

Power Ratio Reduction with OFDM Signals for Broadband Wireless Mobile Communication

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ABSTRACT

With the advancement of technologies in communication field, high data rate in addition to both power efficiency and lower bit error rate have always been areas of interest for scientist. The ever increasing demand of high data rate can be fulfilled by the single carrier modulation along with the tradeoff between the power efficiency and bit error rate. It is very difficult to achieve high data rate for this single carrier modulation with a lower bit error rate performance in the presence of frequency selective fading environment. With considering an advance step towards the multi carrier modulation scheme it is possible to get high data rate in this multipath fading channel without degrading the bit error rate performance. To achieve better performance using multi carrier modulation we should make the subcarriers to be orthogonal to each other i.e. known as the Orthogonal Frequency Division Multiplexing (OFDM) technique. But the great disadvantage of the OFDM technique is its high Peak to Average Power Ratio (PAPR). As we are using the linear power amplifier at the transmitter side so it's operating point will go to the saturation region due to the high PAPR which leads to in-band distortion and out-of-band radiation. This can be avoided with increasing the dynamic range of power amplifier which leads to high cost and high consumption of power at the base station. In this paper, some of the important PAPR reduction techniques which have been compared based on computational complexity, bandwidth expansion, spectral spillage and performance in-band signal distortion and out-of-band radiation.

Keywords: Orthogonal; Frequency; PAPR; CDF.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a modulation technique used in many new and emerging broadband technologies either wired like ADSL (asymmetric digital subscriber line) or wireless as in DAB (digital audio broadcasting), DVB-T (digital video broadcasting-terrestrial), wireless LANs, and so forth. In all these systems, the information data stream is transmitted in parallel on several orthogonal subcarriers, each subcarrier being QAM or PSK modulated. The main advantage of OFDM is its strong

immunity to multipath fading and impulse noise, its great simplification of channel equalization and its low computational complexity implementation based on using Fast Fourier Transform (FFT) techniques.

Apart from lot of advantages, some drawbacks become apparent, while using OFDM in transmission systems. A major obstacle is that the multiplex signal exhibits a very high Peak-to-Average Power Ratio (PAPR).

Large PAPR brings the OFDM signal distortion in the non-linear region of high power amplifier (HPA) and the signal distortion induces the degradation of bit error rate (BER). Moreover, to prevent spectral growth of the multicarrier signal in the form of inter modulation among subcarriers and out-of-band radiation, the transmit power amplifier has to be operated in its linear region. If the HPA is not operated in linear region with large power back-offs, it is impossible to keep the out-of band power below the specified limits. This situation leads to very inefficient amplification and expensive transmitters. The main objective of this paper is to find best data compression technique that can reduce PAPR significantly.

To reduce the PAPR several techniques have been proposed such as Partial Transmit Sequences (PTS) [1,2,3,4], Selective Mapping (SLM) [5,6], clipping [7,8] clipping and filtering [9], Coding [10], tone reservation (TR), Active Constellation Extension (ACE) [11], Phase Optimization[13], Nonlinear Companding transforms and tone injection (TI) [14]. These techniques achieve PAPR reduction at the expense of transmit signal power increase, bit error rate (BER) increase, data rate loss, computational complexity increase, and so on.

II. PAPR IN OFDM SYSTEM

In this section, we review the basics of OFDM system and the definition of PAPR. OFDM signal may be generated by an N-point Inverse Fast Fourier Transform (IFFT) in the transmitter, and the Fast Fourier Transform (FFT) is employed at the receiver to restore the signal. The

PAPR is the relation between the maximum power of a sample in a given OFDM transmit symbol divided by the average power of that OFDM symbol. PAPR occurs when in a multicarrier system the different sub-carriers are out of phase with each other. At each instant they are different with respect to each other at different phase values. When all the points achieve the maximum value simultaneously; this will cause the output envelope to suddenly shoot up which causes a 'peak' in the output envelope. Due to presence of large number of independently modulated subcarriers in an OFDM system, the peak value of the system can be very high as compared to the average of the whole system. This ratio of the peak to average power value is termed as Peak-to- Average Power Ratio.

