Study on the Pricing of Bus in Yanji City
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\textbf{ABSTRACT}

By using the mathematical modeling, this paper analyzes the rationality of fixing price of the buses in Yanji City. And it also gives suggestions of the buses’ prices in Yanji City to promote a harmonious public transport and good social welfare.

\textbf{Keywords}-- Harmonious public transportation, Reasonable pricing, Simple optimization model, Sectional fare

\section{I. INTRODUCTION}

Nowadays, we advocate “the priority of the buses leads to a harmonious society”, so the development of the city’s public transportation has been regarded as a critical point to ease congestion. Ensuring the priority of buses needs realizing the harmony of the buses firstly. Therefore, using and making a rational mode of pricing are of great importance. In our country, most current researches emphasis on analyzing, but few focus on the relevant quantitative analyzing. However, when pricing, it’s a game between the price and the citizens’ affordability. It’s a complex problem to consider from all respects to make a rational price which not only benefits both the passengers and public transport company, and also ensures the company’s normal operation. This essay, based on the problem of the pricing problem of buses in YanJi, analyzes the best price by the simple optimization model.

\section{II. THE RATIONAL ANALYZING OF THE FIXED PRICE OF THE BUSES IN YANJI}

The profit of the public transport company is equal to revenues minus total expenses. Suppose the price of the ticket is $P$, the unit cost is $q$, the actual number of passengers is $x$, total income is $I$, total cost is $C$, then $I = px$, $C = qx$, $X = F(p)$, and it is called demand function which is also the decreasing function of $P$. When $q$ is a constant, both the total income $I$ and total cost $C$ are the functions of the price $P$:

$I(p) = pF(p)$, $C(p) = qF(p)$ . According to the profit = revenues - total expenses, the profit, $U$ can be expressed as $C(p) = qF(p)$ .....(1)

Getting the extremum according to the necessary condition of the function of one variable, make profit $U(p)$ reach the biggest optimal price $P^*$, because

$$\frac{dl}{dp}_{p=p^*} = 0 ,$$

then

$$\frac{dl}{dp}_{p=p^*} = \frac{dC}{dp}_{p=p^*}$$

......(2). Among it, $\frac{dl}{dp}$ is called marginal revenue, $\frac{dC}{dp}$ is called marginal cost. (2) shows the profit reach the biggest number when the marginal revenue is equal with the marginal cost.

If the demand function is a linear function, that is
\[ F(p) = a - bp(a, b > 0) \quad \ldots \ldots \ldots (3) \]

and the unit cost \( q \) is irrelevant to the actual number of passengers \( x \), substitute the total income function, total cost function, and demand function, and (3) into (1), it gets

\[ U(p) = (p - q)(a - bp). \]

Then by using differential methods, it can obtain the optimal price

\[ p^* = \frac{q}{2} + \frac{a}{2b} \quad \ldots \ldots \ldots (4) \]

In the formula (3), \( a \) can be understood as the demand quantity of passengers when the bus is free \((p = 0)\), which called “absolute demand quantity”. represents the range of the decrease of quantity when the price increases one unit. Generally, \( a \) and \( b \) are determined by the statistical data of the price \( P \) and actual number of passengers by using Least-square Method. But because the ticket price cannot always change, it is impossible to get the statistical relationship between the price \( P \) and the actual number of passengers \( X \). Therefore, we could get the diagram through questionnaire, as shown in the following image. (4) shows the optimal price is the sum of the two parts, one is the half of the unit cost \( q \), another is proportional to the “absolute demand quantity”, and inversely proportional to prices’ sensitivity coefficients of the demand quantity of passengers.[1]

Because the unit cost is \( q \) = the salary of one driver/(fixed number of the seats in one bus \( \times \) the number of working turns of one bus \( \times \) the number of the days in one month), through collecting data and factual investigation, we get, fixed number of the seats in one bus = 24, the number of working turns of one bus = 12 turns, the number of the days in one month = 30 days.

\[ q = \frac{3400}{24 \times 12 \times 30} = 0.394 \quad \ldots \ldots \ldots (8) \]

Substitute (7) and (8) into (4), it is

\[ p^* = 0.394 \times 2 + \frac{30}{2 \times 10} = 1.697 \approx 1.7 \]

Therefore, the optimal price is 1.7 yuan, 0.3 less than the price after adjusting. So the price is a little irrational, comparing with 1.7. It is better to set the price around 1.7 yuan.

III. FOUR ANALYSES OF PRICING TOWARD THE GOAL OF HARMONIOUS PUBLIC TRANSPORT

3.1 Plan one: Different Prices for Different Sections

Most city buses’ have long routes, so it’s inappropriate for them to fix only one price for the different sections. To be justice, passengers for various destinations had better to pay different money.[3]

It is

\[ R = Q \times P = \sum p_i q_i \quad (p_i \text{ is the price of the i’s section}, \ q_i \text{ is the total number of the passengers in the i’s section}). \]

The miles passengers for the whole course had to sit for:

\[ M = L = \sum I_i \]

so the price of the average miles is

\[ J = \frac{R}{M} = \frac{\sum p_i q_i}{\sum I_i} \]

In practice, it cannot be infinite, the suitable range of price is from 1 to 3 yuan. And \( p_i \) can also not have infinite numbers of decimal, so it should regularly increase 0.5 yuan for different sections.
3.2 Plan 2: Different Prices for Different Sections under a Growth Pattern

Public transport company is both an enterprise and welfare at the same time. These two properties determine that when pricing, pursuing profit should not be the main focus, instead, the benefits for both the citizens and company must be thoroughly considered. So it leaves a space of saving money which expands along with the time for the public transport company.

