



Machine Breakdown Inventory System with Multi Items and Partial Backlogging

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ABSTRACT

This paper deals a production system with multi items and machine breakdown in which units are subject to deterioration at a time dependent rate. The unit production cost is taken to be a function of the finite production rate which is treated as a decision variable. A partially backlogged shortage has been discussed in this study. The solution procedure is illustrated with the help of numerical examples. The sensitivity of the variable to changes in the values of the parameters of the system is examined.

Keywords-- Multi items, Volume flexible, partial backlogging and machine breakdown

The sale of many products depends upon selling price because of the change in consumers' preferences. In other words, the demand rate and the unit price of items are closely correlated. At certain time of the product cycle, the monotonically decreasing price pattern result is sales increase. This pattern conforms to price-dependent demand. This model is applicable to certain products such as electronic goods or luxury items at specific time of the product cycle when the demand fluctuates with the price of product.

Chung (1997) discussed about the bound of machine breakdown problem. Giri, and Dohi, (2005) presented an exact formulation of EMQ model under a general framework in which the time to machine failure, corrective and preventive repair times all are assumed to follow arbitrary probability distributions. But they ignored the study of fuzzy. Chakraborty *et al.* (2008) presents a generalized economic manufacturing quantity model for an unreliable production system in which the production facility may shift from an 'in-control' state to an 'out-of-control' state at any random time and may ultimately break down afterwards. Singhal *et al.* (2010) developed a volume flexible supply chain system with uncertain lead time and stochastic deterioration. Singh and Urvashi (2010) discussed the effect of machine breakdown with fuzzy demand rate. They considered the volume flexibility with idle time of the inventory system. Widyadana and Wee (2010) develops deteriorating items production inventory models with random machine breakdown and stochastic repair time. The model assumes the machine repair time is independent of the machine breakdown rate. Singhal and Singh (2012) discussed effect of probabilistic backorder on an inventory system with selling price demand under volume flexible strategy. Singhal and Singh (2013) developed volume flexible multi items inventory system with price dependent demand and imprecise environment. Singhal and Singh (2014) deals volume flexible inventory model for probabilistic decaying items with remanufacturing and exponential demand. Singhal *et al.* (2014) presented a probabilistic inventory model for weibull deteriorating items with flexibility and

I. INTRODUCTION

Production, quality and maintenance are the significant parts of any manufacturing firm. In the competitive business environment, managers of manufacturing industries encounter the challenge everyday to produce quality products and to provide better services than before to customers. Due to technological innovations and scientific developments around the world, manufacturing infrastructure is also changing rapidly. Three important factors of the Economic Manufacturing Quantity (EMQ) model have been dealt with prior significance. First it is assumed that the production facility is not perfect reliable. Second, the production rate is not predetermined and fixed in advance. Third, the modern facilities are not free from deterioration due to epoch. As a result, random machine shifts from 'in-control' state to 'out-of-control' state frequently occur during production runs and some percentage of non-conforming items is produced. Further, the process deterioration after a machine shift may result in a machine breakdown in which case the interrupted lot is usually aborted and then the basic EMQ model loses its usefulness. Therefore, from theoretical as well as practical view points, the study of EMQ problem for unreliable manufacturing systems is quite significant and meaningful.

reliability consideration. Singhal and Singh (2015) discussed a modelling of an inventory system with multivariate demand under volume flexibility and learning. Singhal et al. (2016) developed a stochastic partial backlogging inventory models for deteriorating items with time dependent demand and volume elasticity.

An EMQ model for decaying items and selling price demand with volume flexible environment has been developed. Decaying rate is taken as time dependent. In this study, we have discussed the concept of machine breakdown during the production. In such a situation, the demand is met until the inventory level falls below the quantity demanded. When inventory level becomes less than the demand, the concerned

management unit B_i is rendered fully idle. This situation occurs when the customer is a wholesaler having the demand of a big lot size and the concerned management unit can't meet this demand because the stock size is less than the quantity demanded. Therefore, we considered the idle time of each management unit; this idle time leads to an additional cost for the last man hours. We have considered the capital available for manufacturing the items is limited. Shortages are allowed with partial backlogging. Numerical example is provided and sensitivity analysis is carried out to demonstrate the significance of considering profit maximization technique.

