h^2}{2\mu} \right); v_4 = 3 \left( \frac{P_y h^2}{2\mu} \right); v_5 = 0$$

#### 4.4 Governing Equation for Flexible Hose at Radiator Inlet

The governing equation for radiator inlet tube in one-dimension is:

$$\frac{d^2 u}{dy^2} - \frac{P_x}{\mu} = 0 \quad (14)$$

subject to the following initial inlet and boundary conditions:

$$u(y = +1) = 0; u(y = -1) = 0; V_x = 0.018 \frac{m}{s}; P_y = 23 \frac{N}{m^2}$$

Finite element models for 3 and 4 elements mesh are (see Appendix A):

**For 3 elements**

$$u_1 = 0; u_2 = 2 \left( \frac{P_x h^2}{2\mu} \right); u_3 = 2 \left( \frac{P_x h^2}{2\mu} \right); u_4 = 0$$

**For 4 elements**

$$u_1 = 0; u_2 = 3 \left( \frac{P_x h^2}{2\mu} \right); u_3 = 4 \left( \frac{P_x h^2}{2\mu} \right); u_4 = 3 \left( \frac{P_x h^2}{2\mu} \right); u_5 = 0$$

## V. RESULTS AND DISCUSSION

The results and discussion presented here are for temperature distributions along radiator tube length (L) and thickness (S) as well as velocity distributions along radiator tube length (L) and in the radiator inlet hose.

For temperature distribution in the radiator tube length, Fig. 4 is a graph of temperature against tube length comparing finite element solutions for 3 (diamond), 4 (square) and 5 (triangle) elements with analytical solution (cross); the graph reveals that the more the number of elements is increased the more the finite element solution approximates the analytical solutions. Fig. 5 shows temperature distribution long tube length using ANSYS R16.2 software. Temperature is seen to decrease from

150°C (red portion) to approximately 80°C (blue portion), which agrees with the result obtained by analysis. Fig. 6 is a graph of temperature against tube length comparing finite element temperature distribution pattern (diamond) with heat flow distribution pattern (triangle) along the tube length respectively. In Fig. 6, temperature is seen to decrease with increase in length while heat flow increases with increase in length. For temperature distribution along the tube thickness, Fig. 7 is a graph of temperature against tube thickness comparing finite element solutions for 3 (diamond), 4 (square) and 5 (triangle) elements with analytical solution (cross); the graph reveals that the more the number of elements is increased the more the finite element solution approximates the exact solutions. Fig. 8 is a graph of temperature against tube thickness comparing



finite element temperature distribution pattern (diamond) with heat flow distribution pattern (triangle) along the tube thickness respectively with temperature shown to decrease with increase in length while heat flow increases with increase in length.

For velocity distribution in the radiator tube diameter, Fig. 9 is a graph of velocity against tube length comparing finite element solutions for 3 (square) and 4 (diamond) elements with analytical solution (cross); the graph reveals that more the number of elements is increased the more the finite element solution approximates the exact solutions. From Fig. 9, at the tube walls the no-slip

boundary conditions are satisfied with velocity increasing from the wall velocity = 0 to the midsection with velocity = 50.195m/s. While for velocity distribution in inlet hose diameter, in Fig.10 a graph of velocity against inlet hose diameter, 2 elements solutions (triangle), 3 elements solutions (diamond) and 4 elements solutions (circle) are compared with the analytical solution (square).The graph reveals the finite element solution approximates the analytical solution as the number of elements is increased with the no-slip boundary conditions being satisfied at the walls as velocity increases from 0 to the maximum at the midsection with velocity = 669.269m/s.

