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Design of a Controller for MIMO System by using Approximate Model Matching (AMM) Method

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ABSTRACT

This paper presents the design of a controller for MIMO systems. Performance analysis and controller design for SISO systems are easier than that for MIMO systems. These difficulties are due to the coupling or interactions between the various inputs and output variables. An input u_1, from the set u_i, $i = 1, m$; not only affects the output y_1, from the set y_i, $i = 1, p$; but may affect all the other outputs y_i, $i = 2, p$. Similarly, each input u_j, may also affect output y_k, $k \neq j$.

In many cases cross-coupling between the inputs and outputs is low and hence conventional SISO controllers may perform well. If a multivariable system exhibits strong cross-coupling between the various inputs and outputs, a multivariable controller has to be designed for achieving satisfactory performance. A popular approach to deal with control loop interactions is to design non-interacting or decoupling control schemes. The role of decoupler is to decompose a multivariable process into a series of independent single-loop sub-system. If such a controller is designed, complete or ideal decoupling occurs and the multivariable process can be controlled using independent loop controllers.

Keywords-- Controller, Coupling, Time Delay, Decoupling, AMM, Decoupler, Zero Frequency Decoupler

I. INTRODUCTION

A system with Multiple Inputs and Multiple Outputs is known as a MIMO system. Industrial systems like chemical reactors, heat exchangers, distillation columns etc. are some examples of MIMO systems[2]. Performance analysis and controller design for SISO systems are easier than that for MIMO systems. These difficulties are due to the coupling or interactions between the various inputs and output variables. An input u_1, from the set u_i, i=1, mnot only affects the output $y_1$from the set y_i, i=1,p;but may affect all the other outputs y_i, i=2,p; similarly, each input u_j may also affect each output y_k, $k \neq j$.

In many cases, cross-coupling between the inputs and outputs is low and hence conventional SISO controllers may perform well. If a multivariable system exhibits strong cross-coupling between the various inputs and outputs, a multivariable controller has to be designed for achieving satisfactory performance.

II. CONTROLLER DESIGN BY USING APPROXIMATE MODEL MATCHING (AMM) METHOD

A popular approach to deal with control loop interactions is to design non-interacting or decoupling control schemes. The role of decoupler[3] is to convert a multivariable process into several independent single-loop sub-systems. If such a controller is designed and complete or ideal decoupling occurs, the multivariable process can be controlled by using independent single-loop (SISO) controllers [1].

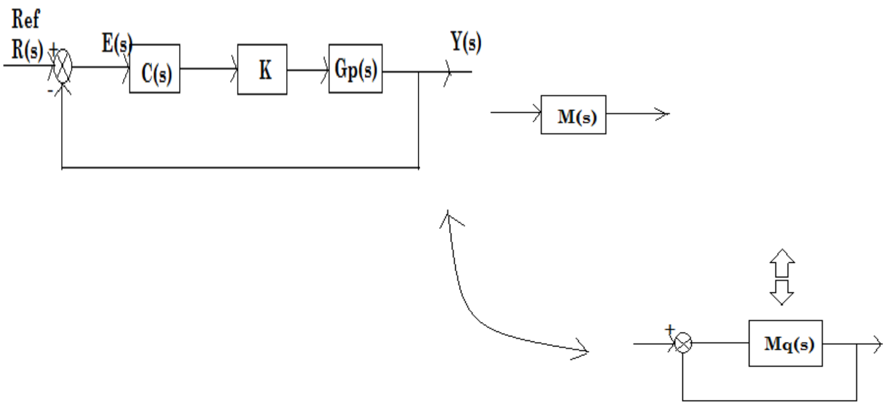

Figure 1.Approximate Model Matching (AMM) method for controller design

Where, the decoupled matrix is then given by:

$$G_{pd}(s) = [K][G_p(s)][M_p(s)] = \begin{bmatrix} m_{11}(s) & 0 \\ 0 & m_{22}(s) \end{bmatrix}$$

