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Reliability Evaluation for Power Distribution System

Dr. Reena Garg
Assistant Professor, YMCA University, Faridabad, Haryana, INDIA

ABSTRACT

In this model, the authors have evaluated availability and profit function for a power distribution system. Since the system is of non-Markovian nature, the supplementary variables have introduced to make the system Markovian. All the failures follow exponential time distribution whereas all the repairs follow general time distribution. System configuration has shown in fig.1. Laplace-transform and supplementary variables technique have used to solve and formulated the mathematical model, respectively. Steady-state behavior of the system and a particular case has also been obtained to improve the practical utility of the model.

Keywords: Markovian process, supplementary variables, steady state behavior, Laplace-transform.

I. INTRODUCTION

The authors have considered a power distribution system, in this model, to evaluate some reliability parameters. The whole system is divided in to two subsystems, namely *A* and *B*, connected in series. The whole system can fail due to failure of its either subsystems, due to environmental reasons and due to human error. The subsystem *A* is power generation system and it has M non-identical power plants connected in parallel redundancy. The subsystem *A* is of 2-out of M: F nature. The subsystem *B* is power distribution system and it contains N- non-identical units (as shown in fig. 1) connected in series. The subsystem *A* is of 1-out of N:F nature. The state-transition diagram has shown in fig.2. Some important reliability parameters together with a numerical example and graphical illustrations have appended at the end to show the practical utility of the model.

II. ASSUMPTIONS

The following assumptions are associated with this model:

(1) Initially, all the components are in operable condition of full efficiency.
(2) Failures are statistically independent.
(3) The subsystem *B* is of nature 1-out-of –M: D.
(4) All the failures follow exponential time distribution where as all the repairs follow general time distribution.
(5) System may be fail due to environmental reasons and human error.
(6) Repair facilities are always available.
(7) After repair system works like a new one and never damages anything.
(8)

III. NOTATIONS

The following notations have been used throughout this model:

$P_{0,0}(t)$:The probability that at time ‘t’, the system is in operable state.