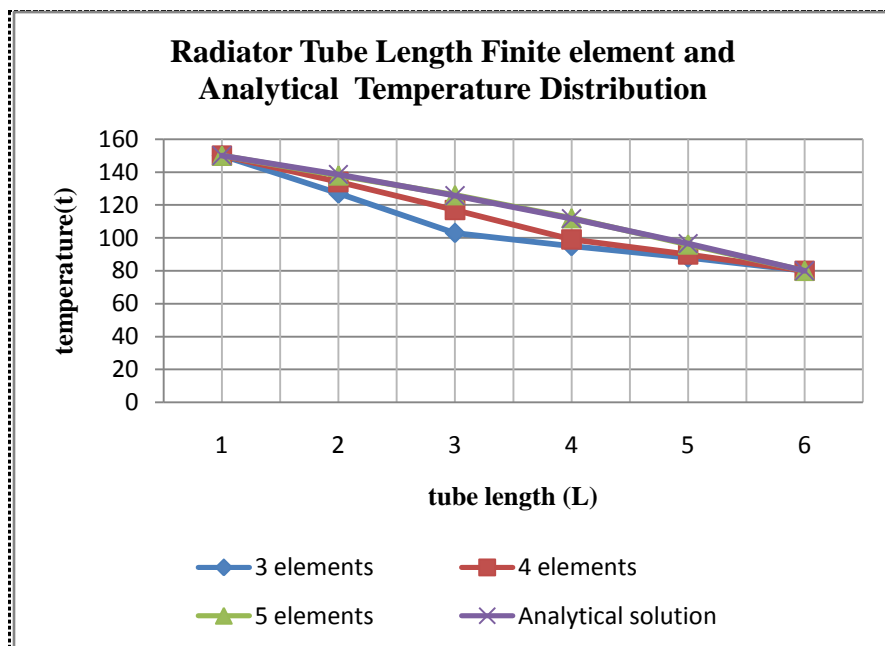


Fig. 4: Finite element and analytical temperature distribution along tube length

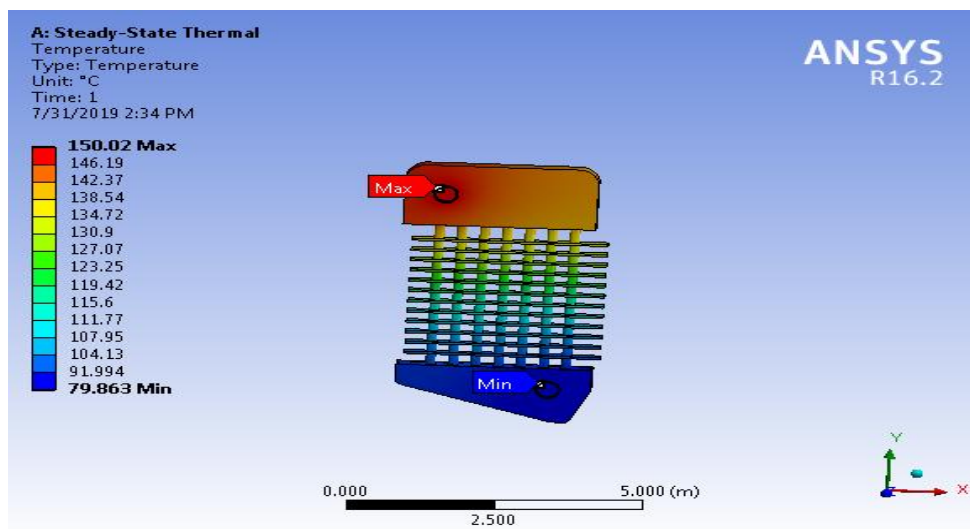


Fig. 5: Finite element ANSYS temperature distribution along tube length



