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On the Robustness of the Characteristics Related to (M\M\1) (∞\FCFS) Queue System Model

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ABSTRACT

Various Queues system have been analyzed in respect of their characteristics using the corresponding probability mass function (p.m.f.). Traffic intensity (denoted byρ) defined as the ratio of the arrival rate to the service rate, is an important parameter of this p.m.f. With the advancement in science and technology over a period, the parameter involved in queues characteristics cannot be considered as constant, here it should be recognized that the investigator has considerable a priori knowledge about the variations in these parameters. Highlighting the point that the basic queue distribution can be updated in respect of prior variations in its mean, the present study deals with the analysis of the Sensitive/ Robust character of (M\M\1): (∞ /FCFS) queue system model when traffic intensity is considered as a random variable. Cost Benefit Analysis is also present in the corresponding situation.

Keywords---Queue system, Traffic intensity, Robustness, Prior and Posterior distribution, Compound distribution

I. INTRODUCTION

Queuing theory is concerned with the statistical description of the behavior of queue with findings, e.g., the probability distribution of the number in the queue from which the mean and variance of the queue length and the probability distribution of the waiting time for a customer, or the distribution of a server's busy periods can be found. In operation research problems involving queues, investigators must measure the existing system to make an objective assessment of its characteristics and must determine how change may be made to system what effect of various kind of change in system characteristics would be, and whether in the light of cost incurred in the system changes should be made to it. A model of the queuing system understudy must be constructed in this kind of analysis and the results of the queuing theory are required to obtained the characteristics of the models, and to assess the effect of changes such as the addition of an extra server or a reduction in mean service time. The characteristics of various queues system have been analyzed by using there respective p.m.f.. The traffic intensity (ρ) defined as the ratio of the arrival rate to the service rate, is an important parameter of this p.m.f. and various type of queue characteristics are defined using this parameter [1,8].

In reliability theory this ratio is also known as availability ratio. The situation become alarming when one is going for queue characteristics of the model of the same nature accomplishing the same task in varying conditions. Obviously, for overcoming the situation, it seems statistical logical to assume variations in traffic intensity represented by known suitable prior distribution. In this regard, on the repeated analysis of various queue system, we have a strong base for collecting prior information showing variations in ρ. For example, in a Barber's shop where the arriving individuals are the customers and the barbers are the server, this intensity may differ in respect of days in a week. Another example is represented by letter arriving at a typist's desk, where the letter is represented the customer and typist represented the server. Following the concept some of the studies like [2, 4,5, 6 and 7] include a vast literature on some queue characteristics in the Bayesian framework in which updating the prior with experimental data has been main concern. However in the Bayesian framework, it should be recognized that the prior do have an impact on the basic distribution and therefore, the present study consider the analysis of the robustness of queue's characteristics in (M\M\1):(∞\FCFS) system when traffic intensity is updating in respect of prior and post prior variations.

II. STATISTICAL BACKGROUND

(a) For developing the procedure, we consider the p.m.f. of a (M\M\1): (∞/FCFS) queue system with poission input and poission output is

$$P_x = f(x,\rho) = (1-\rho).\rho^x; \qquad \text{....(1)}$$

$$\text{where} \quad x = 0,1,2,.... \quad ; \quad 0 \le \rho \le 1$$

Here, (1) denote the probability that there are x units in the queue. Thus the random variable x follows geometric distribution with parameter (1-ρ). The mean and variance of X is given by

$E(x) = \rho/(1-\rho)$

And $V(x)= \rho/(1-\rho)^2$

(b) The investigator's prior belief about ρ , the parameter representing traffic intensity is represented by a beta distribution of first kind with p.d.f.