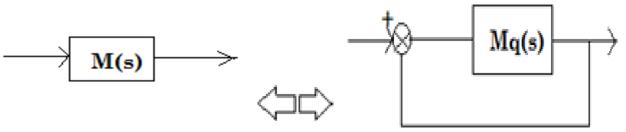

Figure 2. Equivalent closed loop of open loop system

$$\frac{M_q(s)}{1 + M_q(s)} = M(s)$$

$$M_q(s) = M(s) + M(s)M_q(s) \quad M_q(s)[1 - M(s)] = M(s)$$

$$M_q(s) = \frac{M(s)}{1-M(s)} [M_q(s)] = \begin{bmatrix} \frac{m_{11}(s)}{1-m_{11}(s)} & 0 \\ 0 & \frac{m_{22}(s)}{1-m_{22}(s)} \end{bmatrix} \qquad (1)$$

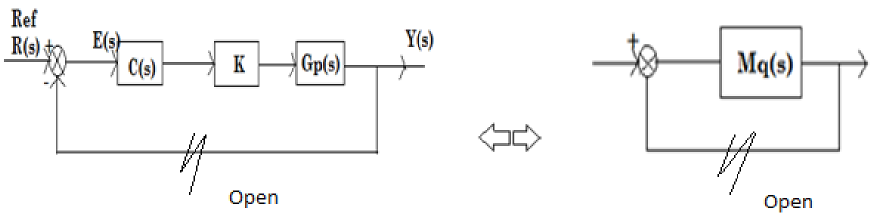

Figure 3. Equivalent diagram for approximate model matching

$$[G_p(s)][K(s)][C(s)] = M_q(s)$$

$$\begin{aligned} C(s) &= \{[G_p(s)][K(s)]\}^{-1}[G_p(s)][K(s)][C(s)] \\ &= [K(s)]^{-1}[G_p(s)]^{-1}[G_p(s)][K(s)][C(s)] \\ &= [K(s)]^{-1}[G_p(s)]^{-1}[M_q(s)] \end{aligned}$$

$$\begin{bmatrix} K_{p1} + \frac{K_{i1}}{s} & 0 \\ 0 & K_{p2} + \frac{K_{i2}}{s} \end{bmatrix} = [K(s)]^{-1}[G_p(s)]^{-1}[M_q(s)]$$

Then by comparing the diagonal equations of both right hand and left-hand matrix the values of K_{p1} *and* K_{i1} can be calculated.

III. DESIGN OF A CONTROLLER FOR A MIMO (4X4) SYSTEM WITHOUT DELAY

Let the plant be given by

$$G(s) = \begin{bmatrix} \frac{-2.2}{7s+1} & \frac{1.3}{7s+1} & \frac{1.7}{7s+1} & \frac{2.4}{6s+1} \\ \frac{-2.8}{9.5s+1} & \frac{4.3}{9.2s+1} & \frac{3.6}{9.7s+1} & \frac{2.5}{8.5s+1} \\ \frac{-2.4}{7.6s+1} & \frac{3.9}{6.5s+1} & \frac{2.5}{4.6s+1} & \frac{4.2}{5.2s+1} \\ \frac{-301}{5.2s+1} & \frac{4.1}{5.4s+1} & \frac{2.2}{4.6s+1} & \frac{2.6}{4.1s+1} \end{bmatrix}$$

$K = [G_P(0)^{-1}]$

$$= \begin{bmatrix} -2.2 & 1.3 & 1.7 & 2.4 \\ -2.8 & 4.3 & 3.6 & 2.5 \\ -2.4 & 3.9 & 2.5 & 4.2 \\ -3.1 & 4.1 & 2.2 & 2.6 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -0.8052 & 0.8272 & -0.6437 & -0.5608 \\ -0.6437 & -0.3475 & -0.7734 & 0.3093 \\ -0.1468 & 1.3363 & -1.4244 & -0.6539 \\ 0.1152 & -0.2292 & 0.4439 & -0.2184 \end{bmatrix}$$

