Mathematical Model for Awarding Jobs to Sub-Contractors

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ABSTRACT

The modelling is considered one of the modern dependable methods in the engineering systems which handle many problems in the engineering sector, from here came the research idea to use mathematical modelling in subcontracts assignment in the productive sector for achieving optimization in lower cost or higher profits, and also to ensure fair distribution of the subcontracts according to the experience and cost and excluding non-qualified subcontractors.

Whereas the study included researches studying, former studies and dependable techniques in the assignment and studying linear programming method according to the assignment system, so the necessary data for the study were collected from Al-Zawra’a Public Company which performed the project of supplying and installation electrical substations, these data represent tenders prices whom are introduced by the subcontractors for all the subcontracts and the assigned prices for them.

Eight mathematical models were constructed which represent different study cases that could happen to any project depending on the assignment method in the Hungarian way whereas external effects were introduced like forcing the committees to make one or more of the contractors deliver more than a contract or excluding one of the contractors, the program (Win qsb) was used to solve the models.

The study showed the most important conclusion that the Hungarian way may not give the best results because it is restricted in the method of assigning one contract to each contractor and the improved model for this way which allow assigning more than one subcontract to one contractor gave better results than the Hungarian way according to the cost and in helping to speed up the achieving of the project.

Keywords:--- linear programming, subcontractors, The modeling, The assignment Method, Hungrain Method

I. INTRODUCTION

The contracting Companies in Iraq doesn’t follow any scientific method for the assignment system, therefore, the possibility of using the mathematical models in assigning the subcontracts by the Hungarian method to find and distribute the subcontracts with the lowest costs and it can be considered the best solution if there is the term of assigning one subcontract for each subcontractor.

II. RESEARCH OBJECTIVE

The research goals can be represented as follows:
1) Using the assignment method in assigning the subcontracts through mathematical model building for achieving the lowest cost paid by the companies or the highest profits which guarantees works distribution on the biggest number of contractors so that each subcontract can be assigned to one subcontractor which leads to faster implementation.
2) Finding the losses resulted from assigning the subcontract to a specific subcontractor as a result of influences and relationships by using the mathematical modelling.
3) Developing the assignment method so that more than one subcontract can be assigned to any subcontractor when he meet the required terms whereas the assignment method (the Hungarian way) is based on the basis of assigning each contract to one contractor.

III. RESEARCH METHODOLOGY

To access to the required goal, the research methodology includes the following stages:
First) The Theoretical Side:
This stage includes planning, studying and reading on what is available from books, resources, publications, Arabic and foreign periodicals in addition to what is got from the internet and also from former studies and researches related to the research subject in order to get the following information:
1) Information related to the subcontracts and there rules, methods and ways of projects assignment and the factors affecting on choosing the contractors and the techniques used in making decision.
2) Information related to the science of the operations research and its characteristics and its methods, mathematical modelling and its types, how to build the mathematical model, the linear programming method and especially the assignment method and how to use it in the subcontracts assignment.

Second) The Practical Side:

The practical side consists of the following stages:
1) Stage of constructing the information base and carry out the information gathering through visiting the sites that belong to the governmental facilities and the companies that work in the country in the present day in order to recognize the dependable methods of the subcontracts assignment, so it has been chosen the project of electricity stations construction for the Iraqi ministry of electricity which is being implemented in many governorates to use its data in the research whereas the project was assigned to a governmental company which in turn subcontracted with subcontractors to implement these stations in which each station was considered as a subcontract.
2) Putting mathematical models to a group of different study cases in which the computer program (Win qsb) was used to obtain the optimization and then to evaluate the results and show and conclude the important information, it was found the mathematical equations for the mathematical model to solve the assignment problems to many study cases whereas the Hungarian method work was developed in order to make it possible to make the qualified subcontractor get more than one subcontract instead of assigning one subcontract to one subcontractor.

