Vandana Publications
IJEMR

Volume-7, Issue-4, July-August 2017
International Journal of Engineering and Management Research
Page Number: 292-296

Sample Mean Deviation (d) Chart Under the Assumption of Moderateness and its Performance Analysis Under Normality Against Moderateness

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ABSTRACT

Moderate distribution proposed by Naik V.D and Desai J.M. ,is a sound alternative of normal distribution, which has mean and mean deviation as pivotal parameters and which has properties similar to normal distribution. Mean deviation (δ) is a very good alternative of standard deviation (σ) as mean deviation is considered to be the most intuitively and rationally defined measure of dispersion. This fact can be very useful in the field of quality control to construct the control limits of the control charts. On the basis of this fact Naik V.D. and Tailor K.S. have proposed 3δ control limits. In 3δ control limits, the upper and lower control limits are set at 3δ distance from the central line where δ is the mean deviation of sampling distribution of the statistic being used for constructing the control chart. In this paper assuming that the underlying distribution of the variable of interest follows moderate distribution, 3δ control limits of sample mean deviation (d) chart are derived. Also the performance analysis of this control chart is carried out with the help of OC curve analysis and ARL curve analysis.

Key words**--** Mean deviation, Standard deviation, Control charts, Normal distribution, Moderate distribution, Statistic, OC function, ARL curve

I. INTRODUCTION

A fundamental assumption in the development of control charts for variables is that the underlying distribution of the quality characteristic is normal. The normal distribution is one of the most important distributions in the statistical inference in which mean (μ) and standard deviation (σ) are the parameters of this distribution. Naik V.D and Desai J.M. have suggested an alternative of normal distribution, which is called moderate distribution. In moderate distribution mean (μ) and mean deviation (δ) are the pivotal parameter and, which has properties similar to normal distribution.

Naik V.D. and Tailor K.S. have proposed the concept of 3δ control limits on the basis of moderate distribution. Under this rule, the upper and lower control limits are set at 3δ distance from the central line where δ is the mean deviation of sampling distribution of the statistic being used for constructing the control chart. Thus in the proposed control charts, under the moderateness assumption, three control limits for any statistic T should be determined as follows.

Central line (CL) = Expected value of T = μ

Lower Control Limit (LCL) = Mean of T - $3\delta_T$

$= \mu - 3\delta_T$

Upper Control Limit (UCL) = Mean of T + $3\delta_T$

$= \mu + 3\delta_T$

Where μ is mean of T and δ_T is the mean error of T.

It is found that since δ provides exact average distance from mean and σ provides only an approximate average distance, 3δ limits are more rational as compared to 3σ limits.

Therefore, in this paper it has been assumed that the underlying distribution of the variable of interest follows moderate distribution and sample mean deviation (d) chart are studied and their 3δ control limits are derived. Also the performance analysis of d chart under the assumption of moderateness against that of normality assumption is carried out through OC curve analysis and ARL curve analysis.

II. (3δ) CONTROL LIMITS FOR SAMPLE MEAN DEVIATION (D) CHART WHEN PROCESS MEAN DEVIATION δ' IS UNKNOWN

An important parameter of dispersion, besides σ or σ^2, is mean deviation δ. As δ is more rational and more efficient estimator than σ, it is suggested that sample mean deviation (d) should be used in lieu of R or s for the purpose of constructing a dispersion chart. The control limits of d chart are derived by assuming that the distribution of d follows moderate distribution. As the value of process M.D. δ' is unknown, its estimator $\bar{d}$ is used. i.e. E (d) = δ'= $\bar{d}$.

Let d is the sample mean deviation, which is defined as follows,

$$d=\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{X}| \quad (1)$$

This d has following important properties.

d is the unbiased estimator of population mean deviation.

i.e. E(d) = δ' (2)

$$V\ (d)=\frac{1}{n}\left(\frac{\pi}{2}-1\right)\delta'^2 \quad (3)$$

$$\text{Hence, S.E.(d)}=\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\delta' \quad (4)$$

$$\text{Mean error = M.E. (d)} = \sqrt{\frac{2}{\pi}}\,\text{(S.E)}$$

$$\therefore \text{M.E.(d)}=\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\delta' \quad (5)$$

Hence on the basis of 3δ criteria the control limits of d chart can be represented as follows.

