Radiation Effects on MHD Boundary Layer Slip Flow Along A Stretching Cylinder with Cross Diffusion

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ABSTRACT:
An analysis of steady MHD axisymmetric laminar boundary layer flow of a viscous incompressible electrically conducting and radiating fluid towards a stretching cylinder in the presence of Soret and Dufour effects with heat and mass transfer is presented. The Rosseland approximation is used to describe the radiative heat transfer in the limit of optically thick fluids. Instead of no-slip conditions at the boundary, partial slip is considered. The governing partial differential equations are non-dimensionalized and transformed into a system of non-linear ordinary differential similarity equations, in a single independent variable. The resulting coupled non-linear equations are solved under appropriate transformed boundary conditions using the Runge-Kutta fourth order method along with the shooting method. The effects of the radiation parameter, magnetic parameter, Soret number and Dufour number and buoyancy parameter on the velocity, temperature and concentration are plotted graphically and discussed.

Keywords---Soret and Dufour effects, MHD, Radiation, Slip flow, heat and mass transfer.

I. INTRODUCTION
Coupled heat and mass transfer in boundary layer flow have enormous applications in many processes and therefore receive a considerable amount of attention from researchers. Heat and mass transfer problems have a great impact on modern-day industries, such as geothermal systems, heat insulation, paper production, drying technology, compact heat exchangers, solar power collectors, catalytic reactors and food industries.

In many heat and mass transfer problems, the Soret and Dufour effects are neglected as they are of a smaller order of magnitude when compared to other effects. Soret effect means species differentiation developing in an initial homogeneous mixture subjected to a temperature gradient. The heat flux induced by a concentration gradient is called Dufour effect. These effects are significant in areas such as hydrology, petrology, chemical reactors, drying processes, geosciences etc. The effects of Soret and Dufour on the heat and mass transfer were developed from the kinetic theory of gases by Chapman and Cowling [1]. Hirschfelder et al [2] presented the necessary formulae to calculate the thermal-diffusion coefficient and the thermal diffusion factor for monatomic gases or for polyatomic gas mixtures. Thermal-diffusion effects on mass transfer investigated by Baron [3]. In the recent years, the convective heat and mass transfer about cylindrical bodies has begun to attract the attention of many researchers as the cylinders have been used in nuclear waste disposal, energy extortion in underground and catalytic beds. Free convective flows driven by temperature and concentration differences have been studied extensively. Chamkha [4] investigated the heat and mass transfer of a MHD flow over a moving permeable cylinder with heat generation or absorption and chemical reaction. Trujillo et al. [5] analyzed the heat and mass transfer process during the evaporation of water from a circular cylinder through CFD modeling. In the context of space technology and in the processes involving high temperatures, the effect of radiation plays a vital role. The radiation conduction interaction on mixed convection from a horizontal circular cylinder using an implicit finite-difference scheme presented by Hossain et.al. [6]. Recently, Swati Mukhopadyaya [7] investigated the steady boundary layer slip flow and heat transfer along a horizontal stretching cylinder in the presence of uniform magnetic field. In that work she showed numerically that the curvature parameter affects the flow field. Later, Hayat et.al. [8] extended this work by considering vertical stretching cylinder with heat and mass transfer.

To the best of our knowledge, the problem of steady MHD axisymmetric laminar boundary layer flow of a viscous incompressible electrically conducting and radiating fluid towards a stretching cylinder in the presence of Soret and Dufour effects with heat and mass transfer has remained unexplored. So, the main objective of this paper is to extend the work of Hayat et.al [8] in three directions: (i) to consider the radiation effects (ii) to consider the Soret effect and (iii) to include the Dufour effect. The governing equations of the flow are solved numerically, and the effects of various flow parameters on the flow field have been discussed. The organization of the
paper is as follows. In Section 2, we describe the model with it governing equations and boundary conditions. In section 3, we present method of solution, In Section 4, we present results and discussion. Finally, in Section 5, we summarize our results and present our conclusions.

