A Predictive Mathematical Model for Water Absorption of Sawdust Ash - Sand Concrete

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ABSTRACT

Saw Dust Ash (SDA) is an industrial waste that has been used by many researchers in concrete to achieve economic and environmental sustainability. In this study, 5% of sand was replaced with SDA to produce concrete with different mix ratios. Scheffe's simplex theory was used for five mix ratios in a {5,2} experimental design which resulted in additional ten mix ratios. Additional fifteen mix ratios were generated from the initial fifteen, for verification and testing. Concrete cubes of 150mmX150mmX150mm were formed using the thirty concrete mix ratios generated, and soaked in water for 24hours. The water absorptions of cubes from each mix ratio were determined with the standard procedure. The results of the first fifteen water absorption values were used for the calibration of the model constant coefficients, while those from the second fifteen were used for the model verification using Scheffe's simplex lattice design. A mathematical regression model was formulated from the results, with which the water absorptions were predicted. The model was then subjected to a two-tailed t-test with 5% significance, which ascertained the model to be adequate and fit with an R2 value of 0.8244. The study also revealed that SDA can replace 5% of sand and promote environmental sustainability without significantly changing the water absorption.

Keywords**--** Saw Dust Ash, Scheffe's Simplex Lattice, Sustainability, Water Absorption of Concrete

I. INTRODUCTION

Water absorption of concrete is the moisture content in the Saturated Surface-Dry (SSD) condition [1]. It is usually considered after 24hours for water with normal temperature, or 5hours for hot water. Water absorption procedure has been described in [2]. However, [2] specifies that the age of the concrete specimen would be 28days days old. Where the age is less than 28days old, the water absorption value will be higher, or lower where the age is more than 28days old. On this note several studies [3]–[7] have been carried out on water absorption of concrete of different kinds. [4] carried out a study by incorporating Fine Bottom Ass (FBA) with Normal Fine (NF) aggregates in cement concrete. They found the water absorption to be between 11.2% and 18.4%. In another study, [5] also investigated the water absorption and found the values to be between 4.1% and 4.7% for conventional concrete, but increased to between 4.5% and 12.1 when between 25% and 75% of coarse aggregate was replaced with Palm Kernel Shell (PKS) for 7, 28, and 56days age of concrete. However, [6] postulate that the water absorption of mortar can be greatly affected by the relative humidity, as samples tested at a 50% relative humidity gave water absorption results that are six time greater than those with 80% relative humidity. Another study carried out by [7] shows that the water absorption of concrete starts decreasing with water-cement ratios greater than 0.55, due to the excess water available for hydration, whereby the hydration products of cement had blocked some pores in the concrete. However, water absorption cannot be used to ascertain the quality of concrete with regards to strength and durability [7], [8]. Nevertheless, water absorption can be used to characterize concrete as it gives an indication of pore structure and surface condition of the concrete. The aim of this work is to predict the water absorption of Saw Dust Ash SDA concrete using the Scheffe's simplex model.

A. Saw Dust Ash in Concrete

Saw Dust Ash is the pulverised form of an industrial waste of saw mills. It has been used in concrete construction for over 30 years [9]. Studies by various authors have shown that the use of SDA as sand replacement could have both economic and environmental benefits. This is so given the environmental issues associated with unsustainable sand mining [10], [11]. Chowdhury et al. [12] credited cement production to be a major source of environmental degradation as every 600kg of cement produced emits about 400kg of CO_2. They therefore replaced 10% of cement with SDA without negatively affecting the chloride permeability and thaw resistance of the concrete, but rather increased the water absorption, and decreased the drying shrinkage. Similarly, [13] replaced 10% of fine aggregate with SDA, which resulted in acceptable tensile, flexural, and compressive strengths as well as reduce the amount of wastes in the environment. In [13] it was found that SDA has a specific gravity of 2.5, water absorption of 0.56%, fineness modulus of 1.78, and bulk dry density of 1300kg/m^3 as against sand with specific gravity of 2.65, water absorption of 0.45%, fineness modulus of 2.21, and bulk dry density of 1512 kg/m^3. After the 10% of sand with SDA, these properties became 2.67, 0.5%, 2.2, and 1436kg/m^3 for specific gravity, water absorption, fineness modulus, and bulk dry density respectively. This is a significant indication that the mixture of sand and SDA did not significantly change the physical properties of sand, making the mixture adequate for a fine aggregate.

The chemical compositions of SDA as found by [13], [14] all indicate that SDA has a high percentage of SiO_2 and small percentages of Al_2O_3 and Fe_2O_3, which are similar to those of sand with high percentage of about 95% SiO_2. Hence SDA can be used with sand as fine aggregate.

B. Scheffe's Simplex Theory

Several authors [15]–[28] have carried out concrete mixture researches with development of mathematical models, most of which were based on Scheffe's Simplex theory.

Scheffe's model is based on the simplex lattice and simplex theory or approach [29]. The simplex approach considers a number of components, q, and a degree of polynomial, m. The sum of all the i^{th} components is not greater than 1. Hence,

$$\sum_{i=1}^{q} x_i = 1 \tag{1}$$

$$x_1 + x_2 + \cdots + x_q = 1 \tag{2}$$

with $0 \le x \le 1$, the factor space becomes S_{q-1}. According to [29] the $\{q,m\}$ simplex lattice design is a symmetrical arrangement of points within the experimental region in a suitable polynomial equation representing the response surface in the simplex region.