An OFDM symbol consists of N sub-carriers by the frequency spacing of Δf . The total band with B will be divided into N equally spaced sub-carriers with all sub-carriers are orthogonal

$$T = \frac{1}{\Delta f}$$

to each other within a time interval of length T . Each sub-carrier can be modulated independently with complex modulation symbol $X_{m,n}$, where m is a time index and n is a subcarrier index. The m -th OFDM block period can be described by equation (1) as:

$$x_m(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_{m,n} g_n(t - mT) \quad (1)$$

Where, $g_n(t)$ is defined through equation (2).

$$g_n(t) = \begin{cases} \frac{\exp(j2\pi n \Delta f t)}{0} & 0 \leq |t| \leq T \\ else & \end{cases} \quad (2)$$

Where $g_n(t)$ is a rectangular pulse applied to each subcarrier. The total continuous time signal $x(t)$ consisting of all the OFDM block is given by equation (3).

$$x(t) = \frac{1}{\sqrt{N}} \sum_{m=0}^{\infty} \sum_{n=0}^{N-1} x_{m,n} g_n(t - mT) \quad (3)$$

Consider a single OFDM symbol ($m = 0$) without loss of generality. This can be shown because there is no overlap between different OFDM symbols.

Since $m = 0$, $X_{m,n}$ can be replaced by X_n . Then, the OFDM signal can be described as follows,

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n \Delta f t} \quad (4)$$

If the band-width of the OFDM signal is $B = N \times \Delta f$ and the signal $x(t)$ is sampled by the sampling time of

$$\Delta t = \frac{1}{B} = \frac{1}{N \Delta f}$$

, then the OFDM signal is in discrete time form and can be written as shown in equation (5).

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n \Delta f k / N}, k = 0, 1, 2, \dots, N-1 \quad (5)$$

PEAK PROBLEM

An OFDM signal consists of an "N" number of independently modulated subcarriers, which can give a very large PAPR when added up coherently. PAPR is the ratio between the maximum power and the average power of the complex signal. Generally, the PAPR for the time domain OFDM signal can be defined as:

$$PAPR = \frac{P_{peak}}{P_{average}} = 10 \log_{10} \frac{\max [|x_n|^2]}{E [|x_n|^2]} \quad (6)$$

Where P_{peak} represents peak output power, $P_{average}$ means average output power. $E[.]$ denotes the expected value, x_n represents the transmitted OFDM signals which are obtained by taking IFFT operation on modulated input symbols X_k . Mathematical, x_n is expressed as:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k W_k^{nk} \quad (7)$$

For an OFDM system with N sub-carriers, the peak power of received signals is N times the average power when phase values are the same. The PAPR of baseband signal will reach its theoretical maximum at $(dB) = 10 \log N$.

Another commonly used parameter is the Crest Factor (CF), which is defined as the ratio between maximum amplitude of OFDM signal (t) and root-mean-square (RMS) of the waveform. The CF is defined as :

$$CF(s(t)) = \frac{\max [|x_n|^2]}{E [|x_n|^2]} = \sqrt{PAPR} \quad (8)$$

In most cases, the peak value of signal $s(t)$ is equals to maximum value of its envelope (t).

CUMULATIVE DISTRIBUTION FUNCTION

The Cumulative Distribution Function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of any PAPR technique. Normally, the Complementary CDF (CCDF) is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold.

By implementing the Central Limit Theorem for a multi-carrier signal with a large number of sub-carriers, the real and imaginary part of the time-domain signals have a mean of zero and a variance of 0.5 and follow a Gaussian distribution. So Rayleigh distribution is followed for the amplitude of the multi-carrier signal, where as a central chi-square distribution with two degrees of freedom is followed for the power distribution of the system.