A. Previous Studies
1) Liu, Wang, (2006) [1] prepared a research that include using of the mathematical model to solve the resource assignment problems in the structural projects which leads to improve the work efficiency.
2) Hasan, (2010) [2] suggested to use the computer with its capabilities to determine and choose the best tender among the tenders given by the contracting companies and comparing between them on the basis of cost or period or quality and also the company efficiency that is calculated according to the mathematical equations for the mathematical model to solve the assignment problems in the similar works that were done by the contractor.
3) Abdul-Hameed, (2011) [3] formulated a model in the linear programming to solve the problem of assigning the workers to the machines and then the model was solved by using the simplex method for three study cases and also they were solved by using the transportation algorithm.
4) Rajeiyan and others, (2013) [4] did a research about solving the transportation problems using linear programming in the Companies costs services in a way that guarantees distributing the Companies products all over the world by achieving biggest benefit or minimizing the work cost.

The Contract:

It means the agreed deal which includes acceptance speech, letter of the tender, terms, specifications and blueprints, schedules and any other documents (if found) listed in the contract agreement or in the assignment letter [5].

The Subcontract:

It means the subcontract agreement and the acceptance speech that is issued about the contractor and the subcontract offer. The terms, specifications, blueprints, quantity schedules and any other documents (if found) for the subcontract are listed in the subcontract agreement or in the acceptance speech that is issued about the contractor [6].

The Contractor:

It means the person or persons or the corporation or the company whose tender is accepted by the employer in writing. It includes the authorized representatives of the contractor and persons who succeed him legally and the persons who are allowed to be disclaimed by the employer [7].

The Subcontractor:

It means any person or corporation or company except the contractor that is mentioned in the contract to perform any part of the works or any person who is subcontracted to perform any part of the contract by the engineer's approval in writing. It includes the authorized representatives of the subcontractor and persons who succeed him legally and the persons who are allowed to be disclaimed by the employer [8].

B. The Factors That Affect in Choosing the Contractors [9]:

1) The technical efficiency and the experience in the required work field.
2) The physical qualifications which include the cash balance, equipments and mechanisms that are owned by the contractor.
3) The confidence and good reputation of the contractor.
4) The efficiency of the executive staff and the good administrative organizing are considered the most important side that precede each of the confidence and good reputation of the contractor in exceptional circumstances case.
5) The similar works that were done by the contractor.

C. The New Techniques Used in Making Decision to Choose the Best Tender in the Structural Projects

First) The Technique of Analytic Hierarchy Process (AHP):

The analytic hierarchy process is considered one of most important methods in making multi criteria decision. The professor (Thomas L. Saaty, professor of mathematics in Pittsburgh University, USA) authored this method in the middle of the Seventies; he also authored more than 30 books about it [10]. The technique of analytic hierarchy process has the following Characteristics [11]:

a) It is easy to be applied through putting a goal or a purpose to reach, in addition to its capability of measuring any kind of problems.
b) It can put an organizational structure between the relationships and the effects through the knowledge of more of the problem.
c) It can make collective or individual decisions and find the difference between the experience and the knowledge for the individuals of the group.
d) Allows the difference between opinions to be existed with the capability of making approximation between the different opinions.
e) The individuals can achieve the rules resulted from logic, sense and passion in an organized method.
f) It can be applied with many applications like the linear programming or the goal programming in the applications of the site dividing and the resources distribution.

Second) Techniques for Order Preference by Similarity to Ideal Solution (TOPSIS):

It is one of the most important essential techniques used in multi criteria decision making (MCDM) which was discovered by the scientist (Hwang and Yoon) in 1981, which it takes in consideration the fastest distance from the negative ideal solution and the nearest distance from the positive ideal solution, this preference is arranged according to the approximation between these distances [12]; this method was used widely in the fields of suppliers choosing applications and evaluating them and to compare between the companies, this hypothesis which is based on the principle that every standard of the comparison standards tends either to increase or decrease in the benefit[12].

Characteristics of this Technique (TOPSIS) [13]:

1) The premise which represents the basis of the choice.
2) It can calculate the value of each of the best and the worst alternative at the same time.
3) It is considered one of the simplest mathematical operations and the easiest one so it can be programmed in any data schedule.
4) All kinds of the standards are taken in consideration in this technique.
5) All the used mathematical operations go in one way which is forward.

