On Quasi Umbilical Submanifold of Co-Dimension-2 of Almost Hyperbolic Manifold

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ABSTRACT

Hypersurfaces immersed in an almost hyperbolic Hermitian manifold studied by Dube and Mishra [3]. Almost hyperbolic hermite manifold have been studied by Dube [2]. Hypersurfaces of almost hyperbolic Hermitian manifold I, II, have been studied by Bhatt and Dube [1]. The purpose of the present paper is to study the Quasi – Umbilical submanifold of co-dimension -2 of an almost hyperbolic manifold. We have obtained the condition for this submanifold, to be W –Quasi umbilic.

I. INTRODUCTION

Let $V_{2n+2}$ be $c^\infty$ - manifold and there exist a vector valued linear function $F$ of differentiability class $c^\infty$ - satisfying .

(1.1) $F^2 X = X$ .

Then $V_{2n-1}$ is said to be an almost hyperbolic manifold and $F$ is said to give an almost hyperbolic structure to $V_{2n+2}$ let the almost hyperbolic manifold $V_{2n+2}$ be endowed with Riemannian metric $g$ satisfying [4].

(1.2) $G(FX, FY) = G(X, Y)$

Then $V_{2n+2}$ is called almost hyperbolic Riemannian manifold . Let $D$ be the Riemannian in almost hyperbolic Riemannian manifold $V_{2n+2}$ if ,

(1.3) $D (F) Y = 0$

$V_{2n+2}$ is called almost hyperbolic manifold . $V_{2n+2}$ be a differentiable submanifold of $V_{2n+2}$ such that .

(1.4) $FBX = BFX + u \otimes P + v \otimes Q$

(1.5) $FP = BU - \lambda q$

(1.6) $Fq = bv - \lambda p$

Where $P$ and $Q$ are two unit normal vector fields to $V_{2n}$ if the fields of type (1.1) , $U$, $V$ are vector fields $u$, $v$, and $\lambda$ is a $c^\infty$ function . Operating equation (1.4)(1.5)(1.6) by $F$ and using equation (1.1)(1.2)(1.4)(1.5)(1.6) and using tangential and normal part ,we have

(a) $f^2 X = X + u(X) U + v(X) V$,

(b) $U(fX) = \lambda v(X)$, $v(fX) = \lambda u(X)$

(1.7)

(c) $fu = - \lambda V$, $fv = - \lambda U$,

(d) $u(U) = -(1 + \lambda^2)$, $u(V) = 0$

(e) $v(U) = -(1 + \lambda^2)$, $v(U) = 0$.

Thus we get the almost hyperbolic contact {f, g, $\eta$, $\xi$} structure[4]. Let $g$ be the induced Riemannian metric in $V_{2n}$ defined by

(1.8)

(a) $g(X,Y) = G(BX, BY)$

$B$ being differential map.

(1.8) (b) $G(BX, P) = 0 = G(BX, Q)$

(1.8) (c) $G(P, P) = 1 = G(Q, Q)$

Then using equation (1.4) and (1.8) in equation (1.2) we get

(1.9) $g(fX, fY) = -g(X, Y) - u(X) u(Y) - v(X) v(Y)$

(1.10) $g(U, X) = u(X)$, $g(V, X) = v(X)$

Thus the submanifold $V_n$ of an almost product and decomposable manifold.

Let $D$ be an affine connection is $V_{2n}$ induced by the Riemannian connexion of almost hyperbolic manifold $V_{2n+2}$ then Gauss and Weingarten’s equation are given by

(1.11) $D_{BX} BY = BD_{X} Y + h(X,Y) P + K(X,Y) Q$

(1.12) $D_{BX} P = -BHX + l(X) Q$

(1.13) $D_{BX} Q = -BKX + l(X) Q$

Where $h$ and $K$ are second fundamental forms and $l$ is the third fundamental form defined by

(1.14) $g(HX, Y) = h(X,Y)$, $g(KX, Y) = K(X,Y)$

$H$ and $K$ being the tensors of type

(1.1) differentiating equation

(1.4)(1.5) and (1.6) and using equations (1.1)(1.2)(1.11)(1.12) and (1.13) we get

(1.15) $D_{UX} Y = h(X, Y) U + k(X, Y) V - u(Y) HX - v(Y) KX$

(1.16) $D_{UX} U = -h(X, Y) U - k(X, Y) V - u(Y) l(X)$

(1.17) $D_{UX} V = -\lambda h(X, Y) - k(X, Y) - v(Y) l(X)$


II. QUASI UMBOILICAL SUBMANIFOLD

Let in submanifold $V_{2n}$

(2.1)(a) $h(X,Y) = \alpha g(X,Y) + \beta w(X)w(Y)$

(2.1)(b) $K(X,Y) = \alpha g(X,Y) + \beta w(X)w(Y)$

Be satisfied in which $\alpha, \beta$ and $\alpha', \beta'$ are scaler functions and $w$ is 1-form if $\alpha \neq 0$, $\beta \neq 0$ and $l \neq 0$, then w-Quasi Umbilical submanifold is called para cylindrical submanifold.

Theorem (2.1): If the submanifold $V_{2n}$ of Co-dimension-2 of an almost hyperbolic manifold $V_{2n+2}$ is w-Quasi umbilical thus we have

(2.3)(a)

$$D_X f(Y) = \alpha g(X,fY) + \beta w(X)w(Y)$$

(2.3)(b)

$$D_X v(Y) = \alpha v(X) + \beta w(X)w(Y)$$

(2.3)(c)

$$\lambda^\prime \alpha g(X,Y)-\beta w(X)w(Y)-\alpha' \beta w(X)w(Y)$$

Theorem (2.3): The Nijenhuis tensor Corresponding to the tensor field $f$ in $V_{2n}$

(2.7) $N(X,Y) = \beta w(X)w(Y)$

Proof: If $N$ be the Nijenhuis tensor Corresponding to the tensor field $f$ in $V_{2n}$

(2.9) $N(X,Y) = \beta (v(X)w(Y)-w(X)v(Y))$}

REFERENCE