The CDF of the amplitude of a signal sample is given by

$$F(z) = 1 - \exp(-z) \quad (9)$$

The CCDF of the PAPR of the data block is desired is our case to compare outputs of various reduction techniques. This is given by

$$P(PAPR > z) = 1 - P(PAPR \leq z) \quad (10)$$

$$= 1 - F(z)^N \quad (11)$$

$$= 1 - (1 - \exp(-z))^N \quad (12)$$

III. OVERVIEW OF PAPR REDUCTION

Block Coding Techniques

The fundamental idea is that of all probable message symbols, only those which have low peak power will be chosen by coding as valid code words for transmission. No introduction of distortion to the signals. If there have N subcarriers, they are represented by $2N$ bits using QPSK modulation and thus 2^{2N} messages. Using the whole message space corresponds to zero bits of redundancy. Using only half of the messages corresponds to one bit of redundancy. The remaining message space is then divided in half again and this

process continues until N bits of redundancy have been allocated which corresponds to a rate one-half code for N carriers. Large PAPR reduction can be achieved if the long information sequence is separated into different sub blocks, and all sub block encoded with System on a Programmable Chip (SOC). The adaptive approach has been adopted in order to reduce hardware for a slight increase in complexity. The coding technique Error Correction (EC) SLM using convolution code proposed by S.H.Han and J.H.Lee has been studied.

Block Coding Scheme with Error Correction

The method is proposed that designed block codes can not only minimize the PAPR, but also give error correction capability. A k bit data block is encoded by a (n, k) block code with a generator matrix 'G' in the transmitter of the system. Followed by the phase rotator vector b to produce the encoded output $x=a.G+b \pmod 2$. After that generator matrix 'G' and the phase rotator vector 'b' are produced; which are used mapping between these symbols combination and input data vector 'a'. The converse functions of the transmitter are executed in the receiver system. The parity check matrix 'H' is achieved from the generator matrix 'G', with an exception that the effect of the phase rotator vector b is removed before calculations of syndromes.

Selected Mapping (SLM)

In the SLM technique, the transmitter generates a set of sufficiently different candidate data blocks, all representing the same information as the original data block, and selects the most favorable for transmission. Each of these alternative input data sequences is made the IFFT operation, and then the one with the lowest PAPR is selected for transmission. A block diagram of the SLM technique is shown in Fig. 1. Each data block is multiplied by V different phase sequences, each of length N , $B(v) = [b_{v,0}, b_{v,1}, \dots, b_{v,N-1}]^T$, $v=0,1,2, \dots, V-1$, resulting in V modified data blocks. To include the unmodified data block in the set of modified data blocks, we set $B(v)$ as the all-one vector of length N . Let us denote the modified data block for the u th phase sequence $X(v) = [X_0 b_{v,0}, X_1 b_{v,1}, \dots, X_{N-1} b_{v,N-1}]^T$, $v=0,1,2, \dots, V-1$. After applying SLM to X , the multicarrier signal becomes

$$x_u(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n b_v e^{j2\pi n \Delta f t} \quad (13)$$

Where $0 \leq t \leq NT$, $v=0,1,2, \dots, V-1$

Among the modified data blocks $X(v)$, $v=0,1,2, \dots, V-1$, only one with the lowest PAPR is selected for transmission. Information about the selected phase sequence should be transmitted to the receiver as side information.

