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Observations on Total the Surface Area of the Cuboid and Polygonal Numbers

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ABSTRACT

In this paper, we evaluate the dimensions of the Cuboid, the ranks of triangular number and Dodecagonal number such that the relation $2\times TSA = 32T_{3,n} - 5T_{12,m}$ is satisfied. Also, we determine the dimensions of the Cuboid, the ranks of pentagonal number and hexagonal number such that the relation $\frac{1}{2}\times TSA = 96T_{5,n} - 32T_{6,m}$ is satisfied.

***Keywords*--** Quadratic equation with four unknowns, Polygonal numbers, Ramanujan equation

I. NOTATIONS USED

$T_{3,n} = \frac{1}{2}\left(n^2 + n\right)$ be a triangular number of rank n.

$T_{5,n} = \frac{1}{2}\left(3n^2 - n\right)$ be a pentagonal number of rank n.

$T_{6,n} = 2n^2 - n$ be a hexagonal number of rank n.

$T_{12,n} = 5n^2 - 4n$ be a dodecagonal number of rank n.

TSA - Total surface area of the Cuboid.

II. INTRODUCTION

Diophantine problems dominated most of the celebrated unsolved Mathematical problems. Certain from physical problems or from immediate Mathematical generalizations and others come from geometric in a variety of ways [1-5]. In [6-18], different relations between the Pythagorean triangles and some polygonal numbers are analyzed. In this communication, we find out the relation between the total surface area of the cuboid and some polygonal numbers. In particular, we evaluate the dimensions of the Cuboid, the ranks of triangular number and Dodecagonal number such that the relation $2\times TSA = 32T_{3,n} - 5T_{12,m}$ is satisfied. Also, we determine the dimensions of the Cuboid, the ranks of pentagonal number and hexagonal number such that the relation $\frac{1}{2}\times TSA = 96T_{5,n} - 32T_{6,m}$ is satisfied.

III. METHOD OF ANALYSIS

Let us consider the cuboid whose length, breath and height be l, b and h respectively.

Then, the Total surface area of the Cuboid $= 2(lb + bh + hl)$.The detailed explanation of the connection between the total surface area of the cuboid, triangular number, dodecagonal number, pentagonal number and hexagonal number are given in the following two sections.

SECTION: 1

The Total Surface Area of the Cuboid, Triangular Number and Dodecagonal Number

Assume that

$$2\times TSA = 32T_{3,n} - 5T_{12,m}$$

which implies that

$$4(lb + bh + hl) = (4n+2)^2 - (5m-2)^2 \quad (1)$$

Introduction of the linear transformations,

$$l = u + v, b = u - v, h = 4u \quad \text{where} \quad u \neq v \neq 0 \quad (2)$$

in (1), we find that

$$(6u)^2 + (5m-2)^2 = (2v)^2 + (4n+2)^2 \quad (3)$$

$$\Rightarrow x^2 + z^2 = y^2 + w^2$$
(4)

where $x = 6u, z = 5m - 2, y = 2v, w = 4n + 2$
(5)

Equation (4) represents the well-known second order Ramanujan equation which is satisfied by

$$\left.\begin{aligned} x &= rp - sq \\ z &= rq + sp \\ y &= rq - sp \\ w &= rp + sq \end{aligned}\right\}$$
(6)

Using (6) in (5), we note that

$$\left.\begin{aligned} u &= \frac{rp - sq}{6} \\ v &= \frac{rq - sp}{2} \\ m &= \frac{rq + sp + 2}{5} \\ n &= \frac{rp + sq - 2}{4} \end{aligned}\right\}$$
(7)

Here, we detect that the values of u, v, m and n are integers when

$$r = 180g + 2, p = 240g + 8, s = 60g + 1, q = 120g + 10$$
(8)

The above choices lead (7) to

$$u = 6000g^2 + 200g + 1$$
(9)

$$v = 3600g^2 + 660g + 6$$
(10)

$$m = 7200g^2 + 552g + 6$$
(11)

$$n = 126000g^2 + 660g + 6$$
(12)

Knowing the values of u and v , the length, breath and height of the cuboid which satisfies (1) are represented by

$$\left.\begin{aligned} l &= 9600g^2 + 860g + 7 \\ b &= 2400g^2 - 460g - 5 \\ h &= 24000g^2 + 800g + 4 \end{aligned}\right\}$$
(13)

Hence, the ranks of triangular number, dodecagonal number and the dimensions of the cuboid, satisfying our hypothesis are pointed out by (11), (12) and (13).

