Nano Star Pre Generalized Continuous Functions in Nano Topological Spaces

Nancy J E1, Vaiyomathi K2
1PG Student, Nirmala College for Women, Coimbatore, Tamil Nadu, INDIA
2Assistant Professor, Nirmala College for Women, Coimbatore, Tamil Nadu, INDIA

ABSTRACT
The purpose of this paper is to introduce, Nano star pre generalized continuous function in Nano Topological Spaces.

Keywords-- Nano topological space, Nano*pg closed, Nano*pg continuous function

I. INTRODUCTION
Continuous function is one of the main concepts of Topology. Balachandran et al. [2] and Mashour et al. [3] have introduced g-continuous and pre-continuous function in topological spaces respectively. Arokiarani [1] introduced generalized pre-continuous functions and generalized pre-irresolutes functions and compared with various stronger forms of the same functions. The notion of Nano topology was introduced by LellisThivagar[5] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and he also defined Nano closed sets, Nano-interior, Nano-closure. Nano continuous functions, Nano open mapping, Nano preopen mapping and Nano Homeomorphism. Nancy et al[4] introduced and studied some properties of Nano star pre generalized closed sets in Nano topological spaces. In this paper, a new class of continuous functions called Nano star pre generalized continuous function is introduced and some of its properties in terms of Ng – closed sets, Ng-closure and Ng-interior are discussed.

II. PRELIMINARIES

Definition:2.1[6]A subset A of \((U,\tau_R(U))\) is called (i) Nano pre generalized closed set (briefly Npg-closed) if \(\text{Npcl}(A)\subseteq V\) whenever \(A\subseteq V\) and \(V\) is Nanopre open in \((U,\tau_R(U))\).

Definition:2.2[7] Let \(U\) be non-empty finite universe of objects and \(R\) be an equivalence relation on \(U\). \(\tau_R(U)\) satisfies the following axioms:

\(\tau_R(U) = \{U, \varphi, L_R(U), U_R(U), B_R(U)\}\). If \(\tau_R(U)\) satisfies the following axioms:

The union of the elements of any subcollection of \(\tau_R(U)\) is in \(\tau_R(U)\).

The intersection of the elements of any finite sub collection of \(\tau_R(U)\) is in \(\tau_R(U)\).

Then \(\tau_R(U)\) is a topology on \(U\) called the nano topology on with respect to \(U\). The elements of \(\tau_R(U)\) are called as nano-open sets.
Definition: 2.5[7]: A subset of \((U, τ_γ(X))\) is called Nano generalized closed set (briefly Ng-closed) if \(\text{Ncl}(A)\subseteq V\) whenever \(A\subseteq V\) and \(V\) is Nano open set in \((U, τ_γ(X))\).

Definition: 2.6[4]: A subset \(A\) of \((U, τ_γ(X))\) is called \(\text{Nano*pre}\) generalized closed set (briefly \(\text{N*pg}\)-closed) if \(\text{Nint}(\text{Npcl}(A))\subseteq V\) whenever \(A\subseteq V\) and \(V\) is \(\text{Nano}\) pre open in \((U, τ_γ(X))\).

Definition: 2.7[3]: Let \((U, τ_γ(X))\) and \((V, τ_R(Y))\) be two nano topological spaces. Then a mapping \(f: (U, τ_γ(X)) \rightarrow (V, τ_R(Y))\) is \(\text{Nano*pre}\) generalized continuous (shortly \(\text{Ng}\)-continuous) on \(U\) if the inverse image of every nano open set in \((V, τ_R(Y))\) is \(\text{Nano*pre}\) generalized open set in \((U, τ_γ(X))\).

Example: 3.5 Let \(U=\{a,b,c\}\) with \(U/R=\{\{a\},\{b,c\}\}\) and \(X=\{a,c\}\) and \(\tau_g(X)=\{\{a\},\{a,c\}\}\) with \(U/R=\{\{a\}\}\) and \(X=\{a\}\) and \(\tau_g(Y)=\{\{b\},\{a,c\}\}\) with \(V/R=\{\{b\}\}\) and \(Y=\{a\}\) and \(Y=\{a\}\) and \(Y=\{a\}\). Define \(f: (U, τ_γ(X)) \rightarrow (V, τ_R(Y))\) as \(f(a)=a, f(b)=b, f(c)=b,\) which is \(\text{Nano*pre}\) generalized continuous. But for the nano open set \(\{a\}\) in \((V, τ_R(Y))\) its inverse \(f^{-1}(\{a\}) = \{a\}\) is not nano open in \((U, τ_γ(X))\). Hence \(f\) is not \(\text{Nano*pre}\) generalized continuous.

Theorem: 3.6 Every nano pre continuous function is \(\text{Nano*pre}\) generalized continuous.

Proof: Let \(f: (U, τ_γ(X)) \rightarrow (V, τ_R(Y))\) be nano pre continuous on \((U, τ_γ(X))\). Since \(f\) is nano pre continuous of \((U, τ_γ(X))\), the inverse image of every nano open set in \((V, τ_R(Y))\) is nano open in \((U, τ_γ(X))\). But every nano pre open set is \(\text{Nano*pre}\) generalized open set. Hence the inverse image of every nano open set \((V, τ_R(Y))\) is \(\text{Nano*pre}\) generalized open set in \((U, τ_γ(X))\). Therefore, \(f\) is \(\text{Nano*pre}\) generalized continuous.