The number of points $C_m^{(q+m-1)}$ has (m+1) equally spaced values of $x_i = 0,\ {}^1/_m,\ {}^2/_m,\ \ldots\ {}^m/_m$. For a 3-component mixture with degree of polynomial 2, the corresponding number of points will be $C_2^{(3+2-1)}$ which gives 6 (eq. 3 or eq. 4 below) with number of spaced values, 2+1 = 3, that is x_i = 0, 1/2, and 1 and design points of (1,0,0), (0,1,0), (0,0,1), (1/2,1/2,0), (1/2,01/2), and (0,1/2,1/2). Similarly, for a $\{5,2\}$ simplex, there will be 15 points with x_i = 0, 1/2, and 1 as spaced values.

The 15 design points are (1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (0,0,0,0,1), (1/2,1/2,0,0,0), (1/2,0,1/2,0,0), (1/2,0,0,1/2,0), (1/2,0,0,0,1/2), (0,1/2,1/2,0,0), (0,0,1/2,1/2,0), (0,0,0,1/2,1/2), (0,1/2,0,1/2,0), (0,0,1/2,0,1/2), (0,1/2,0,0,1/2).

$$N = C_n^{(q+n-1)} \tag{3}$$

$$N = \frac{(q+n-1)!}{(q-1)!(n)!} \tag{4}$$

For a polynomial of degree m with q component variables where eq. (2) holds, the general form is:

$$Y = b_0 + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_j x_k + \cdots + \sum b_{i1,i2\ldots in} x_{i1} x_{i2} x_{in} \tag{5}$$

Where $1 \le i \le q$, $1 \le i \le j \le q$, $1 \le i \le j \le k \le q$, and b_0 is the constant coefficient.

x is the pseudo component for constituents i, j, and k.

When $\{q,m\} = \{5,2\}$, eq. (5) becomes:

$$Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{25}x_2x_5 + b_{34}x_3x_4 + b_{35}x_3x_5 + b_{45}x_4x_5 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2 + b_{55}x_5^2 \tag{6}$$

and eq. (2) becomes

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1 \tag{7}$$

Multiplying eq. (7) by b_0 gives

$$b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 = b_0 \tag{8}$$

Multiplying eq. (7) successively by x_1, x_2, x_3, x_4, and x_5 and making x_1, x_2, x_3, x_4, and x_5 the subjects of the respective formulas:

$$\left.\begin{aligned}
x_1^2 &= x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5\\
x_2^2 &= x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5\\
x_3^2 &= x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5\\
x_4^2 &= x_4 - x_1x_4 - x_2x_4 - x_3x_4 - x_4x_5\\
x_5^2 &= x_5 - x_1x_5 - x_2x_5 - x_3x_5 - x_4x_5
\end{aligned}\right\} \tag{9}$$

Substituting eq. (8) and eq. (9) into eq. (6) we have:

$$\begin{aligned}
Y = {} & b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 + b_1x_1 + b_2x_2\\
& + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{13}x_1x_3\\
& + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + b_{24}x_2x_4\\
& + b_{25}x_2x_5 + b_{34}x_3x_4 + b_{35}x_3x_5 + b_{45}x_4x_5\\
& + b_{11}(x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5) + b_{22}(x_2\\
& - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5) + b_{33}(x_3 - x_1x_3\\
& - x_2x_3 - x_3x_4 - x_3x_5) + b_{44}(x_4 - x_1x_4 - x_2x_4\\
& - x_3x_4 - x_4x_5) + b_{55}(x_5 - x_1x_5 - x_2x_5 - x_3x_5\\
& - x_4x_5)
\end{aligned}$$

$$Y = (b_0 + b_1 + b_{11})x_1 + (b_0 + b_2 + b_{22})x_2 + (b_0 + b_3 + b_{33})x_3 + (b_0 + b_4 + b_{44})x_4 + (b_0 + b_5 + b_{55})x_5 + (b_{12} - b_{11} - b_{22})x_1x_2 + (b_{13} - b_{11} - b_{33})x_1x_3 + (b_{14} - b_{11} - b_{44})x_1x_4 + (b_{15} - b_{11} - b_{55})x_1x_5 + (b_{23} - b_{22} - b_{33})x_2x_3 + (b_{24} - b_{22} - b_{44})x_2x_4 + (b_{25} - b_{22} - b_{55})x_2x_5 + (b_{34} - b_{33} - b_{44})x_3x_4 + (b_{35} - b_{33} - b_{55})x_3x_5 + (b_{45} - b_{44} - b_{55})x_4x_5 \tag{10}$$

Let

$$\left.\begin{aligned}
\beta_1 &= b_0 + b_1 + b_{11}\\
\beta_2 &= b_0 + b_2 + b_{22}\\
\beta_3 &= b_0 + b_3 + b_{33}\\
\beta_4 &= b_0 + b_4 + b_{44}\\
\beta_5 &= b_0 + b_5 + b_{55}\\
\beta_{12} &= b_{12} - b_{11} - b_{22}\\
\beta_{13} &= b_{13} - b_{11} - b_{33}\\
\beta_{14} &= b_{14} - b_{11} - b_{44}\\
\beta_{15} &= b_{15} - b_{11} - b_{55}\\
\beta_{23} &= b_{23} - b_{22} - b_{33}\\
\beta_{24} &= b_{24} - b_{22} - b_{44}\\
\beta_{25} &= b_{25} - b_{22} - b_{55}\\
\beta_{34} &= b_{34} - b_{33} - b_{44}\\
\beta_{35} &= b_{35} - b_{33} - b_{55}\\
\beta_{45} &= b_{45} - b_{44} - b_{55}
\end{aligned}\right\} \tag{11}$$