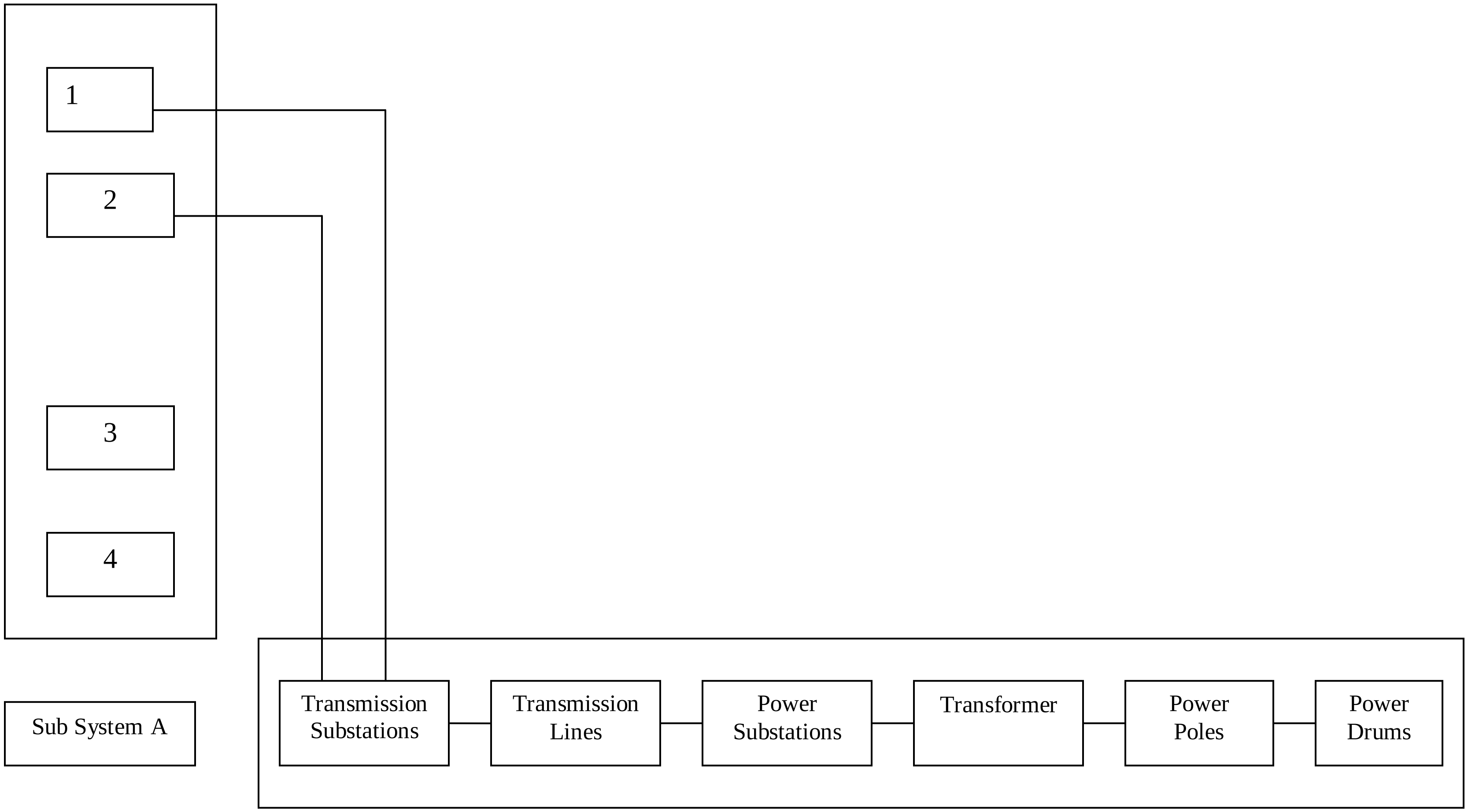


Fig.1 (System Configuration)