Third) The Concept of Mathematical Modelling:

The modelling can be defined as a dimensional form or a sketch or a group of mathematical or numerical equations in which the parts that form the model are simple and clear and similar to the problem that the model was put for, the goal of preparing the model lies in the operation of the prediction and the help in making decision in light of the data that are available from the information related to the problem that is required to be solved [14].

Mathematical models are considered the axis of the practical operating for the operations researches, so the model in general is a translation of the problem solution in a mathematical formula, the model consists of a mathematical formula representing the goal function and it describes the real solution behaviour of the problem, and also the variables represent one of the tools of the model that is required to be solved mathematically but it is required that the model must be bare of reality and doesn't simulate it and whenever the model that was built is accurate, this indicates to the optimization in the denudation and not to be affected with the reality [15].

The Basic Items for Linear Programming [16]:

a) Presence of a certain goal that is required to be achieved (highest profit or lowest cost... etc).
b) Presence of different alternatives to reach the goal.
c) The resources that are used must be limited.

d) There must be a relationship between the variables.
e) The function of the goal and restrictions is expressed by equations or linear Inequalities.

IV. THE ASSIGNMENT PROBLEMS

The assignment problem is considered as a special type of the transportation problems and it is also considered one of the important problems in the field of the operations researches. There are many methods used to solve the assignment problems and the most important one is the Hungarian method which is a mathematica feature that was discovered by the Hungarian scientist (Konig) in 1931 and from there came the name of the method which was developed by each of the scientists (Ford and Volokson) in 1957[17].

The Hungarian method that will be used and developed in this research is an algorithm (matrix) to obtain the optimization which solves the assignment problem in a specific time, every assignment matrix consists of a number of rows and columns so any number that exists in the matrix represents the cost of any individual assignment, the assignment problem differs from the transportation problem in two features:

The assignment problem consists of a square matrix.
The best solution of the problem may be obtained by assignment in the row or the column for the matrix cost [18].

The Data Obtained from the Study Cases:

It has been obtained the data for only 15 of the subcontracts (stations) because of the difficulty of getting the information whereas these data include the tender prices introduced by each contractor for all the contracts as shown in the appendix (A).

A. Case Study 1:

In this case, the number of the subcontractors equals the number of the subcontracts (i.e. m= n, number of the rows equals number of the columns) and here, it has been chosen the data for 15 of the subcontractors that present for 15 of the subcontracts, the principle of assigning indicates that each subcontract is assigned to one subcontractor so as to distribute the works for more than one subcontractor which leads to faster achievement for the project, the model of this case is as follows:

In the beginning, we assume that (m) from the subcontracts and (n) from the subcontractors and (i) represents the number of the subcontract and (j) represents the subcontractor, the subcontractor (j) can perform one subcontract (i) only.

Min cost = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}

Subject to:

\sum_{j=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, m \quad (1)

And this Limitation means that one subcontract is performed exactly by one subcontractor.

\sum_{i=1}^{m} x_{ij} = 1, \quad i = 1, 2, \ldots, n \quad (2)

And this Limitation means that one subcontractor perform exactly one subcontract

\begin{align*}
x_{ij} &= 0 & \text{if the } & i^{th} & \text{ is not assigned to } j^{th} \\
x_{ij} &= 1 & \text{if the } & i^{th} & \text{ is assigned to } j^{th}.
\end{align*}
\[ \sum_{i=1}^{m} x_{ij} \geq 0 \]

(Cij): Is the cost of assigning the subcontract (i) to the subcontractor (j).

(Xij): Is the element (place value) in the matrix of subcontractor-subcontractor.

(Xij) = (zero) and that is in the case of non-assigning the subcontract to the subcontractor.

(Xij) = (1) and that is in the case of assigning the subcontract to the subcontractor.

B. Case Study 2:

In this case, it is assigned two of the subcontracts (i) to two of the subcontractors (j) directly, for example, assigning the subcontract (3) to the subcontractor (4) and assigning the subcontract (6) to the subcontractor (8), the rest of the subcontracts are assigned by the matrix. To solve this problem, as each subcontractor is assigned one subcontract, we eliminate the tenders of the subcontractors (4) and (8) i.e. the columns (3) and (8) from the matrix and we also eliminate the tenders of the subcontractors that present for the contracts (3) and (6).

