



The Analysis of Sparse Representations for the Sequence of Images of Videos

Monika Singh¹, Rashmi²

¹M.Tech (Signal Processing) Department of Electronics AI&R, GGSIPU, INDIA

²M.Tech (ECE) USICT, GGSIPU, INDIA

ABSTRACT

Sparse representation has become very popular in fields of signal processing, image processing computer vision and pattern recognition. Sparse representation also has good reputation in both theoretical and practical applications. Images can be sparsely coded by structural primitives and recently the sparse coding or sparse representation has been widely used to resolve the problems in image resolution applications by the l_0 -norm and l_1 -norm techniques. Sparse representation has been used in pattern classification. Sparse coded a set of bases and classified the signal of a signal based on its coding vector. Sparse representation can be used for the face recognition, the face image is first sparsely coded and then classification is performed by checking the least coding error and also for the pattern recognition. In this paper, using the sparse representations for the analysis of the sequence of images. As per this approach, the sparse representations are used to detect the error in each frame of the video which might have anomalous events. This method gives the error detection rates and elapsed time for the video by using the sparse representation algorithms.

Keywords— Sparse Representations, Greedy Strategy Approximation, Orthogonal Matching Pursuit, Error Detection

I. INTRODUCTION

Sparse representations of signals have received a great deal of attention in recent years. The problem solved by the sparse representation is to search for the most compact representation of a signal in terms of linear combination of atoms in an over complete dictionary. The developments in multi-scale and multi-orientation representation of signals, such as wavelet, ridgelet, curvelet and contourlet transforms are an important incentive for the research on the sparse representation.

Sparse representation has been used in pattern classification. Sparse coded a set of bases and classified the signal of a signal based on its coding vector. Sparse representation can be used for the face recognition, the face image is first sparsely coded and then classification is performed by checking the least coding error and also for the pattern recognition.

Sparse representation by l_0 -norm graph for classification by combined sparse-coding with linear spatial-pyramid matching for image classification. Recently, the sparse representation can be used for computer vision and pattern recognition applications.

Sparse representations are more separable in high-dimensional spaces and used for classification and also regarded for recognition work. Sparse representation not only used for the image recognition and classification point of view but also a practical pose. Sparsity has recently become popular in computer vision for image classification tasks and other tasks because of its simple algorithms based approaches. These approaches are much popular because they need only unlabeled data which are used to produce than labelled data.

Sparsity is very much helpful for the classification of images by few experiments in a supervised or semi-supervised and not in an unsupervised setting.

Sparse representation is a penurious principle that approximated by a sparse linear combinations of basis vectors is very redundant.

1. It is very robust in redundancy because it selects only few basis vectors among the all basis vectors.
2. It is robust for noise, it is non-orthogonal and easy to interpret by its sparseness nature.

There are two techniques for Sparse representation:

1. A basis matrix- learning of sparse coefficients of a new sample is called sparse coding.

2. By training data- learning the basis vector is called dictionary learning.

In dictionary learning by sparse representation is used to represent the input data in the form of a linear combination of basis elements. These basis elements are known as atoms and they combined to form a dictionary. The atoms in a dictionary are not orthogonal. So, the improved representation of the atoms in a dictionary of a signal in sparsity and its representations.

The sparse representations theory has shown that sparse signals can be exactly reconstructed from a small number of elementary signals.

The section II includes the discussion about the sparse representations, section III has been discussed about the greedy algorithms section IV conclude the method of the work.

II. SPARSE REPRESENTATIONS

The sparse representation of natural signals can be achieved by exploiting its sparsity or compressibility. Now, in the mathematics linear representation methods are well followed and recently got popularity. These representations method is the very well developed technique in the field of representative methodology and it gives the very powerful solution to the wide applications i.e., in signal processing, image processing, computer vision and machine learning approaches. These approaches can have the image denoising, inpainting, image deblurring, image restoration, super-resolution, visual tracking, image classification and image segmentation.

Sparse representation can show the wide strength to resolve these problems and resolves easily as compared to other approaches. Sparse representations origin directly from the compressed sensing and this compressed sensing is very popular these days for compressing the data samples or atoms in any dictionary of signals.