The public transport company’s business scale increase as the pace of growth. As the development of the urban and rural integration, the number of passengers become larger. Base on the increase of market demand, the supply of buses is also increase, so it is necessary to leave a space of saving money for the public transport company which will expand as time passes. But this profit space is only used to enlarge the scale and improve the management and service. The growth rate here is a predicted value, it is the growth rate of the size of the budget with this year’s cost base on the current year. Each year, the operating cost of the company keeps a pace of increasing. Because the scale expands every year, the fixed assets and variable costs are more, so the whole operating costs are also increasing at the same time. The here means the increasing rate of this year’s cost compared with last year’s. Then, we get the following pricing model:

\[ R = Q \times P = q_1 p_1 + q_2 p_2 + \ldots + q_n p_n = C(1+a)(1+g) \]

(C is the base year cost, a is the annual growth rate of cost, g is the growth rate of scale base on the cost)

Since the company’s marginal utility from the profit which is larger than the scale expanding needs must smaller than transferring this part of profit to the passengers, especially the marginal utility from vulnerable groups. So, it is already reasonable to suppose each year’s profit is only enough for expanding business scale, transferring the rest profit to the passengers to increase social welfare. Meanwhile, because of principal agency, the company is dynamic to become more efficient in the space with profit of g, and also reduce the cost, gain more productivity, thus increase social welfare.[3]

3.3 Plan 3: Different Prices for Different Sections and Times under a Growth Pattern

In every days’ actual operation, there are passenger peak and lower peak flow. They should be discussed separately according to the time distribution.

To study the pricing model easier, we use these following letters:

- \( p_i \) — the price of the ticket
- \( q_0(t) \) — according to the current price, the number of passengers distribution obeys the time t distribution;
- \( f(p) \) — the influence of the price to the demands of consumers;
- \( R \) — revenue;
- \( C \) — cost;
- \( t \) — time, suppose the buses operate for \( t_1 \sim t_2 \);
- \( T_1 \) — peak period of time \( t_3 < T < t_4, \quad t_5 < T < t_6 \);
- \( T_2 \) — normal period of time;
- \( m \) — the price rate of peak period of time and normal period of time \( m \geq 1 \)
- \( n \) — the price rate of lower peak period of time and normal period of time \( n \leq 1 \)

With current way of charging, passengers in peak period of time is

\[ F(T_1) = \int q_0(t) dt \]

passengers in lower peak period of time is

\[ F(T_2) = \int q_0(t) dt \]

Then the pricing model is:

\[ R = Q \times P = \sum q_i F(T_i) \times f(p_i) + \sum q_i F(T_i) \times f(p_i) + \sum p_i \cdot F(T_i) \times f(p_i) \]

\[ = C(1+a)(1+g) \]

Although this one makes different charging plans according to different periods of time which help to divide the passengers in peak period to lower peak period. Base on the controlling of cost, it not only becomes more comfortable for passengers, but also releases the route’s pressure in peak hours. But it does not consider the passengers’ different affordability of the price.

People from different classes, with various living
standards, has different affordability of the price. So it’s of great importance to fix price according to different people.

3.4 Plan four: Different Prices for Different Sections, Times, and Objects under a Growth Pattern

\[ + \sum \sum x_j p_i y_j [q_i - F(T_1) \times f(m p_i) - F(T_2) \times f(n p_i)] \]
\[ = \sum \sum X_j m p_i y_j F(T_1) \times f(m p_i) + \sum \sum X_j n p_i y_j F(T_2) \times f(n p_i) \]
\[ = C(1 + a)(1 + g) \]

\( x_j \) is the discount for people form the j’s category, \( y_j \) is the proportion of the people from the j’s category under \( p_i \).

According to the four plans, we can see that with the changes of influence conditions, results are the same under a growth pattern.[3]

IV. SUGGESTIONS TO PRICING BASED ON CITIZENS OF YANJI

As the capital of Yanbian, Yanji should combine the harmonious public transport and social welfare. [2]The purpose of harmonious public transport is promoting the balance and unification between people’s traffic consumption and public transport company’s profit. Located in the border area of minority, the majority of Yanbian’s labour force flow to foreign countries, which leaves more elderly people and teenagers, less middle-aged men in the city. To realize social welfare in the city, the government has to investigate more which also increase the price for normal consumers. For young men and middle-aged men who are working, they are indeed more stressful. To be more harmonious between the profit of company and the cost of citizens, it’s better to use the charging method according to sections. The major crowded areas in Yanji are the areas along the Park Road beside Yanbian University, shopping center around the department store which include Henan market and other places. In crowded areas, there are more choices when taking bus, so we can fix the lowest price in these areas, and increase the price regularly when the bus running far from downtown areas.

REFERENCES