So if the matrix consists of (m) from the subcontracts which equals (15) and (n) from the subcontracts which equals (15), after the elimination, the matrix become consisted of (13) from the subcontracts (the rows) and (13) from the columns, and to find the best solution for it, the cost of assigning the subcontracts (3) and (6) to the subcontractors (4) and (8) is added to finally get the best total cost for this problem and subsequently the amount of loss resulted from assigning any subcontract to a subcontractor outside the best assignment policy which happen as a result of the issuance of directing or occurrence of pressure. To calculate the amount of loss, the total cost of this case is subtracted from the total cost of the normal case (i.e. without any direct assignment) and the solution is as follows:

Let \( n = 13, m = 13 \)

Min cost = \( C_{4+} + C_{8+} + (\sum_{i=1}^{13} \sum_{j=1}^{13} C_{ij}X_{ij}) \)

Subject to:

\[ \sum_{j=1}^{m} x_{ij} = 1, \quad j = 1, 2, \ldots, m -----(1) \]

\[ \sum_{i=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n -----(2) \]

\( X_{ij} = 0 \) if the \( i^{th} \) is not assigned to \( j^{th} \).

\( X_{ij} = 1 \) if the \( i^{th} \) is assigned to \( j^{th} \).

\[ \sum_{i=1}^{n} x_{ij} \geq 0 \]

C. Case Study 3:

In this case, the Hungarian method is developed where we study the possibility of getting three contracts instead of one contract for the contractor and that is not stated in the Hungarian method which states as we mentioned before that one job (contract) is assigned to each worker (contractor), and here the opportunity will be provided for all the subcontractors with the possibility of getting three or two or one contract so as to achieve the principle of justice and transparency and to give equal opportunities for the subcontractors to enter the competition and also when all the offers have specifications that are conformable to the terms and requirements. To solve this problem, we assume the following:

1) We have a matrix consists of (15) rows (subcontract) and (15) columns (subcontractor) and to provide the opportunity which is not certain for each subcontractor, it is possible that these subcontractors get three or two or one subcontract or they don’t get any subcontract.

2) The (15) columns is repeated three times to become (45) columns where the first (15) columns are numbered as (1,2,3,..,15) and the columns from (15-30) are numbered as (1’,2’,3’,..,15’) and the columns from (30-45) are numbered as (1”’,2”’,3”’,..,15”’). Because they have the same tenders costs (i.e. the subcontractor no.1 has the columns with the numbers (1,1’,1”) and so on for the rest of the subcontractors.

3) To achieve the square matrix which will be (45x45), it is necessary to add (30) virtual contracts (rows) to achieve the state of balance so we can get a matrix consists of (45) subcontractors and (45) subcontractors.

4) The matrix is solved by the traditional way to reach the best solution.

5) In the case that any contractor for example (1, 1’, 1”) gets the value (1) for three times in the columns, three subcontracts will be assigned to him and if he gets the value (1) for twice, two subcontracts will be assigned to him and if he gets the value (1) for once only, one subcontract will be assigned to him and so on for the rest of the subcontractors and the solution will be as follows:

Let \( n = 45 \)

Min cost = \( \sum_{i=1}^{45} \sum_{j=1}^{45} C_{ij}X_{ij} \)

Subject to:

\[ \sum_{j=1}^{m} x_{ij} = 1, \quad j = 1, 2, 3, \ldots, m = 45 -----(1) \]

\[ \sum_{i=1}^{n} x_{ij} = 1, \quad i = 1, 2, 3, \ldots, n = 45 -----(2) \]

\( X_{ij} = 0 \) if the \( i^{th} \) is not assigned to \( j^{th} \).