Remark:

From (13), we have

$$2 \times TSA = 1244160000g^4 + 67392000g^3 - 43200g^2 - 17280g - 108 \quad (14)$$

Employing the values of m and n as given in (11) and (12), we get

$$32T_{3,n} = 2540160000g^4 + 266112000g^3 + 9590400g^2 + 137280g + 672$$

$$5T_{12,m} = 1296000000g^4 + 198720000g^3 + 9633600g^2 + 154560g + 780$$

Therefore,

$$32T_{3,n} - 5T_{12,m} = 1244160000g^4 + 67392000g^3 - 43200g^2 - 17280g - 108 \quad (15)$$

From (14) and (15), we get

$$2 \times TSA = 32T_{3,n} - 5T_{12,m}$$

Some numerical examples satisfying (1) are tabulated as follows.

g	m	n	l	b	h	$2 \times TSA$	$32T_{3,n} - 5T_{12,m}$
1	7758	13266	10467	1935	24804	1311491412	1311491412
2	29910	51726	40127	8675	97604	$2.044548853 \times 10^{10}$	$2.044548853 \times 10^{10}$

3	66462	115386	88987	20215	218404	$1.025961032 \times 10^{11}$	$1.025961032 \times 10^{11}$
4	117414	204246	157047	36555	387204	$3.228172875 \times 10^{11}$	$3.228172875 \times 10^{11}$
5	182766	318306	244307	57695	604004	$7.860228334 \times 10^{11}$	$7.860228334 \times 10^{11}$
6	262518	457566	350767	83635	868804	$1.626986373 \times 10^{11}$	$1.626986373 \times 10^{11}$

SECTION: 2

The Total Surface Area of the Cuboid, Pentagonal number and Hexagonal number

Consider

$$\frac{1}{2} \times TSA = 96T_{5,n} - 32T_{6,m} \quad (16)$$

which reduces to

$$(lb + bh + hl) = (12n-2)^2 - (8m-2)^2 \quad (17)$$

Introducing the linear transformations

$$l = u+v, b = u-v, h = 4u \text{ where } u \neq v \neq 0 \quad (18)$$

Equation (17) can be written as

$$(3u)^2 + (8m-2)^2 = (v)^2 + (12n-2)^2 \quad (19)$$

$$\Rightarrow x^2 + z^2 = y^2 + w^2 \quad (20)$$

where $x = 3u,\ z = 8m-2, y = v,\ w = 12n-2$ (21)

Comparing (21) and the general solutions to the second order Ramanujan equation (20) which are given in (6), we find that

$$\left.\begin{aligned} u &= \frac{rp - sq}{3} \\ v &= \frac{rq - sp}{1} \\ m &= \frac{rq + sp + 2}{8} \\ n &= \frac{rp + sq + 2}{12} \end{aligned}\right\} \quad (22)$$

We observe that, the values u, v, m and n are integers when

$$r = 144g + 4, p = 192g + 5, s = 48g + 2, q = 96g + 1 \quad (23)$$

Substituting (23) in (22), we have

$$u = 7680g^2 + 416g + 6 \quad (24)$$

$$v = 4608g^2 - 96g - 6 \quad (25)$$

$$m = 2880g^2 + 144g + 2 \quad (26)$$

$$n = 2688g^2 + 144g + 2 \quad (27)$$

Using (24) and (25) in (18), the dimensions of the cuboid which satisfies (16) are indicated by

$$\left.\begin{aligned} l &= 6144g^2 + 160g \\ b &= 1536g^2 + 256g + 6 \\ h &= 15360g^2 + 832g + 12 \end{aligned}\right\} \quad (28)$$

Hence, the ranks of pentagonal number, hexagonal number and the dimensions of the Cuboid satisfying our assumption are expressed by (26), (27) and (28).