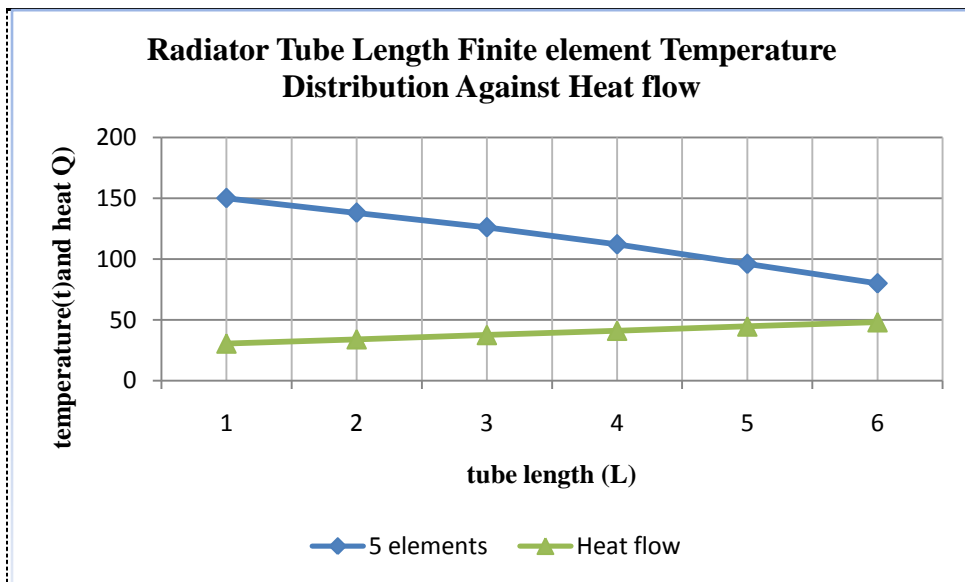


Fig. 6: Temperature and heat distribution along tube length

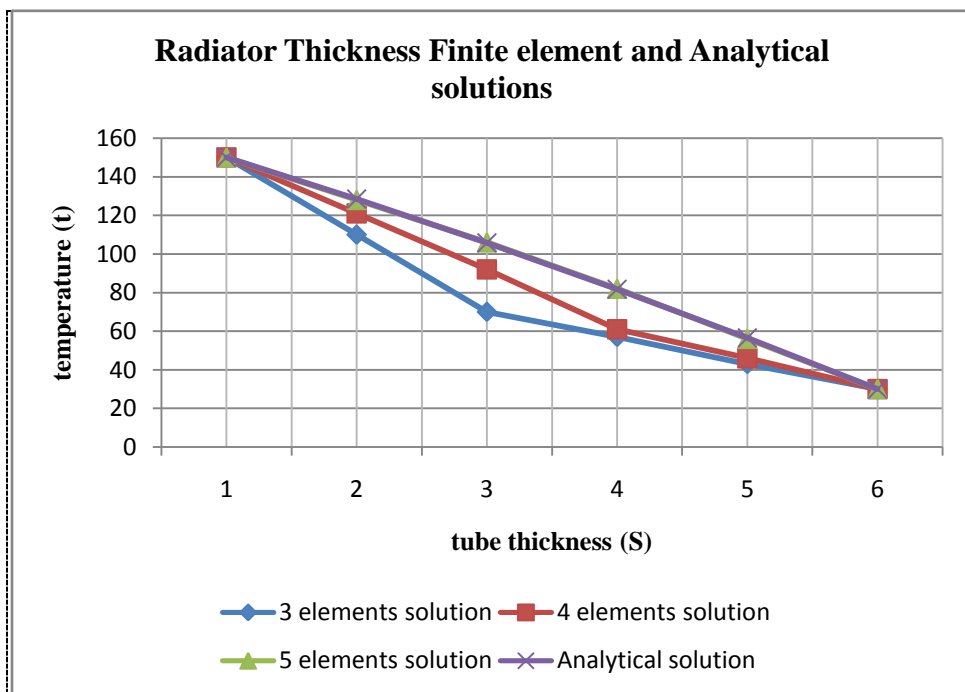


Fig.7: Finite element and analytical temperature distribution along tube thickness