Substituting eq. (11) into eq. (10) gives

$$Y = \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_5x_5 + \beta_{12}x_1x_2 + \beta_{13}x_1x_3 + \beta_{14}x_1x_4 + \beta_{15}x_1x_5 + \beta_{23}x_2x_3 + \beta_{24}x_2x_4 + \beta_{25}x_2x_5 + \beta_{34}x_3x_4 + \beta_{35}x_3x_5 + \beta_{45}x_4x_5 \tag{12}$$

This can be rewritten as:

$$Y = \sum_{i=1}^{5} \beta_i x_i + \sum_{1 \le i \le j \le 5} \beta_{ij} x_i x_j \tag{13}$$

Where the response, Y is a dependent variable (Water Absorption of concrete). Eq. (12) is the general equation for a {5,2} polynomial, and it has 15 terms, which conforms to Scheffe's theory in eq. (3).

Let Y_i denote response to pure components, and Y_{ij} denote response to mixture components in i and j. If x_i =1 and x_j = 0, since $j \neq i$, then

$$Y_i = \beta_i \tag{14}$$

Which means

$$\sum_{i=1}^{5} \beta_i x_i = \sum_{i=1}^{5} Y_i x_i \tag{15}$$

Hence, from eq. (14)

$$\left.\begin{aligned} Y_1 &= \beta_1 \\ Y_2 &= \beta_2 \\ Y_3 &= \beta_3 \\ Y_4 &= \beta_4 \\ Y_5 &= \beta_5 \end{aligned}\right\} \tag{16}$$

According to [29],

$$\beta_{ij} = 4Y_{ij} - 2\beta_i - 2\beta_j \tag{17}$$

Substituting eq. (14)

$$\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \tag{18}$$

II. MATERIALS AND METHODS

Water, cement, sand, SDA, and granite were the materials used to produce the concrete. This means there are five components in the concrete mix. SDA was used to partially replace 5% of the fine aggregate (sand).

The first five concrete mix ratios derived from different mix design methods [26], [27] are given as:

BRE 12 = [0.54 1 1.9475 0.1025 2.95];
BRE 22 = [0.58 1 2.1185 0.1115 3.21];
USBR 22 = [0.58 1 2.2515 0.1185 3.29];
BIS 12 = [0.43 1 1.2065 0.0635 2.88];
ACI 12 = [0.55 1 1.8335 0.0965 3.09]

These can be put in matrix form as follows:

$$S = \begin{bmatrix} 0.54 & 0.58 & 0.58 & 0.43 & 0.55 \\ 1 & 1 & 1 & 1 & 1 \\ 1.9475 & 2.1185 & 2.2515 & 1.2065 & 1.8335 \\ 0.1025 & 0.1115 & 0.1185 & 0.0635 & 0.0965 \\ 2.95 & 3.21 & 3.29 & 2.88 & 3.09 \end{bmatrix} \tag{19}$$

Their corresponding pseudo components are given as:

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

with centre points

X_{12} = [0.5 0.5 0 0 0]; X_{13} = [0.5 0 0.5 0 0];
X_{14} = [0.5 0 0 0.5 0]; X_{15} = [0.5 0 0 0 0.5];
X_{23} = [0 0.5 0.5 0 0]; X_{24} = [0 0.5 0 0.5 0];
X_{25} = [0 0.5 0 0 0.5]; X_{34} = [0 0 0.5 0.5 0];
X_{35} = [0 0 0.5 0 0.5]; X_{45} = [0 0 0 0.5 0.5]

According to [29],

$$S_{ij} = XS_i \tag{21}$$

Substituting,

$$\begin{bmatrix} S_{12} \\ S_{13} \\ S_{14} \\ S_{15} \\ S_{23} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0.5 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0.54 \\ 0.58 \\ 0.58 \\ 0.43 \\ 0.55 \end{bmatrix} \tag{22}$$

This process is repeated for S_{24}, S_{25}, S_{34}, S_{35}, and S_{45}. Similarly, this process is repeated for an additional 15 (control) points that will be used for the verification of the formulated model. The regular pentagons for the actual components with their corresponding pseudo components are given in Figures 1 and 2 respectively. Tables 1 and 2 mix ratio data generated for the main and verification purposes respectively.