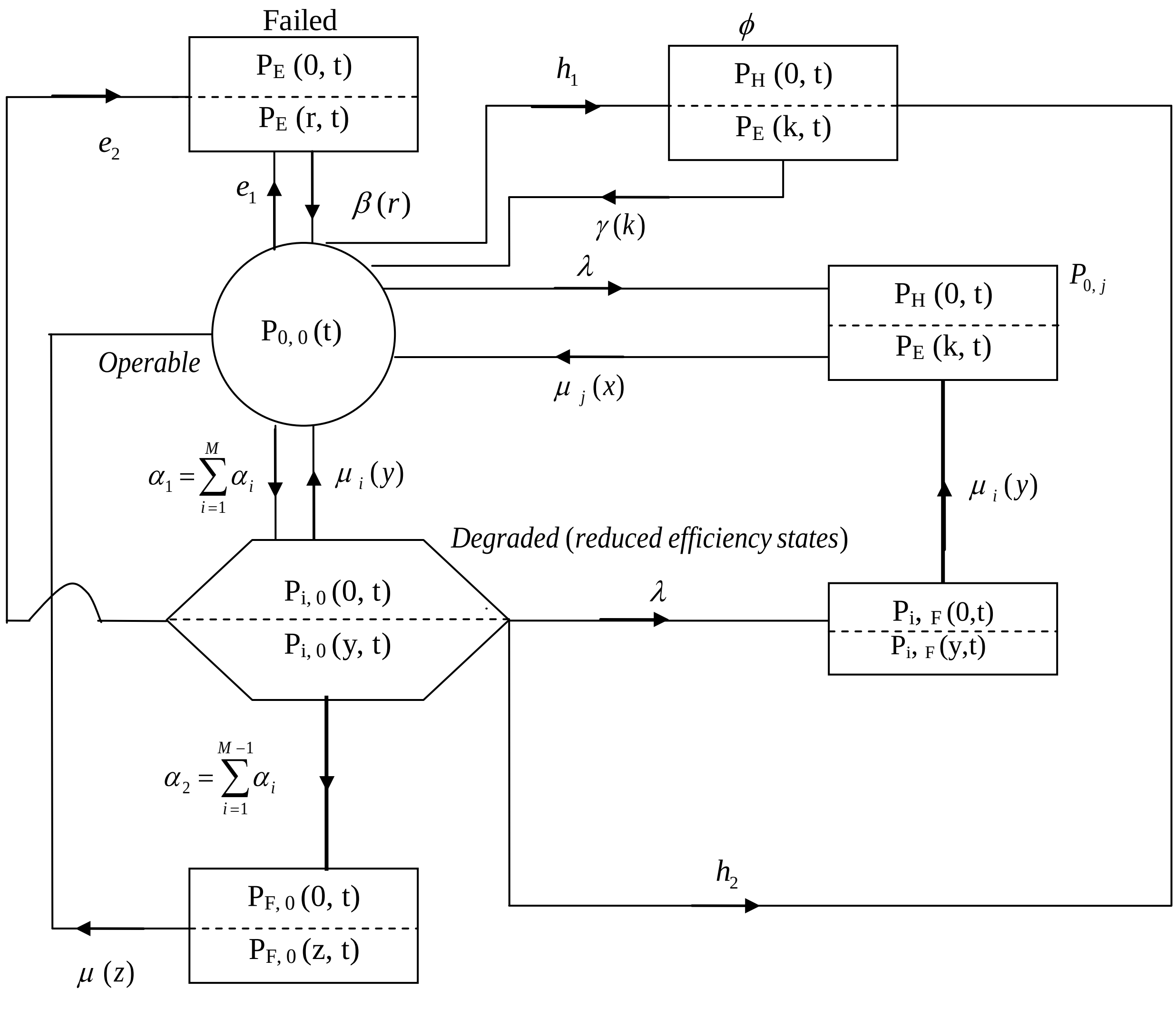


Fig. 2. State-transition Diagram

$P_{0,j}(x,t)\Delta/P_{i,F}(y,t)\Delta$	:The probability that at time 't' the system is in failed state due to failure of (j^{th}-B)/(i^{th}-A and j^{th}-B) component and elapsed repair time lies in the interval (x, x+Δ) / (y, y+Δ) .
$P_{i,0}(y,t)\Delta/P_{F,0}(z,t)\Delta$	:The probability that at time 't' , the system is in degraded/failed state due to failure of i^{th} –A/2-A components and elapsed repair time lies in the interval (y, y+Δ) /(z, z+Δ) .

$P_E(r,t)\Delta/P_H(k,t)\Delta$:The probability that at time 't', the system is in failed state due to environmental reasons/human error and elapsed repair time lies in the interval (r, r+Δ)/(k, k+Δ).

α_i/λ_j : Failure rate of i^{th}-A/j^{th}-B component.

e_1/e_2 : Failure rate due to environmental reasons.

h_1/h_2 : Failure rate due to human error.

$\beta(r)\Delta/\gamma(k)\Delta$: Probability that system will be repaired from environmental failure/human error in time interval (r, r+Δ)/(k, k+Δ) conditioned that it was not repaired up to time r/k.

$\mu_i(y)\Delta/\ \mu_j(x)\Delta/\ \mu(z)\Delta$: Probability that i^{th}-A/j^{th}-B/2A components will be repaired in the time interval (y, y+Δ)/(x, x+Δ)/(z, z+Δ) , conditioned that it was not repaired up to the time y/x/z.

$\overline{F}(s)$: Laplace transform of the function F(t).

$P_{up}(t)/\ P_{down}(t)$: Probability that at time t , the system is in up/down state.

IV. FORMULATION OF MATHEMATICAL MODEL

By the elementary probability consideration and continuity argument, we may obtain the following set of difference-differential equations for the stochastic process, which is continuous in time and discrete in space:

$$\left[\frac{d}{dt}+\lambda+\alpha_1+e_1+h_1\right]P_{0,0}(t)=\int_0^\infty P_{0,j}(x,t)\mu_j(x)dx+\int_0^\infty P_{i,0}(y,t)\mu_i(y)dy+\int_0^\infty P_H(k,t)\gamma(k)dk+$$

$$\int_0^\infty P_E(r,t)\beta(r)dr+\int_0^\infty P_{F,0}(z,t)\mu(z)dz \tag{1}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_j(x)\right]P_{0,j}(x,t)=0 \tag{2}$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\alpha_2+\lambda+h_2+e_2+\mu_i(y)\right]P_{i,0}(y,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\mu(z)\right]P_{F,0}(z,t)=0 \tag{4}$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_i(y)\right]P_{i,F}(y,t)=\lambda\ P_{i,0}(y,t) \tag{5}$$

$$\left[\frac{\partial}{\partial r}+\frac{\partial}{\partial t}+\beta(r)\right]P_E(r,t)=0 \tag{6}$$

$$\left[\frac{\partial}{\partial k}+\frac{\partial}{\partial t}+\gamma(k)\right]P_H(k,t)=0 \tag{7}$$

Boundary Conditions are

$$P_{0,j}(0,t) = \lambda\ P_{0,0}(t) + \int_0^\infty P_{i,F}(y,t)\mu_i(y)dy \tag{8}$$

$$P_{i,0}(0,t) = \alpha_1\ P_{0,0}(t) \tag{9}$$

$$P_{F,0}(0,t) = \alpha_2\ P_{i,0}(t) \tag{10}$$

$$P_{i,F}(0,t) = 0 \tag{11}$$

$$P_E(0,t) = e_1\ P_{0,0}(t) + e_2\ P_{i,0}(t) \tag{12}$$

$$P_H(0,t) = h_1\ P_{0,0}(t) + h_2\ P_{i,0}(t) \tag{13}$$

Initial conditions are

$$P_{0,0}(0) = 1; \qquad \text{Otherwise all state probabilities are zero.} \tag{14}$$

