Observations on Total the Surface Area of the Cuboid and Polygonal Numbers

Pandichelvi .V1, Sivakamasundari .P2
1Assistant Professor, Department of Mathematics, Urumu Dhanalakshmi College, Trichy, INDIA
2Guest Lecturer, Department of Mathematics, BDUCC,Lalgudi, Trichy, INDIA

ABSTRACT

In this paper, we evaluate the dimensions of the Cuboid, the ranks of triangular number and Dodecagonal number such that the relation \(2 \times \text{TSA} = 32T_{3,n} - 5T_{12,m}\) is satisfied. Also, we determine the dimensions of the Cuboid, the ranks of pentagonal number and hexagonal number such that the relation \(\frac{1}{2} \times \text{TSA} = 96T_{5,n} - 32T_{6,m}\) is satisfied.

Keywords-- Quadratic equation with four unknowns, Polygonal numbers, Ramanujan equation

I. NOTATIONS USED

\(T_{3,n} = \frac{1}{2} (n^2 + n)\) be a triangular number of rank \(n\).

\(T_{5,n} = \frac{1}{2} (3n^2 - n)\) be a pentagonal number of rank \(n\).

\(T_{6,n} = 2n^2 - n\) be a hexagonal number of rank \(n\).

\(T_{12,n} = 5n^2 - 4n\) be a dodecagonal number of rank \(n\).

\(\text{TSA}\) - Total surface area of the Cuboid.

II. INTRODUCTION

Diophantine problems dominated most of the celebrated unsolved Mathematical problems. Certain from physical problems or from immediate Mathematical generalizations and others come from geometric in a variety of ways [1-5]. In [6-18], different relations between the Pythagorean triangles and some polygonal numbers are analyzed. In this communication, we find out the relation between the total surface area of the cuboid and some polygonal numbers. In particular, we evaluate the dimensions of the Cuboid, the ranks of triangular number and Dodecagonal number such that the relation \(2 \times \text{TSA} = 32T_{3,n} - 5T_{12,m}\) is satisfied. Also, we determine the dimensions of the Cuboid, the ranks of pentagonal number and hexagonal number such that the relation \(\frac{1}{2} \times \text{TSA} = 96T_{5,n} - 32T_{6,m}\) is satisfied.

III. METHOD OF ANALYSIS

Let us consider the cuboid whose length, breadth and height be \(l, b\) and \(h\) respectively.

Then, the Total surface area of the Cuboid \(= 2(lb + bh + hl)\). The detailed explanation of the connection between the total surface area of the cuboid, triangular number, dodecagonal number, pentagonal number and hexagonal number are given in the following two sections.

SECTION: 1
The Total Surface Area of the Cuboid, Triangular Number and Dodecagonal Number

Assume that

\[2 \times \text{TSA} = 32T_{3,n} - 5T_{12,m}\]

which implies that

\[4(lb + bh + hl) = (4n + 2)^2 - (5m - 2)^2\] (1)

Introduction of the linear transformations,

\[l = u + v, b = u - v, h = 4u\]

where \(u \neq v \neq 0\) (2)

in (1), we find that

\[(6u)^2 + (5m - 2)^2 = (2v)^2 + (4n + 2)^2\] (3)
\[ x^2 + z^2 = y^2 + w^2 \] 

(4) 

where 
\[ x = 6u, z = 5m - 2, y = 2v, w = 4n + 2 \] 

(5) 

Equation (4) represents the well-known second order Ramanujan equation which is satisfied by 
\[
\begin{align*}
x &= rp - sq \\
z &= rq + sp \\
y &= rq - sp \\
w &= rp + sq
\end{align*}
\]

(6) 

Using (6) in (5), we note that 
\[
\begin{align*}
u &= \frac{rp - sq}{6} \\
v &= \frac{rq - sp}{2} \\
m &= \frac{rq + sp + 2}{5} \\
n &= \frac{rp + sq - 2}{4}
\end{align*}
\]

(7) 

Here, we detect that the values of \( u, v, m \) and \( n \) are integers when

**Remark:** 
From (13), we have 
\[
2 \times TSA = 1244160000g^4 + 67392000g^3 - 43200g^2 - 17280g - 108
\]

(14) 

Employing the values of \( m \) and \( n \) as given in (11) and (12), we get 
\[
32T_{3,n} = 2540160000g^4 + 266112000g^3 + 9590400g^2 + 137280g + 672 \\
5T_{12,m} = 1296000000g^4 + 198720000g^3 + 9633600g^2 + 154560g + 780
\]

Therefore, 
\[
32T_{3,n} - 5T_{12,m} = 1244160000g^4 + 67392000g^3 - 43200g^2 - 17280g - 108
\]

(15) 

From (14) and (15), we get 
\[
2 \times TSA = 32T_{3,n} - 5T_{12,m}
\]

Some numerical examples satisfying (1) are tabulated as follows. 

<table>
<thead>
<tr>
<th>( g )</th>
<th>( m )</th>
<th>( n )</th>
<th>( l )</th>
<th>( b )</th>
<th>( h )</th>
<th>( 2 \times TSA )</th>
<th>( 32T_{3,n} - 5T_{12,m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7758</td>
<td>13266</td>
<td>10467</td>
<td>1935</td>
<td>24804</td>
<td>1311491412</td>
<td>1311491412</td>
</tr>
<tr>
<td>2</td>
<td>29910</td>
<td>51726</td>
<td>40127</td>
<td>8675</td>
<td>97604</td>
<td>2.044548853 \times 10^{10}</td>
<td>2.044548853 \times 10^{10}</td>
</tr>
</tbody>
</table>

\[ r = 180g + 2, p = 240g + 8, s = 60g + 1, q = 120g + 10 \] 

(8) 