$$g(\rho) = \frac{1}{B(u,v)} \rho^{u-1}(1-\rho)^{v-1} \qquad \text{....(2)}$$

$$\text{where} \quad 0 \le \rho \le 1 \quad ; u, v > 0$$

With mean = u / (u + v) and variance = $u v / (u + v)^2 (u + v + 1)$

(c) The variations in ρ get neutralized if we consider the compound distribution of X [Johnson (1969)] in view of the distribution of X in (1) and that of ρ in (2). This compound distribution will be

$$f(x,u,v) = \int_0^1 f(x,\rho).g(\rho)\,d\rho$$

$$= \frac{(v).(u)^{[x]}}{(u+v)^{[x+1]}} \quad ; \ (x = 0,1,2,......) \qquad \text{....(3)}$$

the p.m.f. of X in (3) is a well known Inverse polya egenbergar distribution [Johnson (1969)] with E(x) = (u)/(v-1) and $V(x)= u v (u + v - 1) / (v-1)^2(v-2)$

Here, $u^{[x]} = [u(u+1)....(u+x-1)]$ stands for ascending factorial of u. Let the compound distribution in (3) be called as updated basic distribution.

(d) Further let X=($x_1, x_2........x_n$) be random sample of size n from the population (1), then the posterior distribution of ρ , in respect of its prior in (2) will be

$$\prod(\rho/x)=\frac{\rho^{u+v-1}.\,(1-\rho)^{n+v-1}}{B(u+y,n+v)} \quad ....(4)$$

Where $y=\sum_{i=1}^{n}x_i$, here $[\,x_i, i=1,2,....n]$ represents the number of successful operation before the first nature.

(e) Now, the compound distribution of X in (1) taking expectation with respect to posterior distribution of ρ in (4) gives another distribution of X as under

$$f(x,u,v)=\int_0^1 f(x,\rho).\prod(\rho/x).d\rho$$

$$=\frac{(n+v).(y+u)^{[x]}}{(y+u+v+n)^{[x+1]}} \;;\; (x=0,1,2,......) \quad ....(5)$$

The p.m.f. in (5) is well known polyaegenbergar distribution with E(x) = (u + y) / (n + v - 1) and

$V(x)= (u + y) (n + v) (y + u + v + n – 1) / (n + v – 1)^2 (n + v – 2)$

Let the distribution in (5) is termed as predictive basic distribution [Sinha (1986)]. In the compounding process as used in (5), the variations in ρ get neutralized on taking expectation over the function f(x, ρ) with respect to ρ. Variation in ρ is represented by posterior distribution in (4), consequently the updated compound geometric distribution in (5) is interpreted as predictive basic queue system . Thus, in the process, one gets three basic queues distribution are given in (1), (3) and (5) respectively. These distribution are useful for analyzing the queue's characteristics of a model in the following three specific situations.

(i) When the basic queue system model as given in (1) is used in the analysis. Here ρ is treated as constant.

(ii) When the updated basic queue system model, as given in (3) is used in the analysis. Here ρ is treated as a random variable with its prior as given in (2).

(iii) When the predictive basic queue system model as given in (5) is used in the analysis. Here, the parametric variations in ρ are represented by the posterior distribution in (4) which also incorporates experimental data X= (x_1, x_2........x_n) or sample information

III. QUEUE CHARACTERISTICS IN THE CASE OF THREE SPECIFIC SYSTEM MODEL

On using the three queues distribution in (1),(3) & (5), the system queue's characteristics in the corresponding situation have been obtained in the following three sub sections.

3.1 Queues Characteristics in the case of basic queue system model:

(1) The average idle time I(t) for which the system remains idel is

I(t) = P(there is no unit in the queue)

=(1-ρ)times. …(6)

(2) The expected number of the units in the system , Ls is given by

$$Ls = \sum_{x=0}^{\infty} x.P_x = \frac{\rho}{(1-\rho)} \qquad \text{...(7)}$$

(3) The expected queue length , Lq is given by

$$Lq = \sum_{x=1}^{\infty} (x-1).P_x = \frac{\rho^2}{(1-\rho)} \qquad \text{...(8)}$$

(4) The expected length of non empty queue

$$(L \setminus L > 0) = \frac{Ls}{P(\text{an arrival has to wait})} = \frac{1}{(1-\rho)} \qquad \text{...(9)}$$