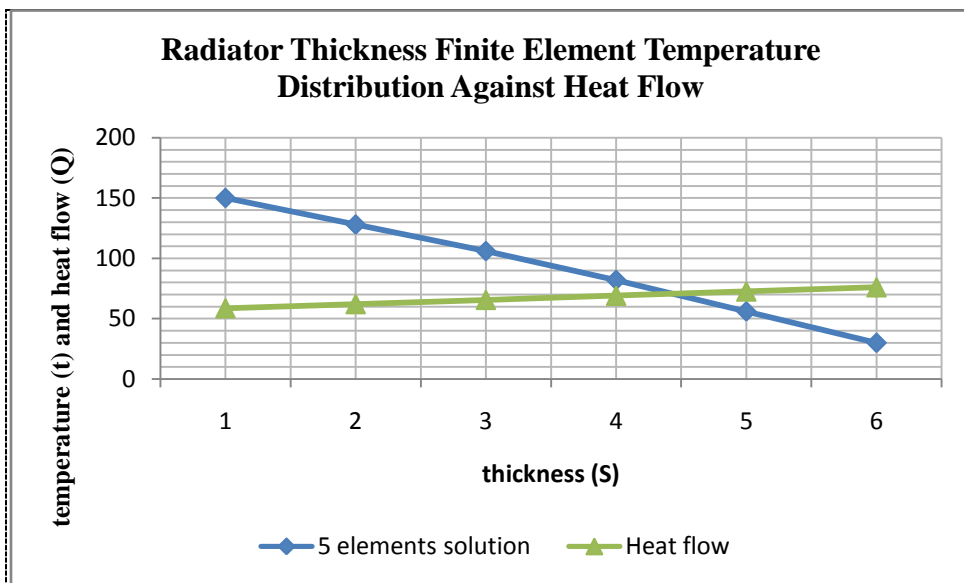


Fig. 8: Temperature and heat distribution along tube thickness

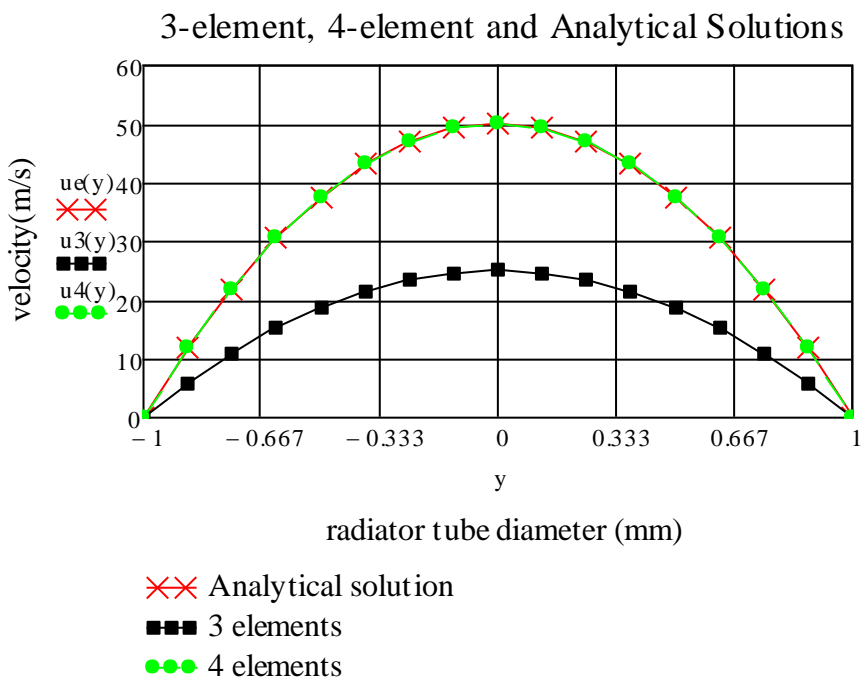


Fig. 9: Velocity against radiator tube diameter

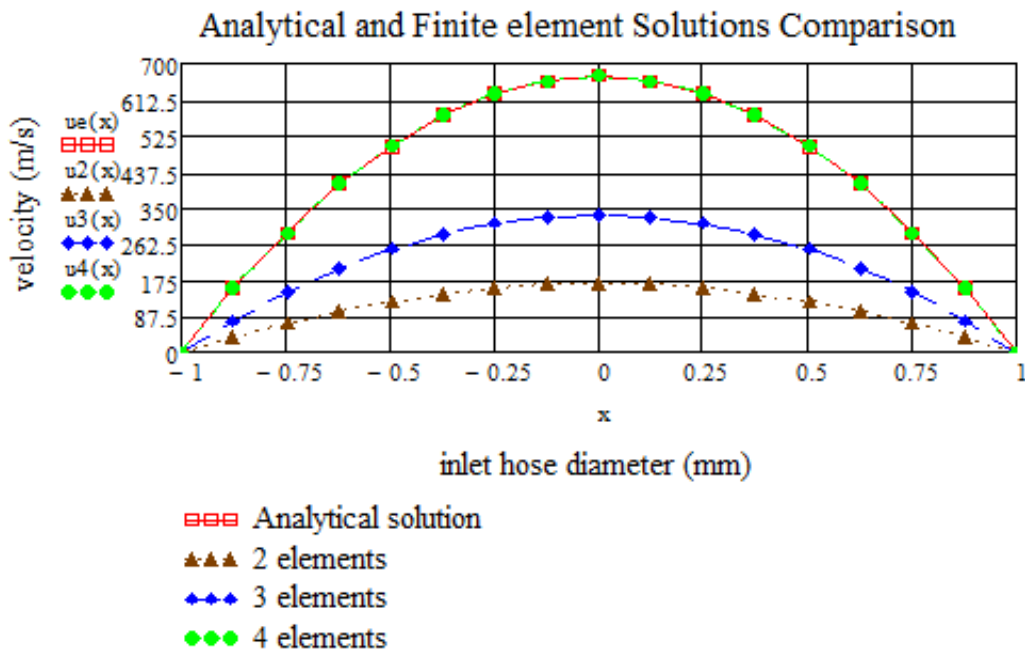


Fig. 10: Velocity distribution radiator inlet hose

For radiator tube, Table 1 shows temperature distribution at the element nodes for 3 (3E), 4(4E) and 5(5E) elements and the corresponding analytical (AS) values. In Table 1, the 5 elements nodal values approximate with those of the analytical nodal point values. Table 2 shows temperature distribution for 5 (5E) elements, the corresponding analytical (AS) values and the heat flux values. Temperature distribution values are seen to decrease with tube length as the heat flux increases. For tube thickness, Table 3 indicates temperature distribution values at element nodes for 3 (3E), 4(4E) and 5(5E) elements and the corresponding analytical (AS) values. In Table 3, the 5 elements nodal values approximate with those of the analytical nodal point values. Table 4 shows temperature

distribution for 5 (5E) elements, the corresponding analytical (AS) values and the heat flux values. Temperature distribution values are seen to decrease with tube length as the heat flux increases.

For radiator tube, Table 5 shows velocity distribution at the element nodes for 3 (3E) and 4(4E) elements and the corresponding analytical (AS) values. In Table 5, the 4 elements nodal values approximate with those of the analytical nodal point values. For inlet hose, Table 6 shows velocity distribution at the element nodes for 3 (3E) and 4(4E) elements and the corresponding analytical (AS) values. In Table 6, the 4 elements nodal values approximate with those of the analytical nodal point values.