II. MATHEMATICAL FORMULATION

A steady axi-symmetric laminar mixed convective boundary layer flow of a viscous incompressible electrically conducting and radiating fluid past along a stretching cylinder subject to a uniform transverse magnetic field in the presence of thermal and concentration buoyancy effects, Soret and Dufour effects is considered. The x-axis is measured along the axis of the cylinder and the r-axis is measured in the radial direction. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small so that induced magnetic field and the Hall effects are negligible. Joule heating and viscous dissipation are not taken into account. No chemical reaction exists. The continuity, momentum and energy equations governing such type of flow are written as

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial r} = 0
\]  

(1)

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{\rho}{\rho c_p} \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} \right) + \frac{g \beta_T (T - T_w)}{\rho} + \frac{g \beta_C (C - C_w)}{\rho}
\]

(2)

\[
\frac{\partial C}{\partial x} + \frac{\partial u}{\partial r} = \frac{D_m}{r} \frac{\partial}{\partial r} \left( \frac{\partial C}{\partial r} \right) + \frac{D_m K_f}{r} \frac{\partial}{\partial r} \left( \frac{\partial C}{\partial r} \right)
\]

(3)

where \(u\) and \(v\) are the components of velocity in the x and y directions respectively, \(\rho\) is the fluid density, \(\sigma\) is the electrical conductivity of the fluid, \(B_0\) is the magnetic induction, \(g\) is the acceleration due to gravity, \(\beta_T\) and \(\beta_C\) are the thermal and concentration expansion coefficients respectively, \(k\) is the thermal conductivity, \(c_p\) is the specific heat at constant pressure, \(q_r\) is the radiative heat flux, \(D_m\) is the solute diffusion coefficient, \(K_f\) is the thermal diffusion coefficient, \(c_s\) is the concentration susceptibility, \(T_m\) is the mean fluid temperature. Also, the last term in equation (3) represents the diffusion-thermal effect i.e., Dufour effect and the last term in equation (4) denotes the thermal – diffusion effect i.e., Soret effect.

The boundary conditions for the velocity, temperature and concentration fields are

\[u = U(x) + vB_1 \frac{\partial u}{\partial r}, \quad v = 0, \quad T = T_w(x) + K_1 \frac{\partial T}{\partial r}, \quad C = C_w(x) + K_2 \frac{\partial C}{\partial r}\] at \(r = R\)

\[u \to 0, T \to T_{\infty}, C \to C_{\infty} \quad \text{as} \quad r \to \infty \]  

(5)

Here \(U(x) = U_0 x\) is the stretching velocity, \(T_w(x) = T_{\infty} + T_0 (\frac{x}{L})^N\) is the prescribed surface temperature, \(C_w(x) = C_{\infty} + C_0 (\frac{x}{L})^N\) is the prescribed surface concentration, \(U_0, T_0\) and \(C_0\) are the reference velocity, temperature and concentration respectively, \(T_{\infty}\) and \(C_{\infty}\) are the ambient temperature and concentration respectively, \(L\) is the characteristic length, \(N\) is the temperature and concentration exponent, \(B_1\) is the velocity slip parameter, \(K_1\) is the thermal slip parameter and \(K_2\) is the concentration slip parameter.

Using Rosseland approximation for radiation, we write

\[q_r = \frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}\]

(6)

where \(\sigma^*\) is the Stefan-Botman constant, \(k^*\) is absorption coefficient.

Assuming that the temperature difference with in the flow is such that \(T^4\) may be expresses in a Taylor series and expanding \(T^4\) about \(T_{\infty}\), the free stream temperature and neglecting higher orders we get

\[T^4 \approx 4T_{\infty}^3 T - 3T_{\infty}^4\]

(7)

Invoking equations (6) and (7), equation (3) can be modified as

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \left( k \frac{16\sigma^* T_{\infty}^3}{3\rho c_p k^*} \right) \frac{\partial}{\partial r} \left( \frac{\partial T}{\partial r} \right) + \frac{D_m K_f 1}{c_s c_p} \frac{\partial}{\partial r} \left( \frac{\partial C}{\partial r} \right)
\]

(8)