$$M_p(s) = \begin{bmatrix} \frac{1}{5s+1} & 0 & 0 & 0 \\ 0 & \frac{1}{7s+1} & 0 & 0 \\ 0 & 0 & \frac{1}{4s+1} & 0 \\ 0 & 0 & 0 & \frac{1}{3s+1} \end{bmatrix}$$

$$= \begin{bmatrix} m_{11}(s) & 0 & 0 & 0 \\ 0 & m_{22}(s) & 0 & 0 \\ 0 & 0 & m_{33}(s) & 0 \\ 0 & 0 & 0 & m_{44}(s)0 \end{bmatrix}$$

$$M_q(s) = \begin{bmatrix} \frac{m_{11}(s)}{1-m_{11}(s)} & 0 & 0 & 0 \\ 0 & \frac{m_{22}(s)}{1-m_{22}(s)} & 0 & 0 \\ 0 & 0 & \frac{m_{33}(s)}{1-m_{33}(s)} & 0 \\ 0 & 0 & 0 & \frac{m_{44}(s)}{1-m_{44}(s)} \end{bmatrix} \quad (2)$$

For s =0.01

$$G_p(s_1) = G_p(0.01) = \begin{bmatrix} -2.0561 & 1.2150 & 1.5888 & 2.2642 \\ -2.5571 & 3.9377 & 3.2817 & 2.3041 \\ -2.2305 & 3.6620 & 2.3901 & 3.9924 \\ -2.9468 & 3.8899 & 2.1033 & 2.4967 \end{bmatrix}$$

$$[G_p(0.01)]^{-1} = \begin{bmatrix} -0.5378 & 0.2497 & 0.5204 & -0.5747 \\ -0.664 & -0.0331 & 0.1526 & 0.3363 \\ 0.2225 & 0.7421 & -0.1024 & -0.7061 \\ 0.1225 & -0.2636 & 0.4625 & -0.2068 \end{bmatrix}$$

For s =0.02

$$G_p(s_1) = G_p(0.02) = \begin{bmatrix} -1.9298 & 1.1404 & 1.4912 & 2.1429 \\ -2.3529 & 3.6318 & 3.0151 & 2.1368 \\ -2.0833 & 3.4513 & 2.2894 & 3.8043 \\ -2.8080 & 3.7004 & 2.0147 & 2.4030 \end{bmatrix}$$

$$[G_p(0.02)]^{-1} = \begin{bmatrix} -0.5667 & 0.2597 & 0.5453 & -0.5888 \\ -0.6488 & -0.0439 & 0.1604 & 0.3637 \\ 0.2477 & 0.7989 & -0.1089 & -0.7589 \\ 0.1292 & -0.2987 & 0.4815 & -0.1957 \end{bmatrix}$$

For s =0.03

$$G_p(s_1) = G_p(0.03) = \begin{bmatrix} -1.8182 & 1.0744 & 1.4050 & 2.0339 \\ -2.1790 & 3.3699 & 2.7885 & 1.9920 \\ -1.9544 & 3.2636 & 2.1968 & 3.6332 \\ -2.6817 & 3.5284 & 1.9332 & 2.3152 \end{bmatrix}$$

$$[G_p(0.03)]^{-1} = \begin{bmatrix} -0.5961 & 0.2690 & 0.5706 & -0.6031 \\ -0.6916 & -0.0553 & 0.1681 & 0.3913 \\ -0.2733 & 0.8745 & -0.1150 & -0.8121 \\ 0.1353 & -0.3344 & 0.5007 & -0.1849 \end{bmatrix}$$

For s =0.04

$$G_p(s_1) = G_p(0.04) = \begin{bmatrix} -1.7188 & 1.0156 & 1.3281 & 1.9355 \\ -2.0290 & 3.1433 & 2.5937 & 1.8657 \\ -1.8405 & 3.0952 & 2.1115 & 3.4768 \\ -2.5662 & 3.3717 & 1.8581 & 2.2337 \end{bmatrix}$$

$$[G_p(0.04)]^{-1} = \begin{bmatrix} -0.6269 & 0.2775 & 0.5962 & -0.6175 \\ -0.7348 & -0.0671 & 0.1758 & 0.4191 \\ 0.2995 & 0.9506 & -0.1207 & -0.8656 \\ 0.1409 & -0.3707 & 0.5200 & -0.1743 \end{bmatrix}$$