Table 1: Radiator tube length finite element and analytical temperature nodal values

Nodes	3E (°C)	4E (°C)	5E (°C)	AS (°C)
0.00	150.000	150.000	150.000	150.000
0.20	127.000	134.000	138.000	138.488
0.40	103.000	117.000	126.000	125.733
0.60	95.000	99.000	112.000	111.733
0.80	88.000	90.000	96.000	96.488
1.00	80.000	80.000	80.000	80.000

3E: 3 elementssolution; 4E: 4 elementssolution; 5E: 5 elements solution;AS: analytical solution

Table 2: Finite element and analytical temperature distribution and heat flow values

Nodes	5E (°C)	AS (°C)	Q(J/m <sup>2</sup> )
0.00	150.000	150.000	30.545
0.20	138.000	138.488	34.035

0.40	126.000	125.733	37.525
0.60	112.000	111.733	41.015
0.80	96.000	96.488	44.505
1.00	80.000	80.000	47.995

5E: 5 elements solution;AS: analytical solution; Q: heat fluxes

**Table 3: Finite element and analytical temperature nodal values**

Nodes	3E (°C)	4E (°C)	5E (°C)	AS (°C)
0.00	150.000	150.000	150.000	150.000
0.06	110.000	121.000	128.000	128.48
0.12	70.000	92.000	106.000	105.733
0.18	57.000	61.000	82.000	81.733
0.24	43.000	46.000	56.000	56.488
0.30	30.000	30.000	30.000	30.000

3E: 3 elementssolution; 4E: 4 elementssolution; 5E: 5 elements solution;AS: analytical solution

**Table 4: Finite element, analytical temperature and heat flow nodal values**

Nodes	5E (°C)	AS (°C)	Q(J)
0.00	150.000	150.000	58.595
0.20	128.000	128.488	62.085
0.40	106.000	105.733	65.575
0.60	82.000	81.733	69.056
0.80	56.000	56.488	72.555
1.00	30.000	30.000	76.045

5E: 5 elements solution;AS: analytical solution; Q: heat fluxes

**Table 5: Radiator tube FE and exact solutions velocity nodal values**

b	3E	4E	AS
1.000	<b>0</b>	<b>0</b>	<b>0</b>
0.875	0.056	0.113	0.113
0.750	<b>0.105</b>	<b>0.210</b>	<b>0.210</b>
0.625	0.146	0.293	0.293
0.500	0.180	<b>0.361</b>	<b>0.361</b>
0.375	0.207	0.413	0.413
0.250	<b>0.225</b>	<b>0.451</b>	<b>0.451</b>
0.125	0.237	0.473	0.473
0.000	<b>0.240</b>	<b>0.481</b>	<b>0.481</b>

**Table 6: Radiator flexible hose FE and Exact velocity nodal values**

b	3E	4E	AS
1.000	<b>0</b>	<b>0</b>	<b>0</b>
0.875	0.056	0.113	0.113
0.750	<b>0.105</b>	<b>0.210</b>	<b>0.210</b>
0.625	0.146	0.293	0.293
0.500	0.180	<b>0.361</b>	<b>0.361</b>
0.375	0.207	0.413	0.413

0.250	<b>0.225</b>	<b>0.451</b>	<b>0.451</b>
0.125	0.237	0.473	0.473
0.000	<b>0.240</b>	<b>0.481</b>	<b>0.481</b>

3E: 3 elementssolution; 4E: 4 elementssolution; AS: analytical solution

## VI. CONCLUSION

From this empirical finite element analysis of radiator temperature and velocity distribution, it is here concluded that:

- Empirical analysis on a radiator was carried out to obtain required parameters.
- Finite element method was used to solve Continuity and Navier Stokes equation.
- FEA was used to determine velocity and temperature distribution in radiator.
- Analytical solutions were derived and compared with FE solutions.

Furthermore, this study on finite element analysis of radiator temperature and velocity distributions revealed that as the number of elements in the analysis are increased FE results tend to converge fast to the analytical results. Also, the finite element nodes values are observed to be the same with analytical values at the nodes, which is a good indication that the FEA method is suitable for determination of temperature distribution. This is further validated by the ANSYS R16.2 results. Consequently, the FEA method can be confidently used to determine how temperature will be distribute during radiator design stage.