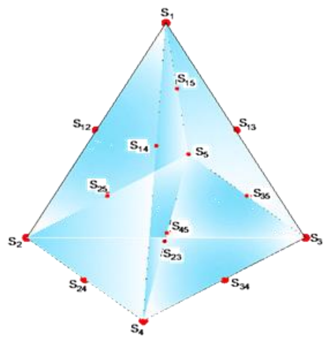

Figure 1: Simplex Plot for Actual Components

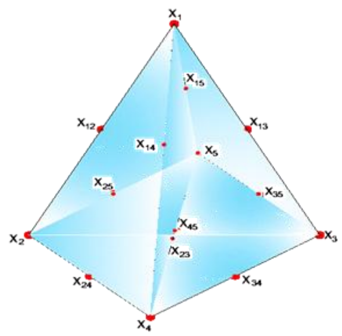

Figure 2: Simplex Plot for Pseudo Components

Table 1 Model Mix Ratios

Sample Points	Actual Components					Response Y_{exp}	Pseudo Components				
	w-c ratio	Cement	Sand	SDA	Granite		w-c ratio	Cement	Sand	SDA	Granite
	S_1	S_2	S_3	S_4	S_5		X_1	X_2	X_3	X_4	X_5
BRE12	0.54	1	1.9475	0.1025	2.95	Y_1	1	0	0	0	0
BRE22	0.58	1	2.1185	0.1115	3.21	Y_2	0	1	0	0	0

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Sample Points	Actual Components					Response Y_{exp}	Pseudo Components				
	w-c ratio S_1	Cement S_2	Sand S_3	SDA S_4	Granite S_5		w-c ratio X_1	Cement X_2	Sand X_3	SDA X_4	Granite X_5
USBR22	0.58	1	2.2515	0.1185	3.29	Y_3	0	0	1	0	0
BIS12	0.43	1	1.2065	0.0635	2.88	Y_4	0	0	0	1	0
ACI12	0.55	1	1.8335	0.0965	3.09	Y_5	0	0	0	0	1
N1	0.56	1	2.033	0.107	3.08	Y_{12}	0.5	0.5	0	0	0
N2	0.56	1	2.0995	0.1105	3.12	Y_{13}	0.5	0	0.5	0	0
N3	0.485	1	1.577	0.083	2.915	Y_{14}	0.5	0	0	0.5	0
N4	0.545	1	1.8905	0.0995	3.02	Y_{15}	0.5	0	0	0	0.5
N5	0.58	1	2.185	0.115	3.25	Y_{23}	0	0.5	0.5	0	0
N6	0.505	1	1.6625	0.0875	3.045	Y_{24}	0	0.5	0	0.5	0
N7	0.565	1	1.976	0.104	3.15	Y_{25}	0	0.5	0	0	0.5
N8	0.505	1	1.729	0.091	3.085	Y_{34}	0	0	0.5	0.5	0
N9	0.565	1	2.0425	0.1075	3.19	Y_{35}	0	0	0.5	0	0.5
N10	0.49	1	1.52	0.08	2.985	Y_{45}	0	0	0	0.5	0.5

Table 2 Control Points

Sample Points	Actual Components					Response Y_{exp}	Pseudo Components				
	w-c ratio S_1	Cement S_2	Sand S_3	SDA S_4	Granite S_5		w-c ratio X_1	Cement X_2	Sand X_3	SDA X_4	Granite X_5
C1	0.558	1	2.0463	0.1077	3.114	YC_1	0.4	0	0.4	0	0.2
C2	0.52	1	1.7537	0.0923	3.078	YC_2	0	0.6	0	0.4	0
C3	0.548	1	2.0083	0.1057	3.018	YC_3	0.8	0	0.2	0	0
C4	0.49	1	1.5713	0.0827	3.012	YC_4	0	0.4	0	0.6	0
C5	0.544	1	1.9019	0.1001	3.006	YC_5	0.6	0	0	0	0.4
C6	0.55	1	2.0425	0.1075	3.208	YC_6	0	0	0.8	0.2	0
C7	0.55	1	1.9589	0.1031	3.03	YC_7	0.6	0.2	0	0	0.2
C8	0.514	1	1.6967	0.0893	3.054	YC_8	0	0.4	0	0.4	0.2
C9	0.548	1	1.8563	0.0977	3.062	YC_9	0.2	0	0	0	0.8
C10	0.46	1	1.4155	0.0745	2.962	YC_{10}	0	0	0.2	0.8	0
C11	0.566	1	2.1071	0.1109	3.182	YC_{11}	0.2	0	0.6	0	0.2
C12	0.544	1	1.9323	0.1017	3.152	YC_{12}	0	0.2	0.4	0.2	0.2
C13	0.58	1	2.1451	0.1129	3.226	YC_{13}	0	0.8	0.2	0	0
C14	0.532	1	1.7651	0.0929	3.072	YC_{14}	0	0.2	0	0.2	0.6
C15	0.536	1	1.8715	0.0985	3.084	YC_{15}	0.2	0.2	0.2	0.2	0.2

A. Water Absorption of Concrete

The procedure used was that described in [2]. Two replicate concrete cubes were made for each of the thirty mix ratios in 150mmX150mmX150mm moulds and allowed to harden. The concrete cubes were removed from the moulds after 24hours and weighed. They were later cured in water for 24hours and weighed again. The water absorption was calculated for each mix ratio with eq. (23) and recorded in Table 3 with the averages determined.

$$\text{Water Absorption}, W_a = \frac{W_s - W_d}{W_d} \qquad (23)$$

where, W_s is the weight of cube soaked in water after 24hours, and W_d is the weight of cube at the SSD state. The water absorption, W_a is in percentage.

III. RESULTS AND DISCUSSIONS

Sieve analysis was carried out on the fine aggregate mixed with 5% SDA as a preliminary investigation. The particle size distribution of the 5% replacement of sand with SDA is shown in Figure 3, and the fineness modulus calculated below.