Central line (C.L) = E (d)

= $\bar{d}$ (6)

Lower control limit (L.C.L) = E (d) - $3\delta_d$

$$= \bar{d} - 3\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\delta'$$

$$= \bar{d} - 3\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\bar{d}$$

$$= \left(1-3\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\right)\bar{d}$$

$$= E'_1\bar{d} \quad (7)$$

$$\text{Where } E'_1 = 1-3\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}$$

Upper control limit (U.C.L) = E (d) + $3\delta_d$

$$= \bar{d} + 3\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\delta'$$

$$= \bar{d} + 3\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\bar{d}$$

$$= \left(1+3\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\right)\bar{d}$$

$$= E'_2\bar{d} \quad (8)$$

$$\text{Where } E'_2 = 1+3\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}$$

Values of E'_1 and E'_2 for different values of n are given in the table 2.

Values of E'_1 and E'_2 for different values of n

n	E'_1	E'_2
2	0.0000	2.2791
3	0.0000	2.0444
4	0.0956	1.9045
5	0.1910	1.8090
6	0.2615	1.7385
7	0.3163	1.6837
8	0.3605	1.6395
9	0.3970	1.6030
10	0.4280	1.5720
11	0.4546	1.5454
12	0.4778	1.5222
13	0.4983	1.5017
14	0.5166	1.4834
15	0.5329	1.4671
16	0.5478	1.4522
17	0.5613	1.4387
18	0.5736	1.4264
19	0.5850	1.4150
20	0.5955	1.4045
21	0.6053	1.3947
22	0.6143	1.3857
23	0.6228	1.3772
24	0.6308	1.3692
25	0.6382	1.3618

Table 1

III. COMPARISON OF PERFORMANCE OF D CHART UNDER THE ASSUMPTION OF NORMALITY WITH 3σ- LIMITS AGAINST THAT UNDER THE ASSUMPTION OF MODERATENESS AND 3δ-LIMITS

There are two commonly used methods for measuring and comparing the performance of control charts. One of them is to determine the Operating Characteristic (OC) curve of the charts and the other one is to determine the average run length (ARL).

It is very helpful to use the operating characteristic (OC) curve of a control chart to display its probability of type-II error. This would be an indication of the ability of the control chart to detect process shifts of different magnitudes. The OC Curve shows the probability that an observation will fall within the control limits given the state of the process. This is very much like finding power curves in hypothesis testing. Another measure of performance that is closely related to OC curve values is the run length. The run length is a random variable and is defined as the number of points plotted on the chart until an out-of-control condition is signaled.

IV. COMPARISON OF PERFORMANCE OF D CHART UNDER THE ASSUMPTION OF NORMALITY WITH 3σ- LIMITS AGAINST THAT UNDER THE ASSUMPTION OF MODERATENESS WITH 3δ-LIMITS THROUGH OC CURVE ANALYSIS

The two types of errors for control charts are defined as follows.

Let α = probability of type-I error of control charts.

= probability that the process is considered to be of out control when it is really in control, and

β = probability of type-II error of control charts.

= probability that the process is considered to be in control when it is really out of control,

Clearly, 1- β = probability of not committing type-II error on control charts. Thus lower value of β and higher value of (1- β) means more effectiveness (better performance) of control charts.

Consider the OC curve for a d chart with the mean deviation δ known and constant. Suppose that the in-control value of mean deviation is δ_0 . If the mean deviation shifts from the in control value- say δ_0 to another value $\delta_1 > \delta_0$, then the probability of not detecting a shift to a new value of σ, say $\sigma_1 > \sigma_0$, on the first sample following the shift is,

$$\beta = P\ \{LCL \le d \le UCL / \delta_1 > \delta_0\} \qquad (9)$$

since mean of d is $\bar{d} = \delta_0$ and mean error is $\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\delta_1$ and the upper and lower 3δ control limits are UCL= $E_2{}'\bar{d}$ and LCL $= E_1{}'\bar{d}$, under the assumption of moderateness, equation (9) can be written as follows,

$$\beta_{md} = \emptyset'\left[\frac{UCL-\bar{d}}{\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\delta_1}\right] - \emptyset'\left[\frac{LCL-\bar{d}}{\sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}\delta_1}\right]$$