By putting, S=0.01,0.02, 0.03 and 0.04 in equation (17) we find

$$M_q(0.01) = \begin{bmatrix} 20 & 0 & 0 & 0 \\ 0 & 14.29 & 0 & 0 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 33.33 \end{bmatrix}$$

$$M_q(0.02) = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 7.143 & 0 & 0 \\ 0 & 0 & 12.5 & 0 \\ 0 & 0 & 0 & 16.665 \end{bmatrix}$$

$$M_q(0.03) = \begin{bmatrix} 6.67 & 0 & 0 & 0 \\ 0 & 4.762 & 0 & 0 \\ 0 & 0 & 8.33 & 0 \\ 0 & 0 & 0 & 11.11 \end{bmatrix}$$

$$M_q(0.04) = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 3.573 & 0 & 0 \\ 0 & 0 & 6.25 & 0 \\ 0 & 0 & 0 & 8.333 \end{bmatrix}$$

For s =0.01

$$\begin{bmatrix} K_{p1}+\frac{K_{i1}}{0.01} & 0 & 0 & 0 \\ 0 & K_{p2}+\frac{K_{i2}}{0.01} & 0 & 0 \\ 0 & 0 & K_{p3}+\frac{K_{i3}}{0.01} & 0 \\ 0 & 0 & 0 & K_{p4}+\frac{K_{i4}}{0.01} \end{bmatrix}$$

$$= \begin{bmatrix} -2.2 & 1.3 & 1.7 & 2.4 \\ -2.8 & 4.3 & 3.6 & 2.5 \\ -2.4 & 3.9 & 2.5 & 4.2 \\ -3.1 & 4.1 & 2.2 & 2.6 \end{bmatrix} \begin{bmatrix} -0.5378 & 0.2497 & 0.5204 & -0.5747 \\ -0.664 & -0.0331 & 0.1526 & 0.3363 \\ 0.2225 & 0.7421 & -0.1024 & -0.7061 \\ 0.1225 & -0.2636 & 0.4625 & -0.2068 \end{bmatrix} \begin{bmatrix} 20 & 0 & 0 & 0 \\ 0 & 14.29 & 0 & 0 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 33.33 \end{bmatrix}$$

$$= \begin{bmatrix} 21.3418 & 0.5224 & -1.2645 & 0.1613 \\ 0.114 & 16.7346 & -0.3332 & -0.1237 \\ -0.0698 & 0.2824 & 25.8170 & 1.9011 \\ -0.2212 & 0.5356 & -0.2590 & 35.6398 \end{bmatrix}$$

$$K_{p1}+\frac{K_{i1}}{0.01} = 21.3418 \quad (3)$$
$$K_{p2}+\frac{K_{i2}}{0.01} = 16.7346 \quad (4)$$
$$K_{p3}+\frac{K_{i3}}{0.01} = 25.8170 \quad (5)$$
$$K_{p4}+\frac{K_{i4}}{0.01} = 35.6398 \quad (6)$$

For s =0.02

$$\begin{bmatrix} K_{p1}+\frac{K_{i1}}{0.02} & 0 & 0 & 0 \\ 0 & K_{p2}+\frac{K_{i2}}{0.02} & 0 & 0 \\ 0 & 0 & K_{p3}+\frac{K_{i3}}{0.02} & 0 \\ 0 & 0 & 0 & K_{p4}+\frac{K_{i4}}{0.02} \end{bmatrix}$$