(5) The probability of minimum queue size being n_0 is given by

$$Q_m = P(x \geq n_0) = \rho^{n_0} \qquad \text{..(10)}$$

(6) Variance of the queue length , V_s is given by

$$V_s = \frac{\rho}{(1-\rho)^2} \qquad \text{...(11)}$$

(7) The Consistency of the queue length, given by its Co-efficient of variation and is given by

$$CV_s = \frac{\sqrt{V_s}}{Ls} * 100$$

$$= \frac{1}{\sqrt{\rho}} * 100 \qquad \text{...(12)}$$

(8) The utilization of the server is given by

$$\eta = 1 - P(x = 0) = \rho \qquad \text{...(13)}$$

(9) Cost Benefit Analysis of the model : For formulating the cost benefit analysis of the system, let us define the profit function as a function of ρ i.e.

$$P_\rho(\rho) = R.Ls - [C_0 + C_1.Ls + C_2.Lq + C_3.I(t)] \qquad \text{...(14)}$$

Where, P (ρ) = Profit function for fixed ρ

R = Revenue earned per unit average queue length

C_0 = Overhead charge

C_1 = Cost or Expenditure per unit average queue length

C_2 = Loss incurred per unit average waiting line length

C_3 = Loss incurred per unit average idle time

3.2 Queues Characteristics in the case of updated basic queue system model:

For analyzing the sensitive character of the (M\M\1): (∞/FCFS) queue system model in respect of its various characteristics when traffic intensity is considered as a random varible with its distribution in (2) , we obtained the various queues characterisics by using the p.m.f. of x in (3) as follows

(1) The average idle time I*(t) for which the system remains idel is

$$I^*(t) = P\text{ (there is no unit in the queue)}$$
$$= (1 - E(\rho))\text{ times.} \quad \ldots(15)$$

(2) The expected number of the units in the system , L_s^* is given by

$$L_s^* = \sum_{x=0}^{\infty} x.f(x,u,v) = \frac{(u)}{(v-1)} \quad \ldots(16)$$

(3) The expected queue length , L_q^* is given by

$$L_q^* = \sum_{x=1}^{\infty} (x-1).f(x,u,v)$$
$$= \frac{u}{(v-1)} + \frac{v}{(u+v)} - 1 \quad \ldots(17)$$

(4) The expected length of non empty queue ,

$$(L^* \backslash L^* > 0) = \frac{L_s^*}{P(\text{an arrival has to wait})}$$
$$= \frac{[u/(v-1)]}{[1 - \frac{v}{(u+v)}]} \quad \ldots(18)$$

(5) The probability of minimum queue size being n_0 is given by

$$Q_m^* = P[x \geq n_0]$$
$$= 1 - \sum_{x=0}^{n_0-1} \frac{v(u)^{[x]}}{(u+v)^{[x+1]}} \quad \ldots(19)$$

(6) Variance of the queue length, V_s* isgivenby

$$V_s* = E[x - E(x)]^2$$
$$= \frac{uv(u+v-1)}{(v-1)^2.(v-2)} \quad \ldots(20)$$

(7) The consistency of the queue length is given by

$$CV^*\{x\} = \frac{\sqrt{V_s^*}}{L_s^*} * 100$$

$$= \sqrt{\frac{v(u+v+1)}{u(v-2)}} * 100 \quad ...(21)$$

(8) The utilization of the server is given by

$$\eta^* = 1 - \frac{v}{(u+v)} \quad ...(22)$$

(9) Cost Benefit Analysis of the model : For formulating the cost benefit analysis of the system ,let us define the profit function as a function of ρ i.e.

$$P_\rho^*(\rho) = R.L_s^* - [C_0 + C_1.L_s^* + C_2.L_q^* + C_3.I^*(t)] \quad ...(23)$$

Where, P*(ρ) = Profit function for fixed ρ

$R, C_0, C_1, C_2,\ C_3$ are defined in (14).

3.3 Queues Characteristics in the case of Predictive basic queue system model:

Having obtained the predictive basic queue distribution in (5), the various queue characterisics by using the p.m.f. of X in (5) as follows.