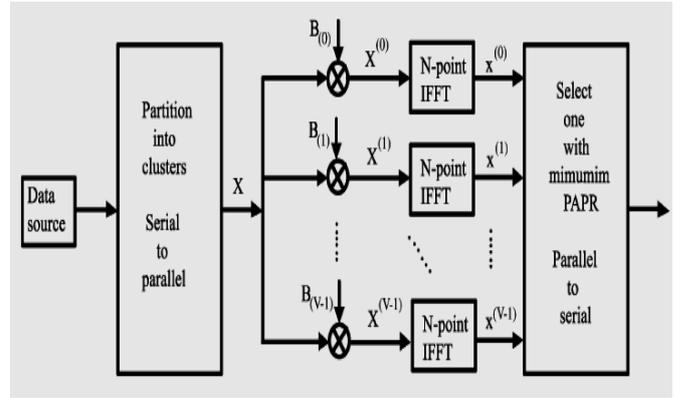


Fig.1 Block Diagram of Selected Mapping Technique

For implementation of SLM OFDM systems, the SLM technique needs IFFT operation and the number of required bits as side information is for each data block. At the receiver, the reverse operation is performed to recover the original data block. This approach is applicable with all types of modulation and any number of subcarriers. The amount of PAPR reduction for SLM depends on the number of phase sequences U and the design of the phase sequences .

Partial Transmit Sequence

PTS is another very effective approach to reduce the PAPR; here an input data block of N symbols is partitioned into disjoint sub blocks. The subcarriers in each sub block are weighted by a phase factor for that sub block. The phase factors are selected such that the PAPR of the combined signal is minimized. Fig. 2 shows the block diagram of the PTS technique. In the ordinary PTS technique input data block X is partitioned into M disjoint sub blocks $X_m = [X_{m,0}, X_{m,1}, \dots,$

$$X = \sum_{m=0}^{M-1} X(m) \quad (14)$$

$X_{m,N-1}]^T$, $m=0,1,2, \dots, M-1$, such that and the sub blocks are combined to minimize the PAPR in the time domain. The L -times oversampled time domain signal of X_m , $m=0,1,2, \dots, M-1$, is denoted $x_m = [x_{m,0}, x_{m,1}, \dots, x_{m,NL-1}]^T$, $m=1,2, \dots, M-1$, is obtained by taking an IFFT of length NL on X_m concatenated with $(L-1)N$ zeros. These are called the partial transmit sequences. Complex phase factors, $b_m = e^{j\theta_m}$, $m=0,1,2, \dots, M-1$, are introduced to combine the PTSs. The set of phase factors is denoted as a vector $b = [b_0, b_2 \dots b_{M-1}]^T$. The time domain signal after combining is given by.

$$\tilde{x}(b) = \sum_{m=0}^{M-1} b_m x_m \quad (14)$$

Where $x'(b) = [x'_0(b), x'_1(b), \dots, x'_{NL-1}(b)]^T$. The objective is to find the set of phase factors that minimizes the PAPR. Therefore, there are two important issues should be solved in PTS: high computational complexity for searching the optimal phase factors and the overhead of the optimal phase factors as side information needed to be transmitted to receiver for the correct decoding of the transmitted bit sequence.

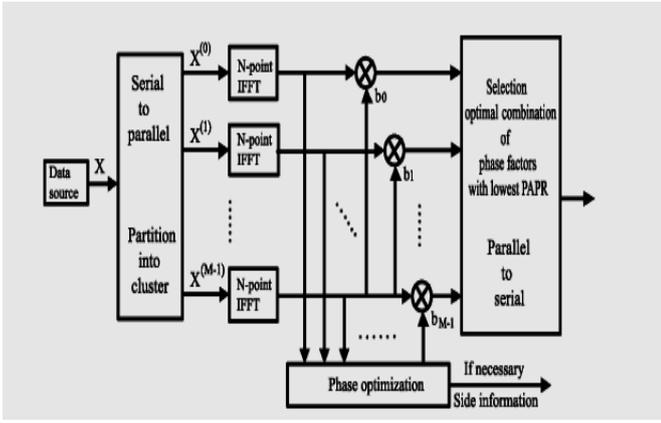


Fig.2 Block Diagram of Partial Transmit Sequence

Companding

The compander consists of compressor and expander. The compressor is a simple logarithm computation. The reverse computation of a compressor is called an expander. In this paper, the compression at the transmit end after the IFFT process and expansion at the receiver end prior to FFT process are used. There are two types of companders that are used here which are described in details in [14]. These two types are μ -law and A-law companders.