$$= \begin{bmatrix} -2.2 & 1.3 & 1.7 & 2.4 \\ -2.8 & 4.3 & 3.6 & 2.5 \\ -2.4 & 3.9 & 2.5 & 4.2 \\ -3.1 & 4.1 & 2.2 & 2.6 \end{bmatrix} \begin{bmatrix} -0.5823 & 0.2900 & 0.4858 & -0.5718 \\ -0.6447 & -0.0532 & 0.1673 & 0.3766 \\ 0.2174 & 0.8969 & -0.2600 & -0.7172 \\ 0.1453 & -0.0352 & 0.5274 & -0.2125 \end{bmatrix} \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 7.143 & 0 & 0 \\ 0 & 0 & 12.5 & 0 \\ 0 & 0 & 0 & 16.665 \end{bmatrix}$$

$$= \begin{bmatrix} 11.3447 & 0.0917 & -0.2584 & 0.1393 \\ 0.1164 & 3.6670 & -0.3176 & -0.1457 \\ -0.0835 & -0.3699 & 13.3361 & 1.8725 \\ -0.2245 & -0.0293 & -0.2559 & 18.9656 \end{bmatrix}$$

$$K_{p1}+\frac{K_{i1}}{0.02} = 11.3447 \quad (7)$$
$$K_{p2}+\frac{K_{i2}}{0.02} = 3.6670 \quad (8)$$
$$K_{p3}+\frac{K_{i3}}{0.02} = 13.3361 \quad (9)$$
$$K_{p4}+\frac{K_{i4}}{0.02} = 18.9656 \quad (10)$$

For s =0.03

$$\begin{bmatrix} K_{p1}+\frac{K_{i1}}{0.03} & 0 & 0 & 0 \\ 0 & K_{p2}+\frac{K_{i2}}{0.03} & 0 & 0 \\ 0 & 0 & K_{p3}+\frac{K_{i3}}{0.03} & 0 \\ 0 & 0 & 0 & K_{p4}+\frac{K_{i4}}{0.03} \end{bmatrix}$$

$$=\begin{bmatrix} -2.2 & 1.3 & 1.7 & 2.4 \\ -2.8 & 4.3 & 3.6 & 2.5 \\ -2.4 & 3.9 & 2.5 & 4.2 \\ -3.1 & 4.1 & 2.2 & 2.6 \end{bmatrix}\begin{bmatrix} -0.5961 & 0.2690 & 0.5706 & -0.6031 \\ -0.6916 & -0.0553 & 0.1681 & 0.3913 \\ -0.2733 & 0.8745 & -0.1150 & -0.8121 \\ 0.1353 & -0.3344 & 0.5007 & -0.1849 \end{bmatrix}\begin{bmatrix} 6.67 & 0 & 0 & 0 \\ 0 & 4.762 & 0 & 0 \\ 0 & 0 & 8.33 & 0 \\ 0 & 0 & 0 & 11.11 \end{bmatrix}$$

$$=\begin{bmatrix} 8.0151 & 0.0971 & -0.2550 & 0.1242 \\ 0.1156 & 6.2916 & -0.3090 & -0.1615 \\ -0.1007 & -0.3786 & 9.1762 & 1.8518 \\ -0.2308 & -0.0294 & -0.2568 & 13.4051 \end{bmatrix}$$

$$K_{p1}+\frac{K_{i1}}{0.03}=8.0151 \quad (11)$$
$$K_{p2}+\frac{K_{i2}}{0.03}=6.2916 \quad (12)$$
$$K_{p3}+\frac{K_{i3}}{0.03}=9.1762 \quad (13)$$
$$K_{p4}+\frac{K_{i4}}{0.03}=13.4051 \quad (14)$$