Fineness modulus,

$$FM = \frac{0.73 + 4.24 + 14.08 + 43.61 + 80.48 + 97.88}{100} = 2.41$$

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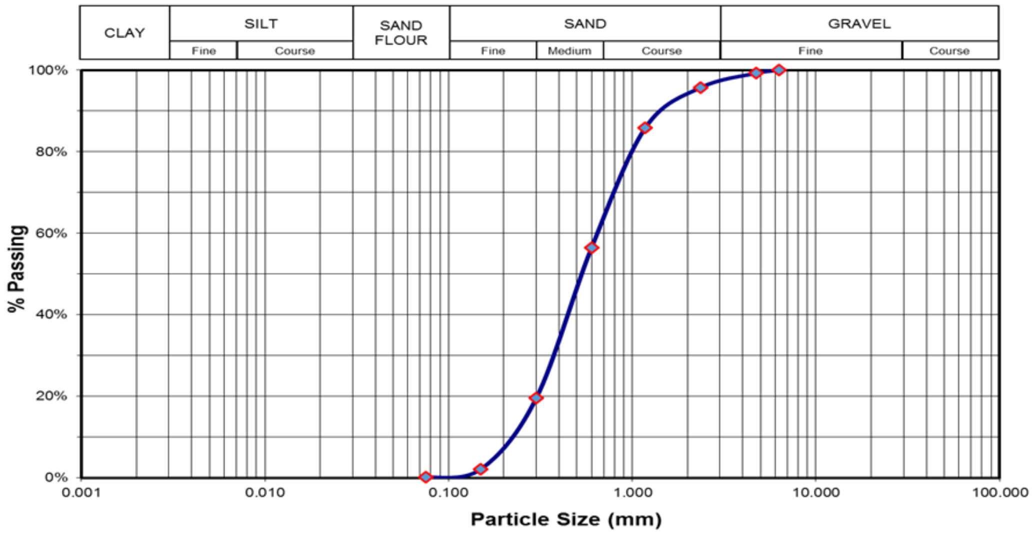

Figure 3: Particle Size Distribution for Fine Aggregate with 5% SDA replacement

The results of the water absorption are shown in Table 3 below.

Table 3 Water Absorption of Concrete

Sample Annotation	Water Absorption						
	Sample A			Sample B			AVERAGE W_a (%)
	Dry mass, W_d (kg)	Soaked mass, W_s (kg)	Absorption, W_a (%)	Dry mass, W_d (kg)	Soaked mass, W_s (kg)	Absorption, W_a (%)	
BRE12	7.2	7.75	7.639	7.21	7.7	6.796	7.218
BRE22	7.4	8	8.108	7.41	8	7.962	8.035
USBR22	7.9	8.2	3.797	7.9	8.19	3.671	3.734
BIS12	7.6	8	5.263	7.5	8	6.667	5.965
ACI12	7	7.45	6.429	7.05	7.5	6.383	6.406
N1	7.1	7.71	8.592	7.14	7.715	8.053	8.322
N2	7.2	7.8	8.333	7.1	7.6	7.042	7.688
N3	7.3	7.8	6.849	7.3	7.795	6.781	6.815
N4	7.25	7.8	7.586	7.25	7.79	7.448	7.517
N5	7.3	7.79	6.712	7.31	7.78	6.430	6.571
N6	7.4	7.82	5.676	7.41	7.81	5.398	5.537
N7	7.45	7.79	4.564	7.4	7.8	5.405	4.985
N8	7.12	7.8	9.551	7.2	7.82	8.611	9.081
N9	7.22	8	10.803	7.2	7.8	8.333	9.568
N10	7.4	8	8.108	7.3	7.75	6.164	7.136
C1	7.3	7.92	8.493	7.31	7.9	8.071	8.282
C2	7.7	8.09	5.054	8.02	8.45	5.324	5.189
C3	7.1	7.88	10.986	7.5	7.82	4.267	7.626
C4	7.95	8.48	6.714	7.3	7.66	4.873	5.794
C5	7.32	7.86	7.406	7.32	7.86	7.404	7.405
C6	7	7.41	5.857	7.05	7.55	7.092	6.475
C7	7	7.59	8.416	7	7.53	7.570	7.993
C8	7.05	7.49	6.191	7.1	7.51	5.746	5.969
C9	7.29	7.81	7.133	7.28	7.8	7.143	7.138
C10	7.1	7.76	9.282	7.2	7.74	7.472	8.377
C11	7.02	7.68	9.402	7.1	7.7	8.451	8.926
C12	7.2	7.74	7.500	7.2	7.75	7.639	7.569
C13	7.5	8	6.667	7.52	8	6.383	6.525
C14	7.56	7.96	5.291	7.57	8.01	5.812	5.552

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Sample Annotation	Water Absorption						
	Sample A			Sample B			AVERAGE W_a (%)
	Dry mass, W_d (kg)	Soaked mass, W_s (kg)	Absorption, W_a (%)	Dry mass, W_d (kg)	Soaked mass, W_s (kg)	Absorption, W_a (%)	
C15	7.54	8.18	8.500	7.45	7.99	7.240	7.870

A. Model Formulation

The coefficients of polynomial from Table 3, eq. (16), and eq. (18) are:
β_1 = 7.218, β_2 = 8.035, β_3 = 3.734, β_4 = 5.965, β_5 = 6.406,
$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2$
$\beta_{12} = 4 * 8.322 - 2 * 7.218 - 2 * 8.035 = 2.782$

Similarly, β_{13} = 8.848, β_{14} = 0.894, β_{15} = 2.82, β_{23} = 2.746, β_{24} = -5.852, β_{25} = -8.942, β_{34} = 16.926, β_{35} = 17.992, β_{45} = 3.802.