III. METHOD OF SOLUTION

To get similarity solutions of equations (1),(2),(3) and (8) subject to the boundary conditions (5), we introduce the following similarity transformations:

\[\eta = \frac{r^2 - R^2}{2R}, \quad \Psi = \sqrt{\frac{U}{u x}} e^{\frac{\eta}{2}}, \quad \Theta(\eta), \quad \Phi(\eta), \quad D^2 = \frac{\sigma B_0^2 L}{\rho U_0}, \quad M = \frac{uL}{v U_0 R^2}, \quad \lambda_T = \frac{g \beta_T (T_w - T_{\infty}) L^2}{u_0^2 x^2}, \quad \lambda_M = \frac{g \beta_C (C_w - C_{\infty}) L^2}{u_0^2 x^2}, \quad \delta = \frac{\lambda T}{\lambda M}, \quad R = \frac{4\sigma^* r^3}{k k^*}, \quad B = B_0 \frac{u_{\infty}}{L}, \quad K_r = K_r \frac{u_{\infty}}{v c_s c_p}, \quad K_c = K_c \frac{u_{\infty}}{v c_s c_p}, \quad Pr = \frac{\nu c_p}{k}, \quad D_f = \frac{D_k c_p C_0}{v c_s c_p}, \quad Sc = \frac{v}{D}, \quad S_r = \frac{D_k c_p C_0}{v_r m c_p}\]

(9)

where \(\Psi\) is the stream function and the velocity components are defined as
\[ u = \frac{1}{r} \frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x} \]  
(10)

which identically satisfies the continuity equation (1).

Using the above similarity transformations, equations (2),(8) and (4) reduce to

\[ (1 + 2M\eta)f'' + 2Mf'' + ff' = (f')^2 - D^2 f' + \lambda_T (\theta + \delta \phi) = 0 \]  
(11)

\[ \left(1 + \frac{4}{3} R \right) (\theta'' + 2M\theta') - Pr/M \theta'' N = f \theta' + PrD_j (\phi'' + 2M \phi') = 0 \]  
(12)

\[ \phi'' (1 + 2M\eta) + 2M\phi' - ScC_i (\phi N - f \phi') + ScS_r (\theta''(1 + 2M\eta) + 2M\theta') = 0 \]  
(13)

and the corresponding boundary conditions are

\[ f' = 1 + B f', \quad f = 0, \quad \theta = 1 + K_T \theta', \quad \phi = 1 + K_C \phi' \text{ at } \eta = 0 \]  
(14)

\[ f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty \]

where the prime denotes differentiation with respect to \( \eta \), \( M \) is the curvature parameter, \( D \) is the magnetic parameter, \( \lambda_T \) is the thermal buoyancy parameter, \( \lambda_M \) is the solutal buoyancy parameter, \( \delta \) is the buoyancy ratio parameter, \( R \) is the radiation parameter, \( Pr \) is the Prandtl number, \( S_r \) is the Soret number, \( D_j \) is the Dufour number, \( Sc \) is the Schmidt number, \( B \) is the velocity slip parameter, \( K_T \) is the thermal slip parameter, \( K_C \) is the concentration slip parameter. The no-slip case is recovered for \( B=0, K_T = 0, K_C = 0 \).

The parameters of practical interest for the present problem are the skin-friction coefficient, the Nusselt number and the Sherwood number, which are given respectively by the following expressions. Knowing the velocity field the skin-friction at the plate can be obtained, which in non-dimensional form is given by

\[ \frac{1}{2} \sqrt{Re} C_f = f'' (0) \]

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non-dimensional form, in terms of Nusselt number is given by

\[ \sqrt{Re} Nu = -\theta' (0) \]

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in non-dimensional form, in terms of Sherwood number, is given by

\[ \sqrt{Re} Sh = -\phi' (0) \]

where \( Re = \frac{u \text{d} \delta}{v} \) is the local Reynolds number.