II. ASSUMPTIONS AND NOTATIONS

The mathematical models are developed on the basis of the following assumptions and notations:

Assumptions

The following assumptions for the proposed model which are as follows:

1. The model is developed for multiple items.
2. Deterioration rate is time dependent.
3. The demand for the item is a downward slopping function of the price.

4. Production rate per unit time is considered as a decision variable.
5. Unit production cost for the i^{th} item ($i=1,2,3,\dots,n$) is a function of the production rate.
6. Invested capital for production is limited.
7. Machine breakdown is considered during the production period.
8. Idle time to the management unit is considered.
9. Shortages are allowed with partial backlogging during the idle time.
10. Backlogging rate is taken as constant.
11. Time horizon is infinite.

Notations

The following notations are assumed in our study:

$Q_i(t)$ The on-hand inventory of the i^{th} item at time t

P_i The production rate per unit time for the i^{th} item

$\theta_i t$ Time dependent deterioration rate $0 < \theta_i < 1$

μ_i The mean time between successive breakdowns of the machines ($A_i, i=1,2,\dots,n$)

$\psi_i(t_i)$ Probability density function of t_i

C Unit purchasing cost with $C < S_p^i$

m_i The mean time of repair of i^{th} machine

τ_i The mean duration of a breakdown of machines ($A_i, i=1,2,\dots,n$)

$\phi_i(\tau_i)$ Probability density function of τ_i

C_h^i The cost of carrying one unit of i^{th} item in inventory per unit time

C_S^i The cost of shortages per unit time of i^{th} item

C_{LS}^i The cost of lost sale per unit time of i^{th} item

D_i The annual demand as a decreasing function of price; $D_i(S_p^i) = \alpha_i (S_p^i)^{-\beta_i}$,

Where $\alpha_i > 0$ and $\beta_i > 1$

$\eta_i(P_i)$ The cost for production of a unit time of the i^{th} item ($i=1,2,\dots,n$) and

$\eta_i(P_i) = (r_i + \frac{g_i}{P_i} + \alpha_i P_i)$, where r_i is material cost, α_i is tool or die cost and g_i

is energy and labor cost

- S_p^i The selling price per unit of i^{th} item
- W_i The cost per unit of idle time of the management unit B_i
- CAP The total capital available for production of all the items

III. MATHEMATICAL FORMULATION

In this model, the production cycle begins with zero stock. Production starts at time $t=0$ and the stock reaches a level $Q_i(t_i) \{i=1,2,\dots,n\}$ at times $t=t_i$ after adjusting demand rates D_i . At times $t=t_i$ machines A_i ,

becomes out of order. Then repairing of machines A_i starts and takes times τ_i to comeback into working state. Here, t_i and τ_i are random variables which follow probability distribution functions ψ_i and Φ_i respectively. The governing equations for the inventory system are as follows:

$$Q_i'(t) + \theta_i t Q_i(t) = P_i - \alpha_i (S_p^i)^{-\beta_i} \quad 0 \leq t \leq t_i \quad \dots(1)$$

$$Q_i'(t) + \theta_i t Q_i(t) = -\alpha_i (S_p^i)^{-\beta_i} \quad t_i \leq t \leq x \quad \dots(2)$$

With the boundary condition $Q_i(0) = 0$ and $Q_i(x) = 0$, where $i=1,2,\dots,n$, solution of the equations (1) and (2) are:

$$Q_i(t) = (P_i - \alpha_i (S_p^i)^{-\beta_i}) \left(t + \frac{\theta_i t^3}{6} \right) e^{-\frac{\theta_i t^2}{2}} \quad 0 \leq t \leq t_i \quad \dots(3)$$

$$Q_i(t) = \alpha_i (S_p^i)^{-\beta_i} \left[(x-t) + \frac{\theta_i}{6} (x^3 - t^3) \right] e^{-\frac{\theta_i t^2}{2}} \quad t_i \leq t \leq x \quad \dots(4)$$