\( X_{ij} = 1 \) if the \( i^{th} \) is assigned to \( j^{th} \).

\[ \sum_{i=1}^{45} x_{ij} \geq 0 \]

VI. WIN QSB PROGRAM

The Program (WIN QSB) (version 2.0) which symbolizes to (Quantitative System for Business) is considered an excellent educating program which is produced by (windows company) and it is one of the most programs that is used in the operations researches to solve many of the big and complex problems with the quantitative nature and it is specialized in solving the linear programming problems in addition to the transportation problems and the assignment and many programs. The intellectual property of this program belongs to the Chinese scientist (chang yei) for a long time ago, and it consists of a group of models that help us in different subjects like the checking in the production operations, simulation, projects evaluation, statistics, quality control and many of the important subjects, total sub-programs included in this program are small programs and they represent models for the operations researches [19].

When the study data were inserted in the program for the three study cases, the results were appeared as shown...
in the schedules and blueprints which are attached herewith which illustrate the cost of the tender that is assigned for each subcontract and the assignment strategy for the different study cases and through the results it was shown the following:

1) The real cost of one subcontract is calculated by dividing the total cost of the project which amounts about (200,000,000,000) two hundred million Iraqi dinars on the number of the subcontracts (the stations) which amounts (40) and this is because that all the subcontracts (the stations) have the same schedules of quantities and technical specifications and the same estimated price and subsequently the cost of one subcontract equals (5,000,000,000) five million Iraqi dinars. To make profits, the company assigned these contracts to subcontractors with a price of (4,200,000,000) four million and two hundred million Iraqi dinars.

2) The total cost of performing (15) subcontracts is (4,200,000,000 x 15 = 63,000,000,000) sixty three million Iraqi dinars.

A. Case Study 1: Through the results that are got for this case and shown in figure (1) and table (1) it is shown that:

1) The total cost of the assignment equals (62,120,000,000) sixty two million and one hundred and twenty million Iraqi dinars which is less than the real total cost of the subcontract assignment which amounts (63,000,000,000) sixty three million Iraqi dinars which in turn achieves the optimization.

2) The areas that lie downward the costs curve and upward the black straight line (represents the cost of the real contract assignment) represent the places of the lowest profits achievement while the areas that lie upward the curve and downward the black straight line represent the places of the highest profits achievement.

3) In this case, each subcontractor has the chance to get one subcontract and subsequently one subcontract will be assigned to each subcontractor.

B. Case Study 2: Through the results that are got for this case and shown in figure (2) and table (II), it is shown that:

1) The total cost equals (54,100,000,000) fifty four million and one hundred million Iraqi dinars in addition to the cost of the two subcontracts that are assigned directly, so the real total cost will become (54,100,000,000 + 4,276,000,000 + 4,276,000,000 = 62,479,000,000) sixty two million and four hundred and seventy nine million Iraqi dinars which is less than the real total cost of the subcontract assignment which amounts (63,000,000,000) sixty three million Iraqi dinars which in turn achieves the optimization.

2) To calculate the amount of loss, the total cost of this case is subtracted from the total cost of the general case (i.e. without direct assignment) and it equals (62,479,000,000 - 62,112,000,000) = (367,000) three hundred and sixty seven thousand Iraqi dinars.

3) The areas that lie downward the costs curve and upward the black straight line (represents the cost of the real subcontract assignment) represent the places of the lowest profits achievement while the areas that lie upward the curve and downward the black straight line represent the places of the highest profits achievement.

4) In this case, each subcontractor has the chance to get one subcontract and subsequently one subcontract will be assigned to each subcontractor.

<table>
<thead>
<tr>
<th>Subcontractors</th>
<th>Sub Contract</th>
<th>Sub Contract x10^6 Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3913</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>3914</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>3950</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>3952</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>4322</td>
</tr>
<tr>
<td>Total Cost x10^6</td>
<td></td>
<td>62120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subcontractors</th>
<th>Sub Contract</th>
<th>Sub Contract x10^6 Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3913</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>3925</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>3990</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>4332</td>
</tr>
</tbody>
</table>
C. Case Study 3:

Through the results that are got for this case and shown in figure (3) and table (III), it is shown that:

1) The total cost equals (59,602,000,000) fifty nine miller and six hundred and two million Iraqi dinars which is less than the real total cost of the subcontract assignment which amounts (63,000,000,000) sixty three miller Iraqi dinars which in turn achieves the optimization.

2) All the costs of the assigned subcontracts lie downward the black straight line (represents the cost of the real subcontract assignment) and it means that there is profit achievement in all the subcontracts.