The above equations (11)-(13) along with boundary conditions (14) are solved by converting them to an initial value problem. We set

\[ f' = z, z' = p, p' = z^2 + D^2 z - 2Mp - f\lambda_T (\theta + \delta \phi)/(1 + 2M\eta) \]

\[ \theta' = q, q' = -\left(\frac{3 + 4R}{3}\right) Mq + Pr(z\theta N - f q) - PrD_j (\theta'' + 2M\theta')/(1 + 2M\eta) \]

\[ \phi' = r, r' = -2Mr + Sc(z\phi N - f r) - ScS_r (q' (1 + 2M\eta) + 2Mq)/(1 + 2M\eta) \]

and

\[ f (0) = 0, f' (0) = 1 + Bf'' (0), f'' (0) = \gamma \]  
(15)

\[ \theta (0) = 1 + K_T \beta, \quad \theta' (0) = \beta, \]  
\[ \phi (0) = 1 + K_C \xi, \quad \phi' (0) = \xi \]  
(16)

In order to integrate (15)-(17) as initial value problems one requires \( f' (0) , \theta' (0) \) and \( \phi' (0) \). But no such values are given at the boundary. The suitable guess values for \( f' (0) \), \( \theta' (0) \) and \( \phi' (0) \) are chosen and then integration is carried out. Comparing the calculated values for \( f', \theta \) and \( \phi \) at \( \eta=6 \) (say) with the given boundary conditions \( f (0) = 0, \theta (0) = 0 \) and \( \phi (0) = 0 \) and adjusting the estimated values, \( f'' (0) \), \theta' (0) \) and \( \phi' (0) \) for better approximations for the solutions are given.

Taking the series of values for \( f' (0) \), \theta' (0) \) and \( \phi' (0) \) and applying the fourth order Runge-Kutta method with step size \( \Delta \eta = 0.01 \), the above procedure is repeated until the results upto the desired degree of accuracy \( 10^{-5} \) are obtained.

IV. RESULTS AND DISCUSSIONS

To analyze the results, numerical computations has been carried out for variations in the governing parameters such as the curvature parameter \( M \), magnetic parameter \( D \), radiation parameter \( R \), Prandtl number \( Pr \), Dufour number \( D_j \), Soret number \( S_r \), Schmidt number \( Sc \), buoyancy ratio parameter \( \delta \), velocity slip parameter \( B \), thermal slip parameter \( K_T \), concentration slip parameter \( K_C \). For illustration of these results, numerical values are plotted in figures 2-22.

In the present study following default parameter values are adopted for computations: \( M=1,D=0.1, R=0.1, Pr=0.7, D_j = 0.1, S_r = 0.1, Sc=0.6, \delta = 1, N=1, \lambda_T = 0.1 \). All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.
Figure 2: Effect of D on the velocity

Figure 3: Effect of D on the temperature

Figure 4: Effect of D on the concentration

Figure 5: Effect of M on the velocity

Figure 2 represents the velocity profiles for different values of the magnetic parameter M. With increasing values of M, fluid velocity is found to decrease. Actually, rate of transport decreases with the increase in M because the Lorenz force which oppose the motion of fluid increases with the increase in M. From Figure 3 we see that the temperature increase with the increase of the magnetic field parameter, which implies that the applied magnetic field tends to heat the fluid and thus reduces the heat transfer from the wall. In Figure 4, the effect of an applied magnetic field is found to increase the concentration and hence increase the concentration boundary layer.
Figures (5)-(7) represent the effect of curvature on the velocity, temperature and concentration. Physically $M=0$ means the cylinder’s outer surface behaves like a stretching flat surface. It means as $M$ approaches to 1, the viscosity effect reduces due to contact area of surface with fluid tends to the tangential position. From Figure 5, it can be observed that the effect of curvature parameter on the velocity field is very small within the region $[0, 0.75]$. This velocity approaches to zero asymptotically within the region $[0.75, \infty)$. In this case, the velocity with in $[0.75, \infty)$ is free stream velocity and in this region, as $M$ increases, the velocity increases. Similar trend is observed for both temperature and concentration i.e., initially both the temperature and concentration decreases and then they increase.

The influence of radiation parameter $R$ on the velocity, temperature and concentration are shown in Figures (8)-(10). The radiation parameter $R$ defines the radiative contribution of conduction heat transfer to thermal
radiation transfer. It is obvious that an increase in the radiation parameter results in increasing velocity and temperature while the concentration of the fluid decreases within the boundary layer.