## RECOMMENDATIONS

There is need for possible further research by using other methods of investigation, geometry of radiator and by considering the convection heat transfer by air current.

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### Appendix A

**(1) Transformation of governing equation into Weak form**

The derivation of element equations for typical element e in the mesh follows these steps (Reddy, 1995):

$$\frac{d}{dy} \left( kA \frac{dT}{dy} \right) + qA = 0 \tag{A1}$$

subject to the following boundary conditions:

$$T(y = 0) = 150^{\circ} C; T(y = H) = 80^{\circ} C \tag{A2}$$

The residual, R, of the governing equation (A1) is:

$$\frac{d}{dy} \left( kA \frac{dT}{dy} \right) + qA = 0 \tag{A3}$$

The Galerkin-weighted residual integral equation (A4) is:

$$0 = \int_{\Omega} w \left[ -kA \left( \frac{d^2T}{dy^2} \right) + Aq(y) \right] d\Omega \tag{A4}$$

The Weak form of the Galerkin-weighted residual integralequation (A5) is:

$$0 = \int_{\Omega} \left[ kA \frac{dw}{dy} \frac{dT}{dy} + wAq \right] d\Omega - wkA \frac{dT}{dy} \Big|_{\Gamma} \tag{A5}$$

The finite element equation is:

$$\begin{aligned} 0 &= \int_{\Omega} \left( kA \frac{d\psi_i^e}{dy} \sum_{j=1}^n T_j^e \frac{d\psi_j^e}{dy} dy + Aq\psi_i^e \right) dy - \left[ kA\psi_i^e \frac{dT}{dy} \right]_L^0 \\ &= T_j^e \sum_{j=1}^n \int_0^h kA \frac{d\psi_i^e}{dy} \frac{d\psi_j^e}{dy} dy + Aq \int_0^h \psi_i^e dy - \left[ kA\psi_i^e \frac{dT}{dy} \right]_L^{0h} \end{aligned} \tag{A6}$$

In matrix form the finite element model is written as:

$$[K^e] \{T^e\} = \{f^e\} - \{Q^e\} \tag{A7}$$

**(2) The analytical solution are:**

$$T(y) = -\frac{qL^2}{2k} + \left[ \frac{y}{L} - \left( \frac{y}{L} \right)^2 \right] \frac{y}{L} + (T_L - T_0) \frac{y}{L} + T_0 \tag{A}$$

$$Q(y) = kA \frac{dT(y)}{dy} = -\frac{qLA}{2} \left[ 1 - 2 \left( \frac{y}{L} \right) \right] \frac{y}{L} - (T_L - T_0) \frac{kA}{L} + T_0 \tag{A9}$$

## Appendix B

### Problem data

#### Empirical values

Observed parameters	Symbols	Quantity	Unit
Tube material	-	Aluminium	
Inlet temperature	$T_L$	150	$^{\circ}\text{C}$
Outlet temperature	$T_O$	80,30	$^{\circ}\text{C}$
Specific heat	$c_p$	4.187	$\text{kJ/kgK}$
Thermal conductivity	$k$	0.66	$\text{W/mK}$
Density	$\rho$	1000	$\text{kg/m}^3$
Dynamic viscosity	$\mu$	$1.793 \times 10^{-3}$	$\text{kg/ms}$

#### Finite element analysis parameters

Observed parameters	Symbols	Quantity	Unit
Specified pressure at tube	$p_y$	0.018	$\text{N/m}^2$
Specified pressure at hose	$p_x$	0.23	$\text{N/m}^2$
Inlet diameter of tube	$D_1$	10	$\text{Mm}$
Outlet diameter of tube	$D_2$	11.25	$\text{Mm}$
Tube thickness	$S$	0.0625	$\text{Mm}$
Element length, tube	$h_y$	0.5	$\text{Mm}$
Element length, hose	$h_x$	0.005	$\text{Mm}$