3) The subcontractors (1) and (2) and (4) and (13) have been assigned three subcontracts for each of them after providing the opportunity for all the subcontractors to get more than one subcontract.

<table>
<thead>
<tr>
<th>Subcontractors</th>
<th>Sub Contract</th>
<th>Sub Contract × 10^6</th>
<th>Cost</th>
<th>Total Cost 10^8</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>10</td>
<td>4255</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>4326</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>4096</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>4254</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>4398</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>4284</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>3973</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>9</td>
<td>4145</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>4209</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Cost 10^8</td>
<td></td>
<td></td>
<td>54,100</td>
<td></td>
</tr>
</tbody>
</table>

TABLE III
Assignment Strategy for the Third Case

Subject to:

\[
\sum_{j=1}^{n} X_{ij} = 1, \quad j = 1, 2, \ldots, m \quad \text{(1)}
\]

\[
\sum_{i=1}^{m} X_{ij} = 1, \quad i = 1, 2, \ldots, n \quad \text{(2)}
\]

\[
X_{ij} = 0 \quad \text{if the } i^{th} \text{ is not assigned to } j^{th}.
\]

\[
X_{ij} = 1 \quad \text{if the } i^{th} \text{ is assigned to } j^{th}.
\]

2) The costs of the tenders that are existed in the matrix are diverted to profits for the subcontractors by subtracting the price in which the original subcontract was assigned to each subcontract which is one price of the cost of the tender that is introduced from each company so that it can be got the profits from each subcontract.

3) The matrix was solved using the program (Win qsb) and the results were obtained as shown in the schedules (1, 2, 3) which are shown above.

4) It was got the profit whom the original company will get for the first case by subtracting the total cost in which the original subcontract was assigned for (15) subcontracts from the total cost resulted from solving the study case by the lowest cost way.

5) Figure (4) shows the results got for the fourth case.

VI. APPROVAL OF THE MODEL

This step is the process of checking the model that was solved in the previous step, and making sure whether the solution matches the required goals or not, and also the process of checking the model increases the exactitude of the solution and refers to the trustiness of the model, and to make sure of the exactitude of the results that are got for the study cases we did a test (checking) to the results for the first study case through the preparation of a study case which is opposite to this case so it was named the fourth study case, it can be done by following the next steps:

1) The goal function is diverted from (Min.) to (Max.), in other word, from a function of a lower cost to a Function of a higher profit, the solution of the problem can be done as follows:

\[
\text{Max cost } = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}
\]

Subject to:

\[
\sum_{j=1}^{n} X_{ij} = 1, \quad j = 1, 2, \ldots, m \quad \text{(1)}
\]

\[
\sum_{i=1}^{m} X_{ij} = 1, \quad i = 1, 2, \ldots, n \quad \text{(2)}
\]

\[
X_{ij} = 0 \quad \text{if the } i^{th} \text{ is not assigned to } j^{th}.
\]

\[
X_{ij} = 1 \quad \text{if the } i^{th} \text{ is assigned to } j^{th}.
\]
The efficiency of the mathematical model was shown through achieving high profits for the original executive company and especially when the goal is to get the lowest cost.

The final results were shown for the study cases as it is stated in the following points:

1) All the costs for the three study cases were less than the real total costs in which the subcontract was assigned and this means achieving the optimization for all the study cases as shown in figure (5).

2) In the third study case, there was the lowest cost (highest profit) because that this case included giving the opportunity for the subcontractors to get more than one subcontract and this means that if the subcontractor is given more than one chance for the subcontract, it leads to increasing in the optimization where the profit increases and the cost decreases and subsequently this means that the developing of the Hungarian method that was done in this research as shown in table (IV) succeeded.

### TABLE IV
The Total Costs for the Study Cases

<table>
<thead>
<tr>
<th>Best Solution of Cases</th>
<th>Total Cost</th>
<th>No. of case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>59,602.00</td>
<td>Case no. 3</td>
</tr>
<tr>
<td>3</td>
<td>62,120.00</td>
<td>Case no. 1</td>
</tr>
<tr>
<td>8</td>
<td>62,479.00</td>
<td>Case no. 2</td>
</tr>
</tbody>
</table>

1) The worst cases are those which includes interventions or assigning the subcontracts directly like the second case and this means that if the subcontracts are assigned directly, it leads to decreasing in the profit of the original company and increasing in the cost (i.e. decreasing in the optimization) as shown in figure no.6 which shows the differentiation among three different cases.

