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Application of Nonlinear Programming to Heat Conduction Model

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ABSTRACT

The design of high-temperature working clothes is a process of theoretical analysis and design of all layers of working clothes based on the premise that human body avoids burns under the high-temperature environment. Steady temperature mathematical model in this paper, through the establishment of system, determine the optimal heat work suit II layer thickness is actually an optimization problem, has been the function relation between the temperature and time conditions, through Fourier heat conduction law, obtained the system ultimately a function of temperature and material thickness, and has set up a nonlinear programming problem, the introduction of the simulated annealing algorithm, and joined the convection and radiation in the algorithm the two factors affect the optimal solution, finally satisfied under the condition of the optimal thickness of the second floor.

Keywords-- High Temperature Working Clothes, Fourier Heat Conduction Law, Nonlinear Programming, Simulated Annealing Algorithm

I. INTRODUCTION

The high-temperature working clothes are usually composed of three layers of fabric materials, denoted as layer I, II and III, in which layer I is in contact with the external environment, and there are gaps between layer III and skin, denoted as layer IV. Therefore, the second layer of overalls plays the most important role in preventing high temperature.

Parameter values of special clothing materials

stratified	density (kg/m^3)	specific heat (J/(kg·°C))	heat conductivity (W/(m·°C))	thickness (mm)
I layer	300	1377	0.082	0.6
II layer	862	2100	0.37	0.6-25
III layer	74.2	1726	0.045	3.6
IV layer	1.18	1005	0.028	0.6-6.4

This paper mainly studies that when the ambient temperature is 65 degrees Celsius, the human body temperature remains unchanged at 37ºC, and the optimal thickness of the second layer is determined, so that when the working time is 60 minutes, the lateral temperature of the skin of the dummy is no more than 47 ºC, and the time of the last period exceeding 44 ºC is no more than 5 minutes.

On the one hand, from the perspective of qualitative analysis, the final temperature of the system should be related to the material's texture and thickness. Therefore, we can study the relationship between the final temperature and the material thickness based on Fourier's law. On the other hand, we obtained the general state equation of heat conduction by using the Fourier heat law and the law of conservation of energy. Through theoretical analysis, we found that the temperature and time of each layer have such a general form:

$$\frac{dT_i}{dt} = K_i T_i (1 - \frac{T_i}{T'}) \qquad (1)$$

Among them, K_i is related to dissipation area, specific heat capacity, medium thickness, and heat conductivity. It is worth noting that this quantitative relation may have certain problems due to the interference of errors and data problems, but its proportional relation is determined,i.e. $K \propto \frac{A_3\lambda_4}{c_4\delta_4 m_4}$.

II. ESTABLISHMENT OF NONLINEAR PROGRAMMING MODEL

1. Since we assume that this process is a stable heat transfer process, we first discuss the heat conduction under the single-layer state. The heat conduction process is caused by temperature difference. It is assumed that the flat wall of a single layer is composed of uniform materials, with thickness of δ and thermal conductivity of λ. The temperature of the two surfaces of the flat wall is T_1 and T_2 respectively, and $T_1 > T_2$. When the heat conduction is stable, the heat conduction is along the x axis direction. Since this is a one-dimensional stable heat conduction problem, the heat conduction differential equation in the flat wall is:

$$\frac{d^2T}{dx^2} = 0 \qquad (2)$$

Its boundary conditions: : $x = 0, T = T_1$ and $x = \delta, T = T_2$, the integral of equation (2) can be obtained:

$$T_2 = C_1 x + C_2 \qquad (3)$$

By substituting the boundary conditions, the temperature distribution in the flat wall can be obtained:

$$T_1 = C_2 \qquad T_2 = C_1\delta + C_2 \qquad (4)$$

In simultaneous equations (3) and (4), we can get:

$$T = \frac{T_2 - T_1}{\delta} x + T_1 \qquad (5)$$

At the same time, the heat flux and heat flux through infinite flat wall can be obtained through Fourier's law:

$$q_x = -\lambda \frac{\partial T}{\partial x} = \frac{T_1 - T_2}{\frac{\delta}{\lambda}} \qquad (6)$$

Thus, it can be obtained:

$$Q = Aq_x = \frac{T_1 - T_2}{\frac{\delta}{\lambda A}} \qquad (7)$$

Among them, A is the area of flat wall, which is also the area of dissipation due to the assumption that the material of the protective suit is uniform in all directions.

2. Based on the above analysis, this paper discusses the establishment and solution of the model in this environment (i.e. multi-layer heat conduction problem) : The design of protective clothing can be regarded as a multi-layer stable heat conduction problem of flat walls. Therefore, the temperature in the above analysis process can still be used to show a linear change rule in the protective clothing. The analysis can be concluded as follows:

$$Q = \frac{T_1 - T_4}{(\frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \frac{\delta_3}{\lambda_3})\frac{1}{A}} \qquad (8)$$

Or:

$$q_x = \frac{T_1 - T_4}{(\frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \frac{\delta_3}{\lambda_3})} \qquad (9)$$

Use the conclusion of equation (9) to solve this problem. In this environment, the initial temperature of the external environment is 65 degrees Celsius, and the human environment is always controlled at 37 degrees Celsius, then the heat required to be transmitted is:

$$q = \frac{28}{\sum_{i=1}^{4} \frac{\delta_i}{\lambda_i}} \qquad (10)$$

Taking the temperature of the fourth layer as the final steady-state temperature of the system, and continuing to use the conclusion of equation (9), the final temperature when the system reaches steady-state can be obtained as:

$$T^{'} = 65 - \frac{28 \times \sum_{i=1}^{3} \frac{\delta_i}{\lambda_i}}{\sum_{i=1}^{4} \frac{\delta_i}{\lambda_i}} \qquad (11)$$

This is the same as our analysis, that is, the steady state temperature of the system is a function of the thickness of the material.