Remark: 3.7 The converse of the above theorem is not true as seen from the following example.

Example: 3.8 Let \(U=\{a,b,c\}\) with \(U/R=\{\{a\}\}\) with \(U/R=\{\{a\},\{b\}\}\) and \(X=\{a\}\) and \(τ_R(Y)=\{\{b\},\{a,c\}\}\) with \(V/R=\{\{b\}\}\) and \(Y=\{a\}\) and \(Y=\{a\}\). Define \(f: (U, τ_γ(X)) \rightarrow (V, τ_R(Y))\) as \(f(a)=a, f(b)=b\), which is \(\text{Nano*pre}\) generalized continuous. But for the nano closed set \(\{b\}\) in \((V, τ_R(Y))\) its inverse \(f^{-1}(\{b\}) = \{b\}\) is not nano pre open in \((U, τ_γ(X))\). Hence \(f\) is nano pre continuous.

Theorem: 3.9 Every nano generalized continuous function is \(\text{Nano*pre}\) generalized continuous.

Proof: Let \(f: (U, τ_γ(X)) \rightarrow (V, τ_R(Y))\) be nano generalized continuous on \((U, τ_γ(X))\). Since \(f\) is nano generalized continuous of \((U, τ_γ(X))\), the inverse image of every nano open set in \((V, τ_R(Y))\) is nano open in \((U, τ_γ(X))\). But every nano open set is \(\text{Nano*pre}\) generalized open set. Hence the inverse image of every nano open set \((V, τ_R(Y))\) is \(\text{Nano*pre}\) generalized open set in \((U, τ_γ(X))\). Therefore, \(f\) is \(\text{Nano*pre}\) generalized continuous.

Remark: 3.10 The converse of the above theorem is not true as seen from the following example.

Example: 3.11 Let \(U=\{a,b,c\}\) with \(U/R=\{\{a\}\}\) with \(U/R=\{\{a\},\{b\}\}\) and \(X=\{a\}\) and \(τ_R(Y)=\{\{b\},\{a,c\}\}\) with \(V/R=\{\{b\}\}\) and \(Y=\{a\}\) and \(Y=\{a\}\). Define \(f: (U, τ_γ(X)) \rightarrow (V, τ_R(Y))\) as \(f(a)=a, f(b)=b,\) which is \(\text{Nano*pre}\) generalized continuous. But for the nano open set \(\{a\}\) in \((V, τ_R(Y))\) its inverse \(f^{-1}(\{a\}) = \{a\}\) is not nano generalized open in \((U, τ_γ(X))\). Hence \(f\) is not nano generalized continuous.

Theorem: 3.12 A function \(f: (U, τ_γ(X)) \rightarrow (V, τ_R(Y))\) is \(\text{Nano*pre}\) generalized continuous iff the inverse image of every nano open set in \((V, τ_R(Y))\) is \(\text{Nano*pre}\) generalized open in \((U, τ_γ(X))\).

Proof: Let \(f: (U, τ_γ(X)) \rightarrow (V, τ_R(Y))\) be \(\text{Nano*pre}\) generalized continuous and \(F\) be nano open in \((V, τ_R(Y))\). Then \(f^{-1}(F)\) is nano closed in \((V, τ_R(Y))\). Since \(f\) is \(\text{Nano*pre}\) generalized continuous.
\( f^{-1}(F^c) \) is nano*pre generalized closed in 
\((U, \tau_R(X))\). But \( f^{-1}(F^c) = (f^{-1}(F))^c \) Therefore, \( f^{-1}(F) \) is nano*pre generalized open in 
\((U, \tau_R(X))\). Thus the inverse image of every nano open set in 
\((V, \tau_R(Y))\) is nano*pre generalized open in 
\((U, \tau_R(X))\). If \( f \) is nano*pre generalized continuous on 
\((U, \tau_R(X))\).

Conversely, Assume that \( f^{-1}(F) \) is nano*pre generalized open in 
\((U, \tau_R(X))\) for each nano open set in 
\((V, \tau_R(Y))\). Let \( G \) be a nano closed set in 
\((V, \tau_R(Y))\). Then \( G^c \) is nano open in 
\((V, \tau_R(Y))\) and by assumption, \( f^{-1}(G^c) \) is nano*pre generalized continuous on 
\((U, \tau_R(X))\). Since \( f^{-1}(G^c) = (f^{-1}(G))^c \) we have \( f^{-1}(G) \) is nano*pre generalized continuous on 
\((U, \tau_R(X))\). Therefore \( f \) nano*pre generalized continuous.

REFERENCES

generalized closed sets in Nano topological spaces.
math.,17(1996), 33-42
forms of weakly open sets,. International Journal of
Mathematics and Statistics Invention, Volume Issue
1,August 2013,PP.31 -37.
Nano Continuity. Mathematical Theory and Modeling.,
3(7): 32
On Nano Generalized continuous function in Nano
Topological Spaces. International Journal of
Mathematical Archive., 6 (6) : 182-186