V. SOLUTION OF MATHEMATICAL MODEL

Taking Laplace Transforms of equations (1) through (13), subjected to (14) and then on solving them one by one , we may obtained the following Laplace transforms of different state probabilities:

$$\overline{P}_{0,0}(s) = \frac{1}{A(s)} \tag{15}$$

$$\overline{P_{0,j}}(s) = \frac{\lambda}{A(s)}[1 + \frac{\alpha_1}{\lambda + \alpha_2 + h_2 + e_2}\{\overline{S_i}(s) - \overline{S}_i(s + \lambda + \alpha_2 + h_2 + e_2)\}]Dj(s) \tag{16}$$

$$\overline{P_{i,0}}(s) = \frac{\alpha_1}{A(s)}\left[D_i(s + \lambda + \alpha_2 + h_2 + e_2)\right] \tag{17}$$

$$\overline{P_{F,0}}(s) = \frac{\alpha_1\alpha_2}{A(s)}\left[D_i(s + \lambda + \alpha_2 + h_2 + e_2)\right]D(s) \tag{18}$$

$$\overline{P_{i,F}}(s) = \frac{\lambda\alpha_1}{A(s)(\lambda + \alpha_2 + h_2 + e_2)}\left[D_i(s) - D_i(s + \lambda + \alpha_2 + h_2 + e_2)\right] \tag{19}$$

$$\overline{P_E}(s) = \frac{1}{A(s)}[e_1 + e_2\alpha_1 D_i(s + \lambda + \alpha_2 + h_2 + e_2)]D_\beta(s) \tag{20}$$

$$\overline{P_H}(s) = \overline{P}_{0,0}(s)[h_1 + h_2\alpha_1 D_i(s + \lambda + \alpha_2 + h_2 + e_2)]D\gamma(s) \tag{21}$$

where,

$$D_A(B) = \frac{1 - \overline{S_A(B)}}{B}, \quad \text{for all A and B.}$$

and A(s)= $s + \lambda + \alpha_1 + h_1 + e_1 - \alpha_1\ \overline{S_i}\ (s + \lambda + \alpha_2 + h_2 + e_2)$

$$- \lambda[1 + \frac{\alpha_1}{\lambda + \alpha_2 + h_2 + e_2}\{\overline{S_i}(s) - \overline{S}_i(s + \lambda + \alpha_2 + h_2 + e_2)\}]\overline{S_j}(s)$$

$$- \alpha_1\alpha_2 D_i(s + \lambda + \alpha_2 + h_2 + e_2)\overline{S}(s)$$

$$- [e_1 + e_2\alpha_1 D_i(s + \lambda + \alpha_2 + h_2 + e_2)]\overline{S}_\beta(s)$$

$$- [h_1 + h_2\alpha_1 D_i(s + \lambda + \alpha_2 + h_2 + e_2)]\overline{S}\gamma(s) \tag{22}$$

Also, it is interesting to note that

$$\overline{P}_{0,0}(s)+\overline{P_{0,j}}(s)+\overline{P}_{i,0}(s)+\overline{P}_{F,0}(s)+\overline{P}_{i,F}(s)+\overline{P_E}(s)+\overline{P_H}(s)=\frac{1}{s} \tag{23}$$

VI. STEADY-STATE BEHAVIOR OF THE SYSTEM

By using Abel's lemma, viz; $\lim_{s\to 0} s\overline{P}(s)=\lim_{t\to\infty} P(t)=P$ (say) ;Provided the limit on R.H.S. exist, we have the following time independent state probabilities from equations (15) through (21):

$$P_{0,0}=\frac{1}{A'(0)} \tag{24}$$

$$P_{0,j}=\frac{\lambda}{A'(0)}[1+\frac{\alpha_1}{\lambda+\alpha_2+h_2+e_2}\{1-\overline{S}_i(\lambda+\alpha_2+h_2+e_2)\}]Tj \tag{25}$$

$$P_{i,0}=\frac{\alpha_1}{A'(0)}\left[D_i(\lambda+\alpha_2+h_2+e_2)\right] \tag{26}$$