$$= \emptyset'\left[\frac{E_2{}'\bar{d}-\bar{d}}{C'_1\delta_1}\right] - \emptyset'\left[\frac{E_1{}'\bar{d}-\bar{d}}{C'_1\delta_1}\right],$$

where $C'_1 = \sqrt{\frac{2}{\pi}}\left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}$

$$= \emptyset'\left[\frac{\bar{d}(E_2{}'-1)}{C'_1\delta_1}\right] - \emptyset'\left[\frac{\bar{d}(E_1{}'-1)}{C'_1\delta_1}\right]$$

$$= \emptyset'\left[\frac{\delta_0(E_2{}'-1)}{C'_1\delta_1}\right] - \emptyset'\left[\frac{\delta_0(E_1{}'-1)}{C'_1\delta_1}\right]$$

$$= \emptyset'\left[\frac{(E_2{}'-1)}{C'_1\lambda_3{}'}\right] - \emptyset'\left[\frac{(E_1{}'-1)}{C'_1\lambda_3{}'}\right] \qquad (10)$$

Where $\lambda_3{}' = \frac{\delta_1}{\delta_0}$ or $\delta_1 = \lambda_3{}'\ \delta_0$ and $\emptyset'$ denotes the standard moderate cumulative distribution.

Similarly, equation (10) can be written as follows under normality assumption,

$$\beta_{nd} = \emptyset\left[\frac{(E_2-1)}{C_1\lambda_3}\right] - \emptyset\left[\frac{(E_1-1)}{C_1\lambda_3}\right], \qquad (11)$$

Where $C_1 = \left[\frac{1}{n}\left(\frac{\pi}{2}-1\right)\right]^{\frac{1}{2}}$ and $\lambda_3 = \frac{\delta_1}{\delta_0}$

Here $\lambda_3{}'$(and λ_3) are chosen in the range [0 1].

To construct the OC curve for d chart under moderateness (or normality) assumption, β-value is plotted against $\lambda_3{}'$ (or λ_3) with various sample sizes n. These probabilities may be evaluated directly from equation (10) and (11).

For different sample sizes n and with three-delta limits (or three-sigma limits), for various values of $\lambda_3{}'$ (orλ_3), β-values are calculated and OC curves are plotted as shown in figure-1.

λ_2(or $\lambda_2{}'$)	n = 3		n = 4		n = 5		n = 8	
	β_{md}	β_{nd}	β_{md}	β_{nd}	β_{md}	β_{nd}	β_{md}	β_{nd}
1.0	0.9807	0.9877	0.9834	0.9947	0.9834	0.9976	0.9834	0.9978
1.2	0.9487	0.9657	0.9540	0.9799	0.9540	0.9886	0.9540	0.9892
1.4	0.9051	0.9333	0.9122	0.9544	0.9122	0.9696	0.9122	0.9714
1.6	0.8577	0.8936	0.8664	0.9205	0.8664	0.9419	0.8664	0.9438
1.8	0.8077	0.8505	0.8172	0.8817	0.8172	0.9079	0.8172	0.9108
2.0	0.7590	0.8081	0.7686	0.8398	0.7686	0.8715	0.7686	0.8740

Table 2

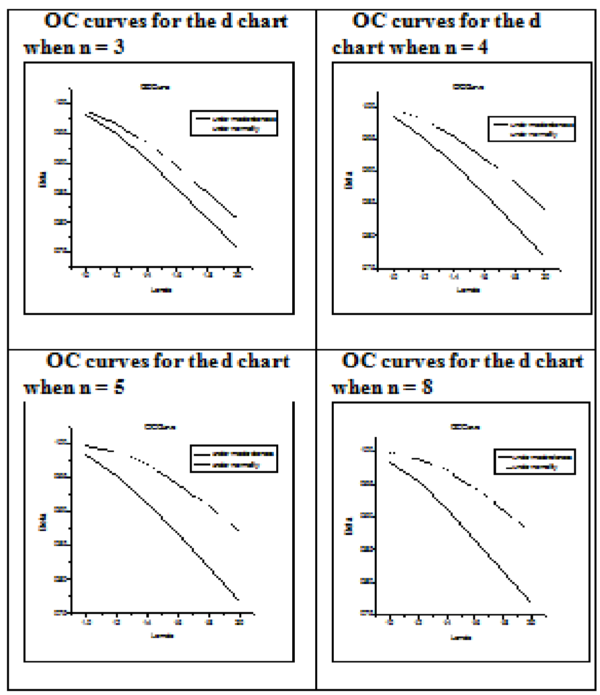


Figure 1

Form the above table 2 and figure 1, it is noticeable that for all values of n when $\lambda_3{}' = \lambda_0{}'(\lambda_3 = \lambda_0)$, i.e even when there is no shift in the value of δ_0(or σ_0), β under moderateness assumption are always smaller than β under normality assumption, which indicates that d-chart under moderateness assumption and 3δ-limits is more effective than that of normality assumption and 3σ-limits.