Substituting the above coefficients into eq. (12) gives

$$\boldsymbol{Y = 7.218x_1 + 8.035x_2 + 3.734x_3 + 5.965x_4 + 6.406x_5 + 2.782x_1x_2 + 8.848x_1x_3 + 0.894x_1x_4 + 2.82x_1x_5 + 2.746x_2x_3 - 5.852x_2x_4 - 8.942x_2x_5 + 16.926x_3x_4 + 17.992x_3x_5 + 3.802x_4x_5} \quad \textbf{(24)}$$

Eq. (24) is the mathematical model to predict the water absorption of concrete using SDA to replace 5% of fine aggregate. Table 4 shows the predictions, while Fig. 4 shows the comparison between the predicted and experimented values of water absorption using the control (verification) data.

Table 4 Experimental and predicted values of Water Absorption of Concrete

Sample Points	Response Y	Pseudo Components					Water Absorption Y_{exp} (%)	Water Absorption Y_{pred} (%)
		w-c ratio X_1	Cement X_2	Sand X_3	SDA X_4	Granite X_5		
BRE12	Y_1	1	0	0	0	0	7.218	7.218
BRE22	Y_2	0	1	0	0	0	8.035	8.035
USBR22	Y_3	0	0	1	0	0	3.734	3.734
BIS12	Y_4	0	0	0	1	0	5.965	5.965
ACI12	Y_5	0	0	0	0	1	6.406	6.406
N1	Y_{12}	0.5	0.5	0	0	0	8.322	8.322
N2	Y_{13}	0.5	0	0.5	0	0	7.688	7.688
N3	Y_{14}	0.5	0	0	0.5	0	6.815	6.815
N4	Y_{15}	0.5	0	0	0	0.5	7.517	7.517
N5	Y_{23}	0	0.5	0.5	0	0	6.571	6.571
N6	Y_{24}	0	0.5	0	0.5	0	5.537	5.537
N7	Y_{25}	0	0.5	0	0	0.5	4.985	4.985
N8	Y_{34}	0	0	0.5	0.5	0	9.081	9.081
N9	Y_{35}	0	0	0.5	0	0.5	9.568	9.568
N10	Y_{45}	0	0	0	0.5	0.5	7.136	7.136
C1	Y_{C1}	0.4	0	0.4	0	0.2	8.282	8.743
C2	Y_{C2}	0	0.6	0	0.4	0	5.189	5.803
C3	Y_{C3}	0.8	0	0.2	0	0	7.626	7.937
C4	Y_{C4}	0	0.4	0	0.6	0	5.794	5.389
C5	Y_{C5}	0.6	0	0	0	0.4	7.405	7.570
C6	Y_{C6}	0	0	0.8	0.2	0	6.475	6.888
C7	Y_{C7}	0.6	0.2	0	0	0.2	7.993	7.534
C8	Y_{C8}	0	0.4	0	0.4	0.2	5.969	5.534
C9	Y_{C9}	0.2	0	0	0	0.8	7.138	7.020
C10	Y_{C10}	0	0	0.2	0.8	0	8.377	8.227

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C11	Y_{C11}	0.2	0	0.6	0	0.2	8.926	8.299
C12	Y_{C12}	0	0.2	0.4	0.2	0.2	7.569	8.148
C13	Y_{C13}	0	0.8	0.2	0	0	6.525	7.614
C14	Y_{C14}	0	0.2	0	0.2	0.6	5.552	5.793
C15	Y_{C15}	0.2	0.2	0.2	0.2	0.2	7.870	7.952

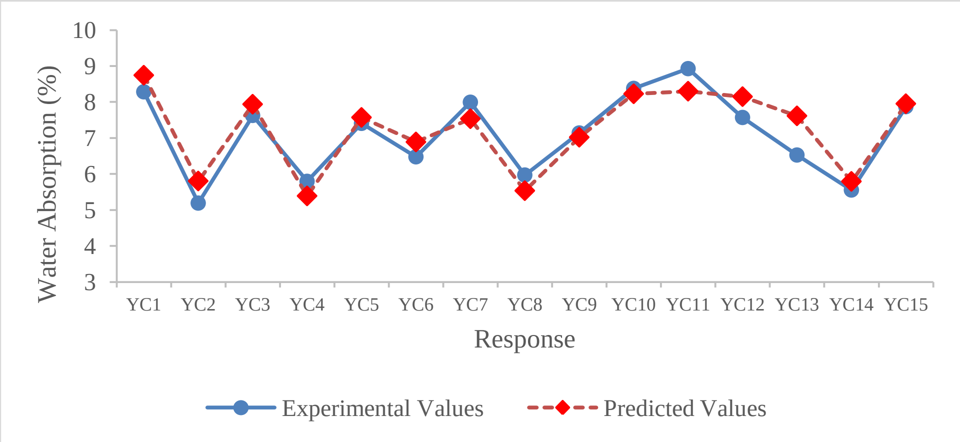

Figure 4: Comparison between Experimental and Predicted Water Absorption

A. *Test of Adequacy of the Model*

A two-tailed student t-test was carried out at 95% confidence level, which implies 100 – 95 = 5% significance. Since it is a two-tailed, significance = 5/2 = 2.5%

Hence significance level = 100 – 2.5 = 97.5%

Let D be difference between the experimental and predicted responses

The mean of the difference,

$$D_a = \frac{1}{n}\sum_{i=1}^{n} D_i \tag{25}$$

The variance of the difference,

$$S^2 = \left(\frac{1}{n-1}\right)\sum_{i=1}^{n}(D - D_a)^2{}_i \tag{26}$$

$$t_{calculated} = \frac{D_a\sqrt{n}}{S} \tag{27}$$

Where n = number of observations with degree of freedom n – 1. Table 5 shows the details of the t-test results.