### TABLE V
Assignment Strategy for the Cases of the reduction and enlargement

<table>
<thead>
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</table>

2) It has been got the same assignment results either the solution is done using the goal function of the lowest tender value (min.) or using the goal function of the highest profit and for all the study cases as shown in schedule (V) which shows the assignment strategy in the cases of the reduction and enlargement for the first study case and the fourth checking case.
VII. CONCLUSIONS AND RECOMMENDATIONS

The researcher devised the following conclusions:

1) The possibility of using the assignment system in the linear programming by the Hungarian method to find and distribute the subcontracts with the lowest costs and it can be considered the best solution if there is the term of assigning one subcontract for each subcontractor.

2) It is obvious that developing the mathematical model for the Hungarian method which includes non-restriction in assigning a subcontractor for each subcontract gives better results than the Hungarian method which restricts the assignment.

3) The possibility of finding the loss resulted from assigning the subcontracts in a random way or under the effect of the outer factors.

4) The possibility of eliminating any contractor who doesn't match the required technical specification for the contract through the matrix that was put directly.

5) Reaching the best solution through development of the assignment system in the Hungarian method so it is possible to assign more than one subcontract to any of the subcontractors.

6) Through the mathematical models it is obvious that when using the method of (Max.) and (Min.) it gives the same results when the calculations of the costs and profits are small.

7) The Companies don't follow any scientific method for the assignment.

8) The possibility of using the mathematical models in assigning the subcontracts even if there is the effect of the other factors through calculating the points for each subcontractor.

9) Assigning the subcontracts for more than one subcontractor using the mathematical modeling helps in speed up in performing the works with the lowest costs.

10) The lowest profits (highest cost) is achieved in the cases where some subcontracts are assigned for subcontractors directly and outside the assignment system and the loss increases whenever the number of the subcontracts that are assigned directly increases.

11) The linear programming that has integers is an excellent way in which the decision makers can use in all of the fields including the subcontracts assignment to reach the best solution by achieving the highest profit or the lowest cost.

12) The possibility of applying the computer program (Win qsb) in solving the mathematical problems related to the projects of the assignment because it characterizes in the ease of use and it solves all of the problems whatever the numbers does it have.

13) The possibility of making adjustments and improving the mathematical model which is prepared to suit with any project weather it was small or big and that requires only providing enough information about the project.

14) In most of the subprojects, the contract of the quantity schedule is currently prevalent.

15) Non-using of the linear programming and the computer programs that are related to it in the projects assignment and especially the subcontracts in most of the governmental companies and the contracts companies.

Through the previous conclusions and based on what the research produced, the researcher recommends the following:

1) Encouraging the companies to use the scientific methods and the modeling in the subcontracts assignment to achieve the highest profits or the lowest costs and guaranteeing the works distribution to achieve faster performing.

2) Using the mathematical modeling (assignment method) to achieve the full control to prevent assigning any subcontract directly under the effect of any circumstance.

3) Conducting training courses for the engineering staff in the departments and governmental companies on using the linear modeling methods to reach the best solution.

VIII. FUTURE RESEARCHES

Continuing this research, the researcher suggests performing the following studies:

1) Developing the mathematical model in order to be used in the assignment for all the projects either they are major or minor.

2) Developing the mathematical model by means of inserting the other standards except the standard of the cost in the subcontracts assignment and calculating them as points instead of the costs numbers to make use of to reach the best solution.

3) Using the linear programming in the scheduling of the subcontracts performing works.

REFERENCES


[2] Hasan, Sada Abdul-Khalilq, "Using the Programming Applications in solving the difficulty in choosing the tenders in the contracts (structural, supplementing)", College of Engineering / Al-Qadisiyah University, the engineering magazine, tome 16 / no.2, 2010.