$$P_{F,0}=\frac{\alpha_1\alpha_2}{A'(0)}\left[D_i(\lambda+\alpha_2+h_2+e_2)\right]T \tag{27}$$

$$P_{i,F}=\frac{\lambda\alpha_1}{A'(0)(\lambda+\alpha_2+h_2+e_2)}\left[T_i-D_i(\lambda+\alpha_2+h_2+e_2)\right] \tag{28}$$

$$P_E=\frac{1}{A'(0)}[e_1+e_2\alpha_1D_i(\lambda+\alpha_2+h_2+e_2)]T_\beta \tag{29}$$

$$P_H=\frac{1}{A'(0)}[h_1+h_2\alpha_1D_i(\lambda+\alpha_2+h_2+e_2)]T\gamma \tag{30}$$

where,

$$A'(0)=\left[\frac{d}{ds}A(s)\right]_{s=0} \quad \text{and} \quad T_j=-\overline{S_i}'(0)etc.$$

VII. PARTICULAR CASE

When all the repairs follow exponential time distribution

Setting $\overline{S_i}(s)=\frac{\mu_i}{s+\mu_i}$ etc. in equations (15) through (21), we obtain the following Laplace transforms of various state probabilities:

$$\overline{P}_{0,0}(s)=\frac{1}{B(s)} \tag{31}$$

$$\overline{P_{0,j}}(s)=\frac{\lambda}{B(s)}[1+\frac{\alpha_1}{\lambda+\alpha_2+h_2+e_2}\{\frac{\mu_i}{s+\mu_i}-\frac{\mu_i}{s+\lambda+\alpha_2+h_2+e_2+\mu_i}\}]\frac{1}{s+\mu_i} \tag{32}$$

$$\overline{P_{i,0}}(s)=\frac{\alpha_1}{B(s)}\frac{1}{s+\lambda+\alpha_2+h_2+e_2+\mu_i} \tag{33}$$

$$\overline{P_{F,0}}(s)=\frac{\alpha_1\alpha_2}{B(s)}\frac{1}{(s+\lambda+\alpha_2+h_2+e_2+\mu_i)(s+\mu)} \tag{34}$$

$$\overline{P_{i,F}}(s)=\frac{\lambda\alpha_1}{B(s)(\lambda+\alpha_2+h_2+e_2)}\left[\frac{1}{s+\mu_i}-\frac{1}{s+\lambda+\alpha_2+h_2+e_2+\mu_i}\right] \tag{35}$$

$$\overline{P_E}(s)=\frac{1}{B(s)}[e_1+e_2\alpha_1\frac{1}{s+\lambda+\alpha_2+h_2+e_2+\mu_i}]\frac{1}{s+\beta} \tag{36}$$

$$\overline{P_H}(s)=\frac{1}{B(s)}[h_1+h_2\alpha_1\frac{1}{s+\lambda+\alpha_2+h_2+e_2+\mu_i}]\frac{1}{s+\gamma} \tag{37}$$

where,

B(s)= $s+\lambda+\alpha_1+h_1+e_1$-

$$\frac{\alpha_1\mu_i}{s+\lambda+\alpha_2+h_2+e_2+\mu_i}-\lambda[1+\frac{\alpha_1}{\lambda+\alpha_2+h_2+e_2}\left\{\frac{\mu_i}{s+\mu_i}-\frac{\mu_i}{s+\lambda+\alpha_2+h_2+e_2+\mu_i}\right\}]$$

$$\frac{\mu_j}{s+\mu_j}-\frac{\alpha_1\alpha_2}{s+\lambda+\alpha_2+h_2+e_2+\mu_i}\frac{\mu}{s+\mu}-\left[e_1+e_2\alpha_1\frac{1}{s+\lambda+\alpha_2+h_2+e_2+\mu_i}\right]\frac{\beta}{s+\beta}$$

$$-\left[h_1+\frac{h_2\lambda_1}{s+\lambda+\alpha_2+h_2+e_2+\mu_i}\right]\frac{\gamma}{s+\gamma} \tag{38}$$