The idle times of the management units $\{B_i, i=1,2,\dots,n\}$ due to a breakdown of the machines $\{A_i, i=1,2,\dots,n\}$ are

$$u_i = 0 \quad \text{if } \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} \geq \tau_i$$

$$u_i = \tau_i - \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} \quad \text{if } \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} < \tau_i$$

The expected cost per breakdown of the machine $\{A_i, i=1,2,\dots,n\}$ during the idle time is given by:

$$E_{ic}^i = W_i \int_0^\infty \left[\int_{\frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}}}^\infty \left(\tau_i - \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} \right) \phi_i(\tau_i) d\tau_i \right] \psi_i(t_i) dt_i$$

$$= \frac{W_i}{\mu_i} \left(\frac{P_i}{\alpha_i (S_p^i)^{-\beta_i}} - 1 \right) \left[\mu_i^2 - 30\theta_i \mu_i^3 - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \left(\frac{2\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}} - \frac{4\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}} \right) \right] \quad \dots(5)$$

The expected shortage cost for i^{th} item during the idle time is

$$E_{Sc}^i = C_S^i \alpha_i (S_p^i)^{-\beta_i} \int_0^\infty \left[\int_{\frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}}}^\infty \left(\tau_i - \frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}} \right) \phi_i(\tau_i) d\tau_i \right] \psi_i(t_i) dt_i$$

$$= C_S^i \left[\alpha_i (S_p^i)^{-\beta_i} m_i - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i + 2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^3 \right] \quad \dots(6)$$

The expected lost sale cost for i^{th} item during the idle time is given by:

$$E_{LSc}^i = C_{LS}^i \int_0^\infty \left[\int_{\frac{Q_i(t_i)}{\alpha_i (S_p^i)^{-\beta_i}}}^\infty (1-\delta) \alpha_i (S_p^i)^{-\beta_i} \phi_i(\tau_i) d\tau_i \right] \psi_i(t_i) dt_i$$

$$= \frac{C_{LS}^i (1-\delta) \alpha_i (S_p^i)^{-\beta_i}}{\mu_i} \left[\mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{m_i \alpha_i (S_p^i)^{-\beta_i}} \left(\mu_i^2 + \frac{\theta_i \mu_i^3}{3} \right) \right] \quad \dots(7)$$

Now, the total inventory of i^{th} item is as follows:

$Inv_i(t_i)$ = Inventory during $[0, t_i]$ + Inventory during $[t_i, x]$

$$\begin{aligned} Inv_i(t_i) &= \left[\int_0^{t_i} C_h^i Q_i(t_i) dt_i + \int_{t_i}^x C_h^i Q_i(t_i) dt_i \right] \\ &= (P_i - \alpha_i (S_p^i)^{-\beta_i}) \left[C_h^i \left(\frac{t_i^2}{2} - \frac{\theta_i t_i^4}{12} \right) \right] + \alpha_i (S_p^i)^{-\beta_i} \left[C_h^i \left(\frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^2 t_i^2}{2 \alpha_i (S_p^i)^{-2\beta_i}} \right. \right. \\ &\quad - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i}) t_i^2}{\alpha_i (S_p^i)^{-\beta_i}} + \frac{t_i^2}{2} - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i t_i^4}{6 \alpha_i (S_p^i)^{-3\beta_i}} - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^4 \theta_i t_i^4}{12 \alpha_i (S_p^i)^{-4\beta_i}} + \frac{\theta_i t_i^4}{12} \\ &\quad \left. \left. + \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i t_i^4}{6 \alpha_i (S_p^i)^{-\beta_i}} \right) \right] \quad \dots(8) \end{aligned}$$