(1) The average idle time $I^{**}(t)$ for which the system remains idel is

$$I^{**}(t) = [\,1 - E(\rho)\,] \text{ times.} \quad ...(24)$$

(2) The expected number of the units in the system , L^{**}_s is given by

$$L^{**}_s = \sum_{x=0}^{\infty} x.f(x,u,v) = \frac{(y+u)}{(n+v-1)} \quad ...(25)$$

(3) The expected queue length , L^{**}_q is given by

$$L^{**}_q = \sum_{x=1}^{\infty} (x-1).f(x,u,v) = \frac{(u+n\bar{x})(u+n\bar{x}+1)}{(n+v-1)(n\bar{x}+u+v+n)}$$

$$= \frac{(u+y)(u+y+1)}{(n+v-1)(y+u+v+n)} \quad ...(26)$$

(4) The expected length of non empty queue ,

$$(L^{**} / L^{**} > 0) = \frac{[(y+u)/(n+v-1)]}{[1 - \frac{(v+n)}{(y+u+v+n)}]} \quad ...(27)$$

(5) The probability of minimum queue size being n_0

is given by

$$Q^{**}_{m} = P[x \geq n_0] = 1 - \sum_{x=0}^{n_0-1} \frac{(v+n)(y+u)^{[x]}}{(y+u+n+v)^{[x+1]}} \quad ...(28)$$

(6) Variance of the queue length is given by

$$V^{**}_{s} = E[x - E(x)]^2 = \frac{(y+u)(v+n)[y+u+v+n-1]}{(v+n-1)^2.(v+n-2)} \quad ...(29)$$

(7) The consistency of the queue length is given by

$$CV^{**} = \frac{\sqrt{V^{**}}}{L^{**}_{s}} * 100 \quad ...(30)$$

(8) The utilization of the server is given by

$$\eta^{**} = 1 - \frac{(n+y)}{(u+v+n+y)} \quad ...(31)$$

(9) For formulating the cost benefit analysis of the system ,let us define the profit function in this case as a function of ρ i.e.

$$P^{**}_{\rho}(\rho) = R.L^{**}_{s} - [C_0 + C_1.L^{**}_{s} + C_2.L^{**}_{q} + C_3.I^{**}(t)] \quad ...(32)$$

Where, $P^{**}_{\rho}(\rho)$ = Profit function for fixed ρ and other constants are defined in (14).

IV. DISCUSSION AND EXAMPLE

Now there are three basic queue systems in (1), (3) and (5) respectively. The queues characteristics in the corresponding situations have been listed in section 3.0. For analyzing the robust character of the queue characteristics when ρ is considered as a random variable, we compare these as given in sub section 3.1 with those given in sub-section 3.2 and 3.3. For introducing statistical validity in such comparison, the parametric values in the three specific situations i.e. (ρ,u,v) are so chosen so that E(X) for the distributions in (1), (3) and (5) are same. Consequently, the reference points for comparison are selected so as to satisfy

$$\frac{\rho}{(1-\rho)} = \frac{u}{(v-1)} = \frac{u+y}{(v+n-1)} \quad ...(33)$$

On using the initial geometric distribution, the respective estimates for queue's characteristics are given in sub section 3.1 and with variations in ρ, have been summarized in table 1. For assumed values of the parameters of the prior distribution, i.e. u,v and y, and also using the expression given in sub-section 3.2 and 3.3, the estimates for various queue's characteristics have been summarized in table 2 and table 3 respectively as expected traffic intensity varies randomly. A comparison of the queue's characteristics obtained in the three situations clearly reveals their non-robust character when the variations in the traffic intensity in the classical queue's system are suspected. In the present data setup, the values for the queue's characteristics tend to be uniformly higher when the prior variations are suspected. The trends in all the three tables clearly highlighted that modified/updated queue's characteristics values are uniformly higher when ρ≤ E(ρ) i.e., (its expected mean variations). On the other hand, when ρ> E(ρ), this updated estimates tend to be higher up to a certain point but thereafter updated queue's characteristics values are uniformly lower . The point at which the trends are reversed is not difficult to estimate. On comparing the variations in co-efficient of variations with respect to ρ and E(ρ) in table-1, 2 and 3, we observe that the estimates tend to be more and more consistent as either ρ or its expected mean variations increases.