VIII. UP AND DOWN STATE PROBABILITIES

We have

$$P_{Up}(t)=(1+c)\exp\{-(\lambda+\alpha_1+e_1+h_1)t\}-c\exp\{-(\lambda+\alpha_2+h_2+e_2)t\} \tag{39}$$

where, $$C=\frac{\alpha_1}{\alpha_2+h_2+e_2-\alpha_1-h_1-e_1} \tag{40}$$

Also, $$P_{down}(t)=1-P_{Up}(t) \tag{41}$$

IX. PROFIT FUNCTION

The profit function for the considered system is given by

$$G(t)=C_1\int_0^t P_{Up}(t)dt-C_2t-C_3$$

where, C_1 and C_2 are revenue and repair costs per unit time and C_3 is system establishment cost per set up. So, have

$$G(t)=C_1\left[\frac{1+C}{\lambda+\alpha_1+e_1+h_1}\left\{1-e^{-(\lambda+\alpha_1+e_1+h_1)t}\right\}-\frac{C}{\lambda+\alpha_2+h_2+e_2}\left\{1-e^{-(\lambda+\alpha_2+h_2+e_2)t}\right\}\right]-C_2t-C_3 \tag{42}$$

where, C is shown in equation (40).

X. NUMERICAL COMPUTATION

For a numerical computation, let us consider the values:
$\lambda = 0.001, \alpha_1 = 0.06, \alpha_2 = 0.07, e_1 = 0.01, e_2 = 0.02, h_1 = 0.03, h_2 = 0.035, C_1 = Rs.7.00\,\textit{per unit time}$
C_2 = Rs. 2.00 per unit time , C_3 = Rs. 10.00 per set up and t = 0,1,2 .. ..
By using these values in equations (39),(41) and (42), we may observe the changes in values of corresponding reliability parameters.

where $C = \frac{.06}{.125-.1} = \frac{.06}{.025} = 2.4$

$P_{Up}(t) = 3.4\, e^{-0.101t} - 2.4\, e^{-0.126t}$ and

$$G(t) = 7\left[\frac{3.4}{.101}\left\{1-e^{-.101t}\right\} - \frac{2.4}{.126}\left\{1-e^{-0.126}\right\}\right] - 2t - 10$$

XI. RESULTS & DISCUSSION

In this model, the authors have evaluated availability and profit function for a power distribution system. Since the system is of non-Markovian nature, the supplementary variables have been introduced to make the system Markovian. These supplementary variables disappear at the solution stage.

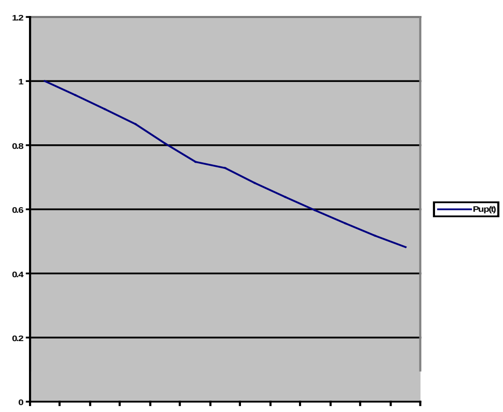
We compute the table-1 and 2 for the considered numerical example. The corresponding graphs have shown in the fig-3 and 4. Table-1 shows that how the availability of the power plant decreases as we make increase in the value of time. Availability decreases approximately in a constant manner.

From table-2 we observe that from *t= 0 to 2* the profit function is negative. This shows that there is no profit up to *t=2* because initially we only have to spend money to establish the system and initial repairs. After *t=2* , we recover the system establishment cost and so it becomes positive. Thus after *t=2* profit function increases in uniform way.

t	$P_{Up}(t)$
0	1.0
1	0.9568
2	0.91286
3	0.86644
4	0.80536
5	0.74786
6	0.7279
7	0.68318
8	0.63962
9	0.59778
10	0.5574
11	0.51904
12	0.48264

Table-1

$P_{up}(t)$



t

Fig 3

t	G(t)
0	-10
1	-5.112112
2	-0.6085219
3	3.6223846
4	8.3526956
5	12.543772
6	14.364086
7	17.298193
8	19.934112
9	22.259432
10	24.311971
11	26.079316
12	27.578826

Table-2

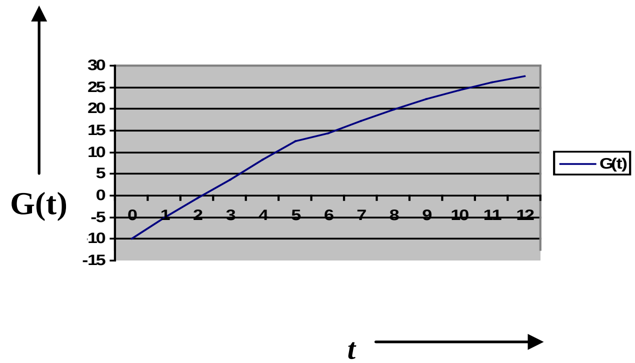


Fig 4

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