As a result, the expected inventory cost for i^{th} item is as follows:

$$\begin{aligned} E_{inc}^i &= \int_0^\infty Inv_i(t_i) \psi_i(t_i) dt_i \\ &= \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{\mu_i} \left[C_h^i (\mu_i^3 - 2\theta_i \mu_i^4) \right] + \alpha_i (S_p^i)^{-\beta_i} \left[C_h^i \left(\frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^2 \mu_i^3}{\alpha_i (S_p^i)^{-2\beta_i}} \right. \right. \\ &\quad - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^3}{\alpha_i (S_p^i)^{-\beta_i}} + \mu_i^3 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}} - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i})^4 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-4\beta_i}} + 2\theta_i \mu_i^4 \\ &\quad \left. \left. - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}} \right) \right] \quad \dots(9) \end{aligned}$$

The, production cost per unit of i^{th} item is given by:

$$\eta_i(P_i) = r_i + \frac{g_i}{P_i} + \alpha_i P_i$$

For that reason, the expected production cost for i^{th} item is given by:

$$\begin{aligned} E_{prc}^i &= \sum_{i=1}^n \int_0^\infty \eta_i(P_i) P_i t_i \psi_i(t_i) dt_i \\ &= \sum_{i=1}^n \left(r_i + \frac{g_i}{P_i} + \alpha_i P_i \right) P_i \mu_i \quad \dots(10) \end{aligned}$$

Then the expected set up cost for i^{th} item, which is as follows:

$$\begin{aligned} SE_{UP}^i &= \sum_{i=1}^n \int_0^\infty S_{UP}^i t_i \psi_i(t_i) dt_i \\ &= \sum_{i=1}^n S_{UP}^i \quad \dots(11) \end{aligned}$$

The reliability of spare parts of a machine follows exponential probability distribution function. The probability density functions of exponential distribution is as follows

$$\psi_i(t_i) = \frac{1}{\mu_i} e^{-t_i/\mu_i}$$

$$\phi_i(\tau_i) = \frac{1}{m_i} e^{-\tau_i/m_i}$$

The expected total profit per breakdown is given by:

ETP(P₁, P₂,..... P_n) = Expected revenue from selling items- Expected production cost - Expected holding cost-Expected cost for idle time-Expected shortages cost-Expected lost sale cost – Expected set up cost

$$= \sum_{i=1}^n [(S_p^i - (r_i + \frac{g_i}{P_i} + H_i P_i)) P_i \mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{\mu_i} \{ C_h^i (\mu_i^3 - 2\theta_i \mu_i^4) \} - \alpha_i (S_p^i)^{-\beta_i} \{ C_h^i (\frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})^2 \mu_i^3}{\alpha_i (S_p^i)^{-2\beta_i}} - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i^3}{\alpha_i (S_p^i)^{-\beta_i}} + \mu_i^3 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i})^3 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-3\beta_i}} - \frac{2(P_i - \alpha_i (S_p^i)^{-\beta_i})^4 \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-4\beta_i}} + 2\theta_i \mu_i^4 - \frac{4(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^4}{\alpha_i (S_p^i)^{-\beta_i}}) \} - \frac{W_i}{\mu_i} (\frac{P_i}{\alpha_i (S_p^i)^{-\beta_i}} - 1) \{ \mu_i^2 - 30\theta_i \mu_i^3 - (P_i - \alpha_i (S_p^i)^{-\beta_i}) (\frac{2\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}} - \frac{4\mu_i^3}{m_i \alpha_i (S_p^i)^{-\beta_i}}) \} - C_S^i \{ \alpha_i (S_p^i)^{-\beta_i} m_i - (P_i - \alpha_i (S_p^i)^{-\beta_i}) \mu_i + 2(P_i - \alpha_i (S_p^i)^{-\beta_i}) \theta_i \mu_i^3 \} - \frac{C_{LS}^i (1 - \delta) \alpha_i (S_p^i)^{-\beta_i}}{\mu_i} \{ \mu_i - \frac{(P_i - \alpha_i (S_p^i)^{-\beta_i})}{m_i \alpha_i (S_p^i)^{-\beta_i}} (\mu_i^2 + \frac{\theta_i \mu_i^3}{3}) \} - S_{UP}^i] \dots(12)$$

Consequently, one has to maximize the profit function:

$$ETP(P_1, P_2, \dots, P_n)$$

Subject to the constraints:

$$\sum_{i=1}^n (r_i + \frac{g_i}{P_i} + \alpha_i P_i) P_i \mu_i \leq CAP$$

$$P_1 \geq D_1, P_2 \geq D_2, \dots, P_n \geq D_n$$

Equation (12) gives an estimate of profit function. For maximization of profit function, differentiate the equation (12) with respect to P₁, P₂,..... P_n. The results are illustrated numerically with the help of the software *Mathematica5.2*.