For s =0.04

$$\begin{bmatrix} K_{p1}+\frac{K_{i1}}{0.04} & 0 & 0 & 0 \\ 0 & K_{p2}+\frac{K_{i2}}{0.04} & 0 & 0 \\ 0 & 0 & K_{p3}+\frac{K_{i3}}{0.04} & 0 \\ 0 & 0 & 0 & K_{p4}+\frac{K_{i4}}{0.04} \end{bmatrix}$$

$$=\begin{bmatrix} -2.2 & 1.3 & 1.7 & 2.4 \\ -2.8 & 4.3 & 3.6 & 2.5 \\ -2.4 & 3.9 & 2.5 & 4.2 \\ -3.1 & 4.1 & 2.2 & 2.6 \end{bmatrix}\begin{bmatrix} -0.6269 & 0.2775 & 0.5962 & -0.6175 \\ -0.7348 & -0.0671 & 0.1758 & 0.4191 \\ 0.2995 & 0.9506 & -0.1207 & -0.8656 \\ 0.1409 & -0.3707 & 0.5200 & -0.1743 \end{bmatrix}\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 3.573 & 0 & 0 \\ 0 & 0 & 6.25 & 0 \\ 0 & 0 & 0 & 8.333 \end{bmatrix}$$

$$=\begin{bmatrix} 6.3563 & 0.1022 & -0.2519 & 0.1124 \\ 0.1307 & 5.1090 & -0.2996 & -0.1732 \\ -0.1032 & -0.3863 & 7.1062 & 1.8368 \\ -0.2202 & -0.0281 & -0.2561 & 10.6251 \end{bmatrix}$$

$$So,\ K_{p1}+\frac{K_{i1}}{0.04}=6.3563 \quad (15)$$
$$K_{p2}+\frac{K_{i2}}{0.04}=5.1090 \quad (16)$$
$$K_{p3}+\frac{K_{i3}}{0.04}=7.1062 \quad (17)$$
$$K_{p4}+\frac{K_{i4}}{0.04}=10.6251 \quad (18)$$

Now by solving equations (3) and (7) we find

$$K_{p1}=1.3476\ and\ K_{i1}=0.19994$$

By solving equations (4) and (8) we find

$$K_{p2}=0.9994\ and\ K_{i2}=0.1574$$

By solving equations (5) and (9), we find

$$K_{p3}=0.8552\ and\ K_{i3}=0.2496$$

By solving equations (6) and (10), we find

$$K_{p4}=2.2914\ and\ K_{i4}=0.3335$$

For the above values of controller parameters K_p and K_i; the Simulink diagram and the responses are shown in Figs. 4 and 5 respectively.

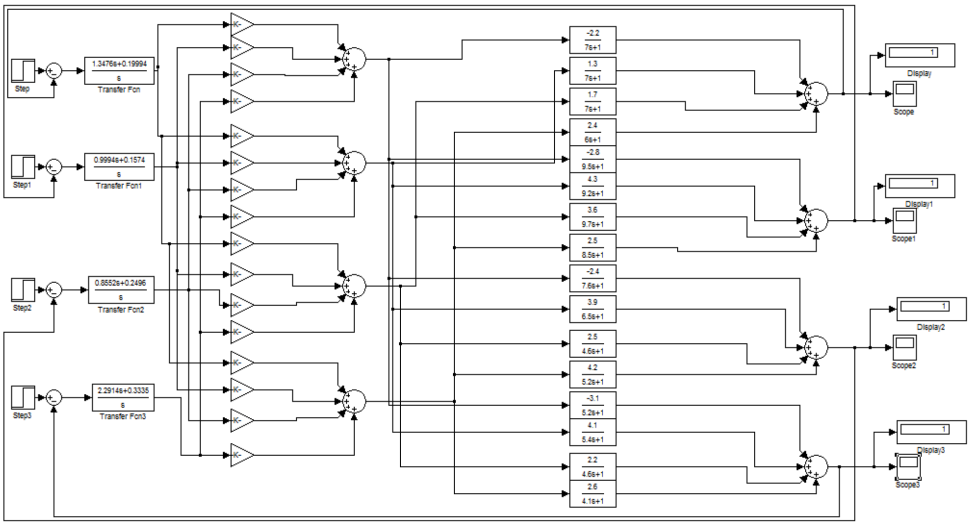

Figure 4. Simulink block diagram for the given system without delay