For cost benefit analysis, the three situations are given in equations (14), (23) and (32), on taking R = Rs.2000/-, C_0 = Rs.300/-, C1 = Rs.800/-, C2 = Rs.200/- and C3 = Rs.150/- and for various values of ρ and its prior variations in profit functions are also shown in table 1,2 and 3. In market analysis, by analyzing the above trends, in respect of profit analysis, one can easily make an economic trade-off in a (M\M\1) (∞\FCFS) queue system model.

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Table-1
Estimates of the Queues Characteristics of the system when Traffic Intensity is Considered a Constant

ρ	L_s	L_q	(L/L>0)	$Q_m(n_0=3)$	V	CV	η	$P_\rho(\rho)$
0.05	0.053	0.0026	1.0526	0.0001	0.0554	444.098	0.050	-210.88
0.15	0.176	0.0264	1.1765	0.0034	0.0276	258.880	0.150	-123.90
0.25	0.333	0.0833	1.3333	0.0156	0.4444	200.190	0.250	-18.96
0.35	0.538	0.1885	1.5385	0.0428	0.8284	169.176	0.350	111.14
0.45	0.818	0.3682	1.8182	0.0911	1.4876	149.104	0.450	280.67
0.55	1.222	0.7622	2.2222	0.1664	2.7160	134.863	0.550	515.28
0.65	1.857	1.2071	2.8571	0.2746	5.3061	124.044	0.650	871.16
0.75	3.000	2.2500	4.0000	0.4218	12.0000	115.470	0.750	1493.75
0.85	5.667	4.8166	6.6667	0.6141	37.7778	108.458	0.850	2846.47

Table-2
Estimates of the Queues Characteristics of the system when Traffic Intensity is considered as a Random Variable

u	v	$E(\rho)$	L^*_s	L^*_q	(L*/L*>0)	$Q^*_m(n_0=3)$	V^*	CV^*	η^*	$P^*_\rho(\rho)$
15	85	0.15	0.179	0.029	1.193	0.00396	0.21	259.64	0.15	-233.5
25	75	0.25	0.338	0.088	1.352	0.01705	0.464	201.53	0.25	-39.50
35	65	0.35	0.546	0.196	1.506	0.03424	0.872	171.03	0.35	203.5
45	55	0.45	0.833	0.383	1.851	0.0944	1.550	163.33	0.45	525.5
55	45	0.55	1.250	0.701	2.273	0.1704	2.943	137.24	0.55	977.5
65	35	0.65	1.911	1.261	2.940	0.2790	5.904	127.15	0.65	1373.5
75	25	0.75	3.125	2.375	4.166	0.4260	14.012	119.78	0.75	2922.5
85	15	0.85	6.077	5.227	7.149	0.6173	49.538	115.93	0.85	5609.5

Table-3
Estimates of the Queues Characteristics of the system when Traffic Intensity is considered as a Random Variable with Posterior prior as its Distribution

u	v	$\rho = E(\rho)$	y	L^{**}_s	L^{**}_q	(L**/L**>0)	$Q^{**}_m(n_0=3)$	V**	CV**	η**	$P^{**}_\rho(\rho)$
15	85	0.15	33.57	0.467	0.147	1.474	0.0331	0.698	263.04	0.316	88.32
25	75	0.25	56.76	0.869	0.399	1.878	0.1751	1.661	148.31	0.462	535.7
35	65	0.35	80.93	1.380	0.785	2.392	0.1942	3.364	132.91	0.576	1083.5
45	55	0.45	106.67	2.049	1.344	3.062	0.3017	6.422	123.68	0.669	1792.5
55	45	0.55	135.05	2.968	2.156	4.002	0.4153	12.156	117.47	0.745	2747.9
65	35	0.65	168.23	4.319	3.384	5.337	0.5311	23.840	113.05	0.809	4138.5
75	25	0.75	212.5	6.534	5.423	7.556	0.6473	51.518	109.85	0.864	6403.7
85	15	0.85	291.42	11.071	9.592	12.101	0.7664	141.742	107.53	0.915	11029.3