The above choices lead (7) to 
\[
u = 6000g^2 + 200g + 1
\]

(9) 

\[
v = 3600g^2 + 660g + 6
\]

(10) 

\[
m = 7200g^2 + 552g + 6
\]

(11) 

\[
n = 126000g^2 + 660g + 6
\]

(12) 

Knowing the values of \( u \) and \( v \), the length, breadth and height of the cuboid which satisfies (1) are represented by 
\[
\begin{align*}
l &= 9600g^2 + 860g + 7 \\
b &= 2400g^2 - 460g - 5 \\
h &= 2400g^2 + 800g + 4
\end{align*}
\]

(13) 

Hence, the ranks of triangular number, dodecagonal number and the dimensions of the cuboid, satisfying our hypothesis are pointed out by (11), (12) and (13).
\[ \frac{1}{2} \times \text{TSA} = 96T_{5,n} - 32T_{6,m} \] (16)

which reduces to

\[ (lb + bh + hl) = (12n - 2)^2 - (8m - 2)^2 \] (17)

Introducing the linear transformations

\[ l = u + v; b = u - v; h = 4u \]

where \( u \neq v \neq 0 \)

Equation (17) can be written as

\[ (3u)^2 + (8m - 2)^2 = (v)^2 + (12n - 2)^2 \] (19)

\[ \Rightarrow x^2 + z^2 = y^2 + w^2 \] (20)

where \( x = 3u, z = 8m - 2, y = v, w = 12n - 2 \) (21)

Comparing (21) and the general solutions to the second order Ramanujan equation (20) which are given in (6), we find that

\[
\begin{align*}
\frac{u}{3} &= \frac{rp - sq}{3} \\
\frac{v}{1} &= \frac{rq - sp}{1} \\
\frac{m}{8} &= \frac{rq + sp + 2}{8} \\
\frac{n}{12} &= \frac{rp + sq + 2}{12}
\end{align*}
\] (22)

**Remark:**

From (28), we observe that

\[ \frac{1}{2} \times \text{TSA} = 509607936g^4 + 58392576g^3 + 2433024g^2 + 43776g + 288 \] (29)

Using the values of \( m \) and \( n \) as given in (26) and (27), we get

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>66462</td>
<td>115386</td>
<td>88987</td>
<td>20215</td>
</tr>
<tr>
<td>4</td>
<td>117414</td>
<td>204246</td>
<td>157047</td>
<td>36555</td>
</tr>
<tr>
<td>5</td>
<td>182766</td>
<td>318306</td>
<td>244307</td>
<td>57695</td>
</tr>
<tr>
<td>6</td>
<td>262518</td>
<td>457566</td>
<td>350767</td>
<td>83635</td>
</tr>
</tbody>
</table>

We observe that, the values \( u, v, m \) and \( n \) are integers when

\[ r = 144g + 4, p = 192g + 5, s = 48g + 2, q = 96g + 1 \] (23)

Substituting (23) in (22), we have

\[ u = 7680g^2 + 416g + 6 \] (24)

\[ v = 4608g^2 - 96g - 6 \] (25)

\[ m = 2880g^2 + 144g + 2 \] (26)

\[ n = 2688g^2 + 144g + 2 \] (27)

Using (24) and (25) in (18), the dimensions of the cuboid which satisfies (16) are indicated by

\[
\begin{align*}
\frac{l}{6} &= 6144g^2 + 160g \\
\frac{b}{1536} &= 256g + 6 \\
\frac{h}{15360} &= 832g + 12
\end{align*}
\] (28)

Hence, the ranks of pentagonal number, hexagonal number and the dimensions of the Cuboid satisfying our assumption are expressed by (26), (27) and (28).
96T_{5,n} = 1040449536g^4 + 111476736g^3 + 4405248g^2 + 76032g + 482
32T_{6,m} = 530841600g^4 + 53084160g^3 + 1972224g^2 + 32256g + 194

Therefore,

96T_{5,n} - 32T_{6,m} = 509607936g^4 + 58392576g^3 + 2433024g^2 + 43776g + 288 \quad (30)

From (29) and (30), we get

\frac{1}{2} \times TSA = 96T_{5,n} - 32T_{6,m}

Some numerical examples satisfying (16) are tabulated as follows

<table>
<thead>
<tr>
<th>g</th>
<th>m</th>
<th>n</th>
<th>l</th>
<th>b</th>
<th>h</th>
<th>96T_{5,n} - 32T_{6,m}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3026</td>
<td>2834</td>
<td>12608</td>
<td>3596</td>
<td>32408</td>
<td>570477600</td>
</tr>
<tr>
<td>2</td>
<td>11810</td>
<td>11042</td>
<td>49792</td>
<td>13324</td>
<td>126232</td>
<td>8630687520</td>
</tr>
<tr>
<td>3</td>
<td>26354</td>
<td>24626</td>
<td>111552</td>
<td>29196</td>
<td>281496</td>
<td>4.28768712 \times 10^{10}</td>
</tr>
<tr>
<td>4</td>
<td>46658</td>
<td>43586</td>
<td>197888</td>
<td>51212</td>
<td>498200</td>
<td>1.342358602 \times 10^{11}</td>
</tr>
<tr>
<td>5</td>
<td>72722</td>
<td>67922</td>
<td>308800</td>
<td>79372</td>
<td>776344</td>
<td>3.258650767 \times 10^{11}</td>
</tr>
</tbody>
</table>

IV. CONCLUSION

In this paper, we find the dimensions of the cuboid, the ranks of polygonal numbers such that the total surface area of the cuboid is equal to linear combination of certain polygonal numbers. In this manner, one may consider the relation for other geometrical figures together with two dimensional, three dimensional and four dimensional figurate numbers.

REFERENCES