Remark:

From (28), we observe that

$$\frac{1}{2} \times TSA = 509607936g^4 + 58392576g^3 + 2433024g^2 + 43776g + 288 \quad (29)$$

Using the values of m and n as given in (26) and (27), we get

$$96T_{5,n} = 1040449536g^4 + 111476736g^3 + 4405248g^2 + 76032g + 482$$

$$32T_{6,m} = 530841600g^4 + 53084160g^3 + 1972224g^2 + 32256g + 194$$

Therefore,

$$96T_{5,n} - 32T_{6,m} = 509607936g^4 + 58392576g^3 + 2433024g^2 + 43776g + 288 \quad (30)$$

From (29) and (30), we get

$$\frac{1}{2} \times TSA = 96T_{5,n} - 32T_{6,m}$$

Some numerical examples satisfying (16) are tabulated as follows

g	m	n	l	b	h		$96T_{5,n} - 32T_{6,m}$
1	3026	2834	12608	3596	32408	570477600	570477600
2	11810	11042	49792	13324	126232	8630687520	8630687520
3	26354	24626	111552	29196	281496	$4.28768712 \times 10^{10}$	$4.28768712 \times 10^{10}$
4	46658	43586	197888	51212	498200	$1.342358602 \times 10^{11}$	$1.342358602 \times 10^{11}$
5	72722	67922	308800	79372	776344	$3.258650767 \times 10^{11}$	$3.258650767 \times 10^{11}$

IV. CONCLUSION

In this paper, we find the dimensions of the cuboid, the ranks of polygonal numbers such that the total surface area of the cuboid is equal to linear combination of certain polygonal numbers. In this manner, one may consider the relation for other Geometrical figures together with two dimensional, three dimensional and four dimensional figurate numbers.

REFERENCES

[1] Carmichael, R.D., 1959, The theory of Numbers and Diophantine Analysis, Dover Publications, New York.

[2] Dickson, L.E., 2005, History of the theory numbers, Vol.II, Dover Publications, New York.

[3] John,H., Conway and Richard K.Guy, 1995,The Book of Numbers, Springer Verlag, New York.

[4] Mordell, L.J., Diophantine Equations, Academic press, London (1969).

[5] Sierpinski.W, 2003, Pythagorean triangles, Dover publications, INC, Newyork.

[6] Gopalan. M.A. and Devibala. S., 2006, On a Pythagorean problem, Acta Ciencia Indica, Vol. XXXII M, No 4, 1451-1452.

[7] Gopalan. M.A.and Gnanam. A., 2007, Pairs of Pythagorean triangles with equal perimeters, Impact J.Sci.Tech., Vol 1(2), 67- 70.

[8] Gopalan. M.A.and Leelavathi. S, 2007, Pythagorean triangle with 2(area/perimeter) as a cubic integer, Bulletin of Pure and Applied Science, Vol.26E (No.2), 197-200.

[9] Gopalan. M.A.and Gnanam. A., 2007, A special Pythagorean problem, Acta Ciencia Indica, Vol.XXXIII M, No 4, 1435-1439.

[10] Gopalan. M.A., Gnanam. A.and Janaki. G., 2007, A Remarkable Pythagorean problem, Acta Ciencia Indica, Vol. XXXIII M, No 4, 1429- 1434.

[11] Gopalan. M.A. and Janaki. G., 2008, Pythagorean triangle with area/perimeter as a special polygonal number, Bulletin of Pure and Applied Science, Vol.27E (No.2) ,393-402.

[12] Gopalan. M.A.and Sangeetha. G., 2010, Pythagorean triangle with perimeter as triangular number, GJ-AMMS,Vol. 3, No 1- 2,93-97.

[13] Gopalan. M.A. and Gnanam. A, 2010, Pythagorean triangles and Polygonal numbers, International Journal of Mathematical Sciences, Vol. 9, No. 1-2, 211-215.

[14] Gopalan. M.A and Sivakami. B., 2012, Pythagorean triangle with hypotenuse minus 2(area/ perimeter) as a square integer, Archimedes J.Math., Vol. 2(2), 153-166.

[15] Gopalan. M.A., Manjusomanath and K.Geetha, 2013, Pythagorean triangle with Area / perimeter as a special polygonal number, IOSR-JM, Vol.7 (3), 52-62.

[16] Gopalan. M.A. and Geetha. V., 2013, Pythagorean triangle with area/perimeter as a Special polygonal number, IRJES, Vol.2(7),28- 34.

[17] Gopalan . M.A., Sangeetha. V. and Manjusomanath, 2013, Pythagorean triangle and Polygonal number, Cayley J.Math., Vol 2(2), 151-156.

[18] Gopalan. M. A., Vidhyalakshmi. S., Thiruniraselvi, Presenna. N., R., 2015,"On Pairs of Pythagorean Triangles –I", IOSR Journal of Mathematics, Vol.11, Issue 1, Ver. IV, 15 -17.