Figure 11: Effect of Pr on the velocity

Figure 12: Effect of Pr on the temperature

Figure (11)-(12) illustrate the velocity and temperature profiles for different values of Prandtl number Pr. It is found that an increase in the value of Pr leads to a decrease in both temperature as well as concentration. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Fluids with lower Prandtl number will possess higher thermal conductivities (and thicker boundary layer structures) so that heat can diffuse from the wall faster than for higher Pr fluids (thinner boundary layers).

Figure 13: Effect of Df on the velocity

Figure 14: Effect of Df on the temperature

Figure 15: Effect of Df on the concentration

For different values of the Dufour number Df, the velocity, temperature and concentration profiles are plotted in figures (13)-(15). The Dufour number Df defines the contribution of the concentration gradients to the thermal energy flux in the flow. It is found that an increase in the Dufour number causes a rise in the velocity and temperature while the concentration of the fluid decreases.
throughout the boundary layer. For $D_f < 1$, the temperature profiles decay smoothly from the plate to the free stream value. However, for $D_f > 1$, a distinct overshoot exists near the plate, thereafter the profile falls to zero at the edge of the boundary layer. For variation in Dufour number $D_f$ the concentration changes slightly.

The velocity, temperature and concentration profiles for different values of Soret number $S_r$ are presented in Figures (16)-(18). The Soret number $S_r$ signifies the effect of the temperature gradients inducing significant mass diffusion effects. It is noticed that an increase in the Soret number $S_r$ results in an increase in the velocity, concentration while decrease in the temperature within the boundary layer. Therefore, we can understand that the influence of Soret as well as Dufour effects is greatly effective in the study of mixed convection problems.

The influence of the Schmidt number $S_C$ on the concentration is plotted in Figure 19. The Schmidt number is the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydromagnetic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the concentration boundary layer. This causes a reduction in concentration profiles.
The velocity profiles are presented in Figure 20 for the variation of velocity slip parameter $B$. With the increasing $B$, the velocity is found to decrease. This feature prevails up to certain heights and then the process is slowdown. When slip occurs, the flow velocity near the stretching wall is no longer equal to the stretching velocity of the wall. With the increase in $B$ such slip velocity increases and consequently fluid velocity decreases because under the slip condition, the pulling of the stretching wall can only partly transmitted to the fluid. $B=0$ corresponds to the no-slip case.

Figures (21) and (22) depict the effect of thermal slip parameter $K_T$ and concentration slip parameter $K_C$ on the temperature and concentration respectively. From Figure (21), it can be observed that, initially the temperature decreases with thermal slip parameter $K_T$ but after a certain distance $\eta$ from the stretching wall, such feature is smeared out. With the increase of thermal slip parameter $K_T$, less heat is transferred to the fluid from the stretching wall and so temperature is found to decrease. Similar trend is observed for concentration. i.e., with the increasing concentration slip parameter $K_C$, the concentration is found to decrease.

V. CONCLUSIONS

The present study gives the numerical solution for study MHD axi-symmetric heat and mass transfer slip flow of a viscous incompressible electrically conducting and radiating fluid along a stretching cylinder in the presence of cross diffusion and radiation. The resulting partial differential equations were transformed into a set of highly non-linear ordinary differential equations using similarity transformations. Numerical solutions of these equations are obtained by fourth order Runge-Kutta method along with shooting technique. A comprehensive set of graphical results for the velocity, temperature and concentration is presented and their dependence on some physical parameters is discussed. Based on the present investigation, the following observations are made:

1. An increase in magnetic parameter decreases the velocity but increases the temperature as well as the concentration.
2. The curvature parameter of the stretching cylinder plays a vital role by affecting the velocity, temperature and concentration.
3. An increase in radiation leads to an increase in the velocity and temperature. However, there is a decrease in concentration.
4. With an increase in Dufour number both the velocity and temperature increases where as the concentration decreases.
5. With an increase in Soret number both the velocity and concentration increases whereas the temperature decreases.
6. Prandtl number reduces the thermal boundary layer.

REFERENCES