μ -law Companding

The μ -law compander employs the logarithmic function at the transmitting side.

In general a μ law compression characteristic:

$$y = \frac{V \log_e \left(\frac{1 + \mu |x|}{V} \right)}{\log_e (1 + \mu)} \text{sgn}(x) \quad (15)$$

where μ is the μ -law parameter of the compander, where x : input signal.

V : is the maximum value of the signal x .

μ : parameter controls the amount of compression.

The maximum value of output y is the same maximum of input x is equal V .

For normalized input signal with $|x| \leq 1$, the characteristic becomes:

$$y = \frac{V \log(1 + \mu |x|)}{\log(1 + \mu)} \text{sgn}(x) \quad (16)$$

The μ -law expander is the inverse of the compressor:

$$x = \frac{V}{\mu} \left(e^{|y| \log \frac{(1+\mu)}{V}} - 1 \right) \text{sgn}(y) \quad (17)$$

A-law Companding

The characteristic of this compander is given by:

$$y = \begin{cases} \frac{1 + \ln A |x|}{1 + \ln A} \text{sgn}(x) & \frac{1}{A} \leq |x| \leq 1 \\ \frac{A |x|}{1 + \ln A} \text{sgn}(x) & 0 \leq |x| \leq \frac{1}{A} \end{cases} \quad (18)$$

where A: parameter controls the amount of compression

AMPLITUDE CLIPPING AND FILTERING

The simplest and most widely used technique of PAPR reduction is to basically clip the parts of the signals that

are outside the allowed region. For example, using HPA with saturation level below the signal span will automatically cause the signal to be clipped.

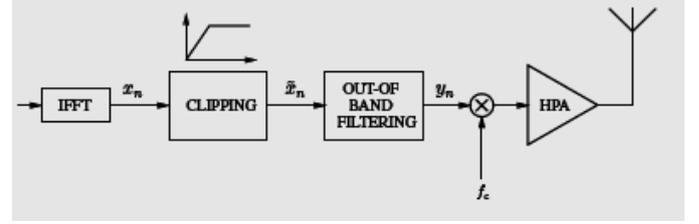


Fig.3. The OFDM transmitter including clipping scheme.

Generally, clipping is performed at the transmitter. However, the receiver needs to estimate the clipping that has occurred and to compensate the received OFDM symbol accordingly. Typically, at most one clipping occurs per OFDM symbol, and thus the receiver has to estimate two parameters: location and size of the clip. However, it is difficult to get these information. Therefore, clipping method introduces both in band distortion and out of band radiation into OFDM signals, which degrades the system performance including BER and spectral efficiency.

Filtering can reduce out of band radiation after clipping although it cannot reduce in-band distortion. However, clipping may cause some peak regrowth so that the signal after clipping and filtering will exceed the clipping level at some points. To reduce peak regrowth, a repeated clipping-and-filtering operation can be used to obtain a desirable PAPR at a cost of computational complexity increase.

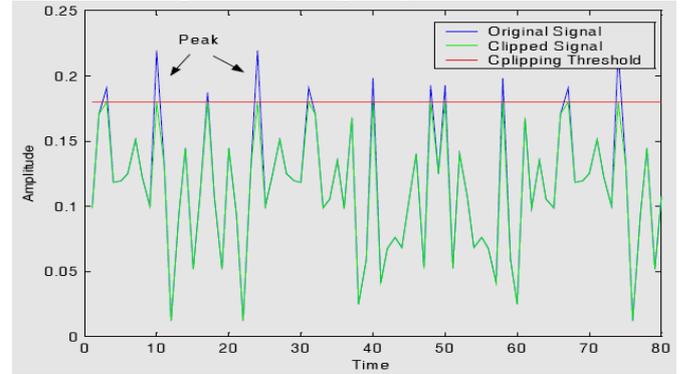


Fig.4 OFDM Signal Amplitude

As improved clipping methods, peak windowing schemes attempt to minimize the out of band radiation by using narrowband windows such as Gaussian window to attenuate peak signals

A threshold value of the amplitude is set in this case to limit the peak envelope of the input signal. Signal having values higher than this pre-determined value are clipped and the rest are allowed to pass through un-disturbed.