IV. NUMERICAL ILLUSTRATIONS

The following numerical study has been used to find the optimal solution of the model.

The common input parameters are: δ=0.4,

Item 'i'	μ _i	r _i	g _i	H _i	α _i	β _i	C _{LS} ⁱ
1	4	280	2700	0.02	7500	1.9	23
2	5	220	2300	0.03	7000	2.0	25
Item 'i'	θ _i	W _i	m _i	S _P ⁱ	C _S ⁱ	C _h ⁱ	
1	0.05	40	0.60	60	13	15	
2	0.04	45	0.72	50	15	15	

Optimum solutions are as follows: The optimal values of P₁ & P₂ are 8.2683 and 6.4187 respectively. The optimal net profit is 245781.

V. SENSITIVITY ANALYSIS

In order to study how the parameters effect on the optimal solution of production and the profit, the sensitivity analysis for all parameters has been conducted.

Table: Sensitivity Analysis of Optimal Solution w.r.t. System Parameters

Parameter	% Change	-30%	-20%	-10%	10%	20%	30%
$W_1 \& W_2$	P_1	+0.92	+0.64	+0.31	-0.31	-0.63	-0.89
	P_2	+0.51	+0.32	+0.14	-0.15	-0.32	-0.52
	Profit	+7.08	+4.62	+2.74	-2.67	-4.58	-7.02
$\mu_1 \& \mu_2$	P_1	+11.82	+6.79	+2.96	-2.73	-5.52	-8.27
	P_2	+10.67	+7.26	+3.83	-3.47	-5.12	-8.65
	Profit	+94.45	+79.63	+40.61	-38.56	-62.18	-78.34
$m_1 \& m_2$	P_1	-1.32	-0.84	-0.37	+0.28	+0.64	+0.89
	P_2	-0.75	-0.38	-0.20	+0.18	+0.34	+0.52
	Profit	-10.23	-6.79	-2.89	+2.68	+5.56	+6.78
$S_{1p} \& S_{2p}$	P_1	+25.2	+15.2	+5.7	-5.6	-15.3	-25.7
	P_2	+38.31	+27.33	+17.45	-17.45	-27.32	-38.34
	Profit	+40.35	+36.48	+30.56	-30.56	-36.48	-40.36

VI. OBSERVATIONS

1. Production rate is less sensitive and the profit is moderately sensitive to change in the parameter of cost of ideal time of management.
2. The production rate and the profit are increases with the decreases of the mean time of successive breakdowns and the production rate and the profit are decreases with the increases of the mean time of successive breakdowns.
3. If the mean time to repair of a machine is increases, then the production rate and the profit are decreases and if the mean time to repair of a machine is decreases then the production rate and the profit are increases.
4. Production rate and the profit are moderately sensitive to change in the parameter of selling price.

VII. CONCLUSIONS

This paper deals a volume flexible manufacturing inventory model for deteriorating items with multi items and machine breakdown. Demand rate is taken as selling price dependent. Usually, a reduced price encourages a customer to buy more. Production rate is taken as decision variable. Shortages are allowed with partial backlogging. This model has been solved numerically with sensitivity analysis. From the analysis of this model, it has been observed that (i) The cost of ideal time of management units is indirectly proportional to the production rate and the profit. (ii) Mean time of successive breakdowns is reversely proportional to the production rate and the profit. (iii) The mean time to repair gives the reverse effect on the production rate and the profit. This model is much more realistic and

practical and can be extended to include the delay in payments, inflation, stochastic demand and stochastic backlogging rate.

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