V. COMPARISON OF PERFORMANCE OF D CHART UNDER THE ASSUMPTION OF NORMALITY WITH 3σ- LIMITS AGAINST THAT OF THE ASSUMPTION OF MODERATENESS WITH 3δ-LIMITS THROUGH ARL CURVE ANALYSIS

For a control chart the average run length (ARL) is the average number of points required to be plotted before a point indicates an out of control condition, that means when ARL is small, the chart is considered to be more effective. If the process observations are uncorrelated, then for any control chart, the ARL can be calculated easily from,

$$ARL=\frac{1}{p}=\frac{1}{1-\beta} \qquad (12)$$

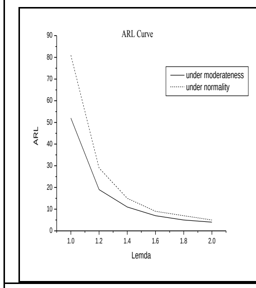
Where p is the probability that any point exceeds the control limits. This equation can be used to evaluate the performance of the control chart.

To construct the ARL curve for the d chart, ARL is plotted against the magnitude of the shift with various sample sizes n. To measure the effectiveness of the control charts under both the assumption, viz moderateness and normality, the probabilities(β) may be evaluated directly from equations (10) and (11) and values of ARL are calculated from equation (12) and ARL curves are plotted. For different sample sizes n and with 3δ limits (or 3σ limits), for various values of $\lambda_2{}'$ (or λ_2), ARLs are calculated and ARL curves are plotted as shown below.

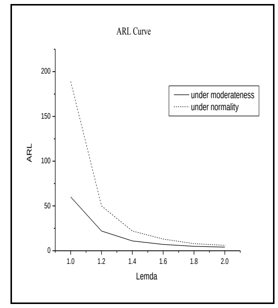
λ_3 (or $\lambda_3{}'$)	n = 3		n = 4		n = 5		n = 8	
	ARL$_{md}$	ARL$_{nd}$	ARL$_{md}$	ARL$_{nd}$	ARL$_{md}$	ARL$_{nd}$	ARL$_{md}$	ARL$_{nd}$
1.0	52	81	60	189	60	417	60	455
1.2	19	29	22	50	22	88	22	93
1.4	11	15	11	22	11	33	11	35
1.6	7	9	7	13	7	17	7	18
1.8	5	7	5	8	5	11	5	11
2.0	4	5	4	6	4	8	4	8

Table 3

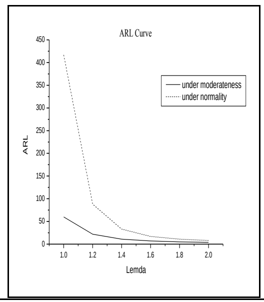
ARL curves for the d chart when n = 3



ARL curves for the d chart when n = 4



ARL curves for the d chart when n = 5



ARL curves for the d chart when n = 8

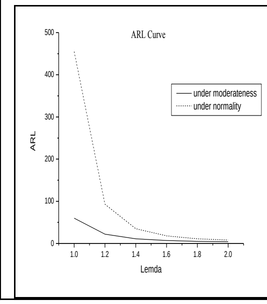


Figure 2

From the figures 2, it is clear that for all values of n, even when there is up to 100% shift in the value of δ_0(or σ_0), ARL under moderateness assumption are always smaller than ARL under normality assumption, which indicates that d chart under moderateness assumption is more effective than that of under normality assumption.

VI. SUMMARY

On the basis of OC curves analysis and ARL curves analysis, it is found that d chart under moderateness assumptions and having 3δ limits rather than 3σ limits are always more effective (perform better) than the charts under normality assumptions and having usual 3σ limits.

So it is recommended that the control charts under moderateness assumption should be used for the best results.

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