For amplitude clipping that is

$$B(x) = \begin{cases} x & |x| \leq T \\ A e^{j\varphi(x)} & |x| < A \end{cases} \quad (19)$$

where,

$B(x)$ = The amplitude value after clipping.

x = the initial signal value.

A = the threshold set by the user for clipping the signal and it is a positive real number.

The problem in this case is that due to amplitude clipping distortion is observed in the system which can be viewed as another source of noise. This distortion falls in both in-band and out-of-band. Filtering cannot be implemented to reduce the in-band distortion and an error performance degradation is observed here. On the other hand spectral efficiency is hampered by out-of-band radiation. Out-of-band radiation can be reduced by filtering after clipping but this may result in some peak re-growth. A repeated filtering and clipping operation can be implemented to solve this problem. The desired amplitude level is only achieved after several iterations of this process.

Interleaving Technique

The notion that highly correlated data structures have large PAPR can be reduced, if long correlation pattern is broken down. The basic idea in adaptive interleaving is to set up an initial terminating threshold. PAPR value goes below the threshold rather than seeking each interleaved sequence. The minimal threshold will compel the adaptive interleaving (AL) to look for all the interleaved sequences. The main important of the scheme is that it is less complex than the PTS technique but obtains comparable result. This method does not give the assurance result for PAPR reduction.

Tone Reservation (TR)

The main idea of this method is to keep a small set of tones for PAPR reduction. This can be originated as a convex problem and this problem can be solved accurately. Tone reservation method is based on adding a data block and time domain signal. A data block is dependent time domain signal to the original multicarrier signal to minimize the high peak. This time domain signal can be calculated simply at the transmitter of system and stripped off at the receiver. The amount of PAPR reduction depends on some factors such as number of reserved tones, location of the reserved tones, amount of complexity and allowed power on reserved tones. This method explains an additive scheme for minimizing PAPR in the multicarrier communication system. It shows that reserving a small fraction of tones leads to large minimization in PAPR even using with simple algorithm at the transmitter of the system without any additional complexity at the receiver end. Here, N is the small number of tones, reserving tones for PAPR reduction may present a non-negligible fraction of the available bandwidth and resulting in a reduction in data rate. The advantage of TR method is that it is less complex, no side information and also no additional operation is required at the receiver of the system.

Tone Injection (TI)

Tone Injection (TI) method has been recommended by S. H. Muller and J. B. Huber [9]. This technique is based on general additive method for PAPR reduction. Using an additive method achieves PAPR reduction of multicarrier signal without any data rate loss. TI uses a set of equivalent constellation points for an original constellation points to reduce PAPR. The main idea behind this method is to increase the constellation size. Then, each point in the original basic constellation can be mapped into several equivalent points in the extended constellation, since all information elements can be mapped into several equivalent constellation points. These additional amounts of freedom can be utilized for PAPR reduction. The drawbacks of this method are; need to side information for decoding signal at the receiver side, and cause extra IFFT operation which is more complex.

IV. CONCLUSIONS

OFDM is a very attractive technique for communications due to its spectrum efficiency and channel robustness. One of the serious drawbacks of OFDM systems is that the composite transmit signal can exhibit a very high peak power when the input sequences are highly correlated. Although a lot of PAPR reduction techniques have been proposed in the recent years. In this paper some PAPR reduction techniques for multicarrier transmission have been discussed. Many techniques to reduce the PAPR have been proposed all of which have the potential to provide substantial reduction in PAPR at the cost of loss in data rate, transmit signal power increase, BER increase, computational complexity increase and so on. No specific PAPR reduction technique has been the best solution for all multicarrier transmission system. It has been suggested that the PAPR reduction technique should be carefully chosen according to various system requirements.

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