Compressed sensing concepts tell that if a signal is compressed or sparse, the original signal can be reconstructed by exploiting a few measured values and these values are very less in other theories. The compressed sensing theory suggests the original signal can be reconstructed by the small portion of fourier transform coefficients. Many algorithms have been proposed for the problems of many fields. Moreover, compressed sensing theory has the three concepts are sparse representations, encoding measuring and reconstructing algorithms

Let $x_1, x_2, \dots, x_n \in \mathbb{R}^d$ be all the n known samples and matrix $X \in \mathbb{R}^{d \times n}$ ($d < n$), which is constructed by identified samples, is the measurement matrix or the basis dictionary

and should also be an over-completed dictionary. Each column of X is one sample and the probe sample is $y \in \mathbb{R}^d$, which is a column vector. Thus, if all the known samples are

used to approximately represent the probe sample, it should be expressed as:

$$y = x_1\alpha_1 + x_2\alpha_2 + \dots + x_n\alpha_n \quad (1)$$

where α_i ($i=1; 2; \dots; n$) is the coefficient of x_i and Eq.1 can be rewritten into the following equation for convenient description:

$$y = X\alpha \quad (2)$$

where matrix $X=[x_1, x_2, \dots, x_n]$ and $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_n]^T$. However, this problem is an underdetermined linear system of equations and the main problem is how to solve it. From the viewpoint of linear algebra, if there is not any prior knowledge or any constraint imposed on the representation solution, equation 2 is an ill-posed problem and will never have a unique solution. That is, it is impossible to utilize equation 2 to uniquely represent the probe sample y using the measurement matrix X . To alleviate this difficulty, it is feasible to impose an appropriate regularizer constraint or regularizer function on representation solution. The sparse representation method demands that the obtained representation solution should be sparse. Hereafter, the meaning of 'sparse' or 'sparsity' refers to the condition that when the linear combination of measurement matrix is exploited to represent the probe sample, many of the coefficients should be zero or very close to zero and few of the entries in the representation solution are differentially large. The sparsest representation solution can be acquired by solving the linear representation system 2 with the l_0 -norm minimization constraint [1]. Thus problem 2 can be converted to the following optimization problem:

$$\hat{\alpha} = \arg \min \|\alpha\|_0 \quad s.t. y = X\alpha \quad (3)$$

Where $\|\cdot\|_0$ refers to the number of nonzero elements in the vector and is also viewed as the measure of sparsity. Moreover, if just k ($k < n$) atoms from the measurement matrix X are utilized to represent the probe sample, problem (3) will be equivalent to the following optimization problem:

$$y = X\alpha \quad s.t. \|\alpha\|_0 \leq k \quad (4)$$

Equation 4 is called the k -sparse approximation problem. Because real data always contains noise, representation noise is unavoidable in most cases. Thus the original model can be revised to a modified model with respect to small possible noise by denoting

$$y = X\alpha + s \quad (5)$$

where $s \in \mathbb{R}^d$ refers to representation noise. With the presence of noise, the sparse solutions of equation 3 and equation 4 can be approximately obtained by resolving the following optimization problems:

$$\hat{\alpha} = \arg \min \|\alpha\|_0 \quad s.t. \|y - X\alpha\|_2^2 \leq \varepsilon \quad (6)$$

In this paper, sparse representation is used by the greedy pursuit algorithms via orthogonal matching pursuit (OMP).

III. GREEDY STRATEGY APPROXIMATION

The idea of the greedy strategy is to determine the position based on the relationship between the atom and probe sample, and then to use the least square to determine the amplitude value. Greedy algorithms can obtain the local optimized solution in each step in order to address the problem. However, the greedy algorithm can mostly produce the global optimal solution or an approximate overall solution. Addressing sparse representation with l_0 -norm regularization, i.e. problem 3, is an NP hard problem. The greedy strategy gives an idea to obtain an approximate sparse representation solution. The greedy strategy cannot directly solve the optimization problem and it only seeks an approximate solution for problem.3.

A. MATCHING PURSUIT ALGORITHM

The matching pursuit (MP) algorithm [4] is the earliest and representative method of the greedy strategy by using the greedy algorithms to approximate the different problem 3 or 4. The Matching Pursuit algorithm is to iteratively choose the best suited atom from the dictionary based on a certain similarity measurement to approximately determine the sparse solution. As taking an example of the sparse decomposition with a vector sample y over-complete dictionary D , the detailed algorithm describes as follows:

Suppose that the initialized representation residual is $R_0 = y$, $D = [d_1, d_2, \dots, d_N] \in \mathbb{R}^{d \times N}$ and each sample in dictionary D is an l_2 -norm unity vector, i.e. $\|d_i\| = 1$. To approximate y , Matching Pursuit first chooses the best matching atom from D and the selected atom should satisfy the following condition:

$$|\langle R_0, d_{l_0} \rangle| = \sup |\langle R_0, d_i \rangle| \tag{7}$$

where l_0 is a label index from dictionary D . Thus y can be decomposed into the following equation:

$$y = \langle y, d_{l_0} \rangle d_{l_0} + R_1 \tag{8}$$

So

$$y = \langle y, d_{l_0} \rangle d_{l_0} + R_1$$

where $\langle y, d_{l_0} \rangle d_{l_0}$ represents the orthogonal projection of y onto d_{l_0} , and R_1 is the representation residual by using d_{l_0} to represent y . considering the fact that d_{l_0} is orthogonal to R_1 , Eq. IV.2 can be rewritten as

$$\|y\|^2 = |\langle y, d_{l_0} \rangle|^2 + \|R_1\|^2 \tag{9}$$

To obtain the minimum representation residual, the MP algorithm iteratively gures out the best matching atom from the over-completed dictionary, and then utilizes the representation residual as the next approximation target until the termination condition of iteration is satisfied. For the t -th iteration, the best matching atom is d_{l_t} and the approximation result is found from the following equation:

$$R_t = \langle R_t, d_{l_t} \rangle d_{l_t} + R_{t+1} \tag{10}$$

where the d_{l_t} satisfies the equation:

$$|\langle R_t, d_{l_t} \rangle| = \sup |\langle R_t, d_i \rangle| \tag{11}$$

Clearly, d_{l_t} is orthogonal to R_{t+1} , and then

$$\|R_t\|^2 = |\langle R_t, d_{l_t} \rangle|^2 + \|R_{t+1}\|^2 \tag{12}$$

For the n -th iteration, the representation residual kR_n , where k is a very small constant and the probe sample y can be formulated as:

$$y = \sum_{j=1}^{n-1} \langle R_j, d_{l_j} \rangle d_{l_j} + R_n \tag{13}$$

If the representation residual is small enough, the probe sample y can approximately satisfy the following equation:

$$y \approx \sum_{j=1}^{n-1} \langle R_j, d_{l_j} \rangle d_{l_j} \tag{14}$$

Where $n \ll N$. Thus, the probe sample can be represented by a small number of elements from a large dictionary. In the context of the specific representation error, the termination condition of sparse representation is that the representation residual is smaller than the presupposed value. More detailed analysis on matching pursuit algorithms can be found in the literature [63].

B. ORTHOGONAL MATCHING PURSUIT ALGORITHM

The orthogonal matching pursuit (OMP) algorithm [2], [3] is an improvement of the MP algorithm. The OMP employs the process of orthogonalization to guarantee the orthogonal direction of projection in each iteration. It has been verified that the OMP algorithm can be converged in limited iterations [2]. The main steps of OMP algorithm have been summarized in Algorithm 1.

Results

OMP, "normal mode"		
Iteration, Residual		
25,	3.60e+00	
50,	1.14e+00	
72,	7.67e-15	
Residual reached desired size (7.67e-15 < 1.00e-12)		
Elapsed time is 0.117625 seconds.		
Error is 7.98e-16 (oracle error is 7.98e-16)		
OMP, "slow/testing mode"		
Iteration, Residual,		Error
25,	3.60e+00,	4.75e-01
50,	1.14e+00,	1.54e-01
72,	7.74e-15,	7.98e-16
Residual reached desired size (7.74e-15 < 1.00e-12)		
Elapsed time is 0.132387 seconds.		
Error is 7.98e-16 (oracle error is 7.98e-16)		
This algorithm can detect the error in each frame of the video.		

IV. CONCLUSION

This approach of sparse representations includes the detection of errors in images of videos by using the existing algorithms of sparsity which is based on greedy pursuit algorithms. This paper has discussed the techniques in which the detection problem has been formulated in related work. The discussion is done on the sparse representation for detection in image processing has the error detection in images of video also for the detection of the normal and anomalous detection. I hope to provide the better results of technique in the field of video images error detection and elapsed time detection.

REFERENCES

- [1] D. L. Donoho and M. Elad, "Optimally sparse representation in general (nonorthogonal) dictionaries via ℓ_1 minimization," *Proc. Nat. Acad. Sci. USA*, vol. 100, no. 5, pp. 2197_2202, 2003.
- [2] J. A. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Trans. Inf. Theory*, vol. 53, no. 12, pp. 4655_4666, Dec. 2007.
- [3] Y. C. Pati, R. Rezaifar, and P. S. Krishnaprasad, "Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition," in *Proc. 27th Asilomar Conf. Signals, Syst., Comput.*, Nov. 1993, pp. 40_44.
- [4] A Survey of Sparse Representation: Algorithms and Applications ZHENG ZHANG^{1,2}, (Student Member, IEEE), YONG XU^{1,2}, (Senior Member, IEEE), JIAN YANG³, (Member, IEEE), XUELONG LI⁴, (Fellow, IEEE), AND DAVID ZHANG⁵, (Fellow, IEEE)