Table 5 Student t-test for water absorption of Concrete

Sample	Absorption (%)		t-test		
	$Y_{experimental}$	$Y_{predicted}$	$D=Y_{exp}-Y_{pred}$	D_a-D	$(D-D_a)^2$
C1	8.282	8.743	-0.461	0.344	0.118
C2	5.189	5.803	-0.614	0.497	0.247
C3	7.626	7.937	-0.311	0.194	0.037
C4	5.794	5.389	0.405	-0.522	0.273
C5	7.405	7.570	-0.165	0.048	0.002
C6	6.475	6.888	-0.413	0.296	0.087
C7	7.993	7.534	0.459	-0.576	0.332
C8	5.969	5.534	0.435	-0.552	0.305
C9	7.138	7.020	0.118	-0.235	0.055
C10	8.377	8.227	0.150	-0.267	0.072
C11	8.926	8.299	0.627	-0.744	0.554
C12	7.569	8.148	-0.579	0.462	0.213
C13	6.525	7.614	-1.089	0.972	0.944
C14	5.552	5.793	-0.241	0.124	0.015

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Sample	Absorption (%)		t-test		
	$Y_{experimental}$	$Y_{predicted}$	$D=Y_{exp}-Y_{pred}$	D_a-D	$(D-D_a)^2$
C15	7.870	7.952	-0.082	-0.035	0.001
TOTAL			-1.761		3.257
AVERAGE D_a			-0.117		

$$S^2 = \frac{3.257}{15-1}$$
$$S^2 = 0.233$$
$$S = \sqrt{0.233} = 0.482$$
$$t_{calculated} = \frac{-0.117\sqrt{15}}{0.482}$$
$$t_{calculated} = -0.943$$

From the t-table, with v = 15 – 1 = 14, and β = significance level. $t_{(0.975,14)} = 2.145$

Since $t_{calculated} < t_{(0.975,14)}$, and lies between -2.145 and 2.145, therefore there is no significant difference between the experimental and predicted responses, H_0 is accepted, and H_a is rejected. The model is confirmed to be adequate.

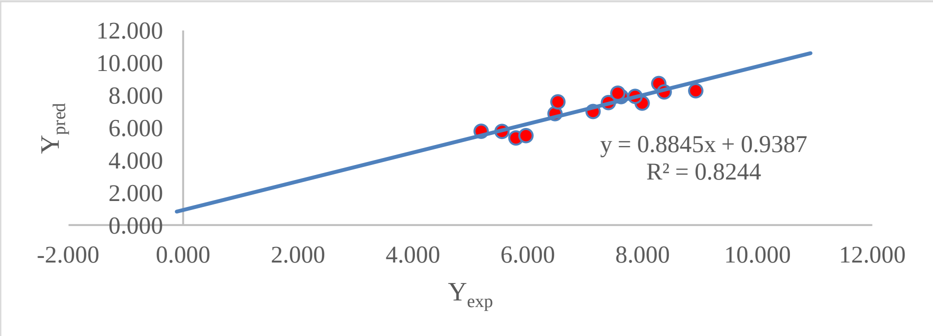

Figure 5: Scatterplot of Predicted vs. Experimental Water Absorption

The R^2 value of 0.8244 in Figure 5 indicates that the experimental results are highly correlated to the predicted results. This is also an indication that the model is fit and adequate.

IV. CONCLUSIONS

After successfully replacing fine aggregate with 5% SDA in a concrete blend, the water absorptions determined from the laboratory were between 3.734% and 9.568% which are less than the maximum acceptable values. The concrete was batched from five different mix ratios. A multiple regression model was generated from the resulting water absorption experimental values, using Scheffe's simplex theory for a {5,2} simplex lattice. A two-tailed student t-test was carried out at 5% significance level, which confirmed the adequacy of the derived model with an excellent fit, given an R^2 value of 0.8244. The results also confirmed that SDA is a suitable material to replace a small fraction (about 5%) of fine aggregate in a bid to promote environmental sustainability.