After determining the functional relationship between the steady-state temperature of the system and the thickness of the material, we need to add the factor of time for analysis. We introduce the concept of "retardation factor" （1-$\frac{T}{T^{'}}$） and derive the temperature distribution function of the fourth layer.

For the surface temperature of the dummy, its initial value is 37 degrees Celsius, which is substituted into equations (1) and (11). The following functional relationship can be obtained:

$$T / (1 + (28 - 28 \times \sum_{i=1}^{3} \frac{\delta_i}{\lambda_i} / \sum_{i=1}^{4} \frac{\delta_i}{\lambda_i}) / (37 \times e^{kt})) = 65 - 28 \times \sum_{i=1}^{3} \frac{\delta_i}{\lambda_i} / \sum_{i=1}^{4} \frac{\delta_i}{\lambda_i} \qquad (12)$$

Among them, $K \propto \frac{A_3 \lambda_4}{c_4 \delta_4 m_4}$,

This question is actually an optimization problem, to determine the optimal thickness of the second layer if the conditions are met. In practice, when other conditions are consistent, the thinner the protective suit, the lower the design cost, while also convenient for people wearing the protective suit to work. The environment requires that the time for the last period exceeding 44 DCS should not exceed 5 minutes, that is, the temperature should be less than 44 DCS at the 55th minute. Therefore, according to the functional relation obtained by our analysis, the problem can be transformed into the following single-objective nonlinear programming problem, that is,

Objective function: $\min \quad \delta_2$

Constraints: $\begin{cases} T(60) \le 47 \\ T(55) \le 44 \\ 0.6 \le \delta_2 \le 25 \end{cases}$

Where, is the function relation derived from equation (22).

III. SOLUTION OF NONLINEAR PROGRAMMING MODEL

Description of simulated annealing algorithm:

The simulated annealing algorithm refers to the process of metal smelting. Statistical mechanics showed that different structures of particles in materials correspond to different energy levels of particles. At high temperatures, the particles are higher in energy and can move freely and rearrange. At low temperatures, the particle is low in energy. If you start at high temperatures and cool down very slowly (this process is called annealing), the particles can reach thermal equilibrium at each temperature. That is to say, when the temperature is high enough, it is more likely to occur a certain cooling process with energy difference. When the temperature is low enough, the probability of cooling is smaller. This is an annealing process.

Compared with other greedy algorithms to solve the optimal value, the simulated annealing is less likely to fall into the process of only obtaining the local optimal solution, which always accepts a poor solution with a certain probability, and the solving process of this probability is:

$$P(x(K) \to x') = \begin{cases} 1 & if\ f(x') < f(x) \\ e^{\frac{f(x') - f(x(K))}{T_i}} & else \end{cases}$$

What this means is that if the value of the generated function is smaller than the previous one, accept it as the new solution.

It is not always possible to jump out of this local optimal solution to approach the global optimal solution. The calculation of this probability jump out is obviously borrowed from the actual annealing process in production and life, so the algorithm is called simulated annealing. When the parameters are set, it is easier to approach the global optimal solution.

The detailed algorithm steps are described below:

(1) first initialize the model: set the initial temperature T(sufficiently large), and set the initial solution state S(the starting point of algorithm iteration), and the number of iterations of each T value num.

(2) k=1 Do steps (3) to 6.

(3) under constraint conditions, a new solution S 'is generated for the objective function every time.

(4) calculate the increment dE= RND (S ')- RND (S), where RND (S) is the evaluation function, and initialize the random seed at the beginning to prevent the result of each run from being the same.

(5) ifdE<0, S 'is accepted as the new current solution, and a better solution is obtained after movement; otherwise, S' is accepted as the new current solution by probability exp(dE/T).

(6) if the termination condition is met, output current solution as the optimal solution and terminate the program. The termination condition is set to terminate the algorithm when the temperature is reached and stop searching.

(7) T decreases gradually to achieve cooling annealing, 0 The larger r, the slower the cooling; The smaller r, the faster the temperature drops, and then we go to step 2. In the process of the algorithm, the better solution is obtained after every move, and the algorithm is accepted. Instead of a better solution, let's use the probability function to accept the move. In order to keep it stable, the longer it takes to set it, the less likely it is to jump around, and the attenuation is set, so the global optimal solution is not hard to get.

IV. RESULTS ANALYSIS

After multiple iterations, we found that the final result was: the minimum value was 13.006240.In c language, the exhaustive method was adopted for verification, and the output results were imported into the file for sorting. The results were found to be 12.9500 with minimal error, indicating that the simulated annealing method had a good effect.

V. CONCLUSION

Along with society's progress and development, the high temperature operation cannot be avoided, by using mathematical model study of hot work clothing constantly to reasonably and effectively optimize the design of the heat work clothing, save manpower and financial resources, this article has been the general relation between the temperature and time of each layer, on the basis of using the method of partial differential equation is derived for the temperature and time and the relation between the thickness of the material, in the end, the model of the simulated annealing algorithm to solve for the inspection result and optimization, finally got satisfy the conditions of minimum thickness, either against the environment of high temperature and can give high temperature workers on a portable work clothes, Provide effective help in the design of high temperature work clothes.

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