REFERENCES

[1] M. S. Mamlouk & J. P. Zaniewski. (2011). *Materials for Civil and Construction Engineers*. (3rd ed.). New Jersey: Pearson Prentice Hall.
[2] BS.1818. (1983), *Part 122: Testing concrete*. London: British Standard Institute.
[3] F. F. Udoeyo, R. Brooks, C. Utam, P. Udo-inyang, & E. C. Ukpong. (2010). Effect of non-standard curing methods on the compressive strength of laterized concrete. *ARPN Journal of Engineering and Applied Sciences*, *5*(2), 6–20.
[4] H. K. Kim, J. H. Jeon, & H. K. Lee. (2012). Flow, water absorption, and mechanical characteristics of normal- and high-strength mortar incorporating fine bottom ash aggregates. *Construction and Building Materials*, *26*(1), 249–256.
[5] N. M. Noor *et al.* (2017). Compressive strength, flexural strength and water absorption of concrete containing palm oil kernel shell. In: *IOP Conf. Series: Materials Science and Engineering 271*.
[6] J. Castro, D. Bentz, & J. Weiss. (2011). Cement & concrete composites effect of sample conditioning on the water absorption of concrete. *Cement and Concrete Composites*, *33*(8), 805–813.
[7] S. H. Alsayed & M. A. Amjad. (1996). Strength, water absorption and porosity of concrete. *Journal of King Saud Universing - Engineering Sciences*, *8*(1), 109–119.
[8] S. P. Zhang & L. Zong. (2014). Evaluation of relationship between water absorption and durability of concrete materials. *Advances in Material Sciences and Engineering*. Available at: https://www.hindawi.com/journals/amse/2014/650373/.
[9] C. Marthong. (2012). Sawdust Ash (SDA) as partial replacement of cement. *International Journal of Engineering Research and Applications*, *2*(4), 1980–1985.

[10] P. Peduzzi. (2014). Sand, rarer than one thinks. Article reproduced from United Nations Environment Programme (UNEP) Global Environmental Alert Service (GEAS). *Environmental Development, 11*, 208–218.
[11] UNEP. (2019). *Sand and sustainability: Finding new solutions for environmental governance of global sand resources*. Available at:
https://wedocs.unep.org/handle/20.500.11822/28163.
[12] S. Chowdhury, M. Mishra, & O. Suganya. (2015). The incorporation of wood waste ash as a partial cement replacement material for making structural grade concrete : An overview. *Ain Shams Engineering Journal*, *6*, 429–437.
[13] M. Mageswari & B. Vidivelli. (2009). The use of sawdust ash as fine aggregate replacement in concrete. *Journal of Environmental Research and Development*, *3*(3), 720–726.
[14] S. T. Tyagher, J. T. Utsev, & T. Adagba. (2011). Suitability of saw dust ash-lime mixture for production of sandcrete hollow blocks. *Nigerian Journal of Technology*, *30*(1), 1–6.
[15] M. E. Onyia. (2017). Optimisation of the cost of lateritic soil stabilized with quarry dust. *International Journal of Scientific Engineering and Research, 8*(9), 1400–1413.
[16] C. U. Anya. (2015). Models for predicting the structural characteristics of sand-quarry dust blocks. *Ph.D Thesis, Department of Civil Engineering, University of Nigeria*, Nsukka.
[17] C. E. Okere, D. O. Onwuka, S. U. Onwuka, & J. I. Arimanwa. (2013). Simplex-based concrete mix design. *IOSR Journal of Mechnical and Civil Engineering*, *5*(2), 46–55.
[18] E. M. Mbadike & N. N. Osadebe. (2014). Five component concrete mix optimization of aluminum waste using Scheffe's theory. *International Journal of Computer Engineering and Research, 4*(4), 23–31.
[19] E. M. Mbadike & N. N. Osadebe. (2013). Application of Scheffe's model in optimization of compressive strength of lateritic concrete. *Journal of Civil Engineering and Construction Technology, 4*(9), 265–274.
[20] L. Brown, A. N. Donev, & A. T. Biessett. (2015). General blending models for data from mixture experiments. *Technometrics, 57*(4), 449–456.
[21] O. M. Ibearugbulem, L. O. Ettu, J. C. Ezeh, & U. C. Anya. (2013). A new regression model for optimizing concrete mixes. *International Journal of Scientific Engineering and Research Technology*, *2*(7), 1735–1742.
[22] Y. M. Gamil & I. H. Bakar. (2015). The Development of mathematical prediction model to predict resilient modulus for natural soil stabilised by POFA-OPC additive for the use in unpaved road design. *In: Soft Soil Engineering International Conference*, pp. 1–11.
[23] P. N. Onuamah. (2015). Development-and-optimization-of-mechanical-strength-model-of-cement-laterite-sand-hollow-sandcrete-blocks. *International Journal of Science Engineering and Research, 6*(5), 645–655.
[24] P. N. Onuama. (2015). Prediction-of-the-compressive-strength-of-concrete-with-palm-kernel-aggregate-using-the-artificial-neutral-networks-approach. *International Journal of Science Engineering and Research, 6*(6), 962–969.
[25] N. N. Osadebe, C. C. Mbajiorgu, & T. U. Nwakonobi. (2007). An optimization model development for laterized-concrete mix proportioning in building constructions. *Nigerian Journal of Technology*, *26*(1), 37–46.
[26] K. M. Oba, O. O. Ugwu, & F. O. Okafor. (2019). Development of a model to predict the flexural strength of concrete using SDA as partial replacement for fine aggregate. *International Journal of Science Engineering and Research*, *10*(5), 1216–1224.
[27] K. M. Oba, O. O. Ugwu, & F. O. Okafor. (2019). Development of Scheffe's model to predict the compressive strength of concrete using SDA as partial replacement for fine aggregate. *International Journal of Innovative Technology and Exploring Engineering, 8*(8), 2512–2521.
[28] K. M. Oba. (2019). A mathematical model to predict the tensile strength of asphalt concrete using quarry dust filler. *International Journal of Science Engineering and Research*, *10*(2), 1491-1498.
[29] H. Scheffe. (1958). Experiments with mixtures. *J. R. Stat. Ser. B*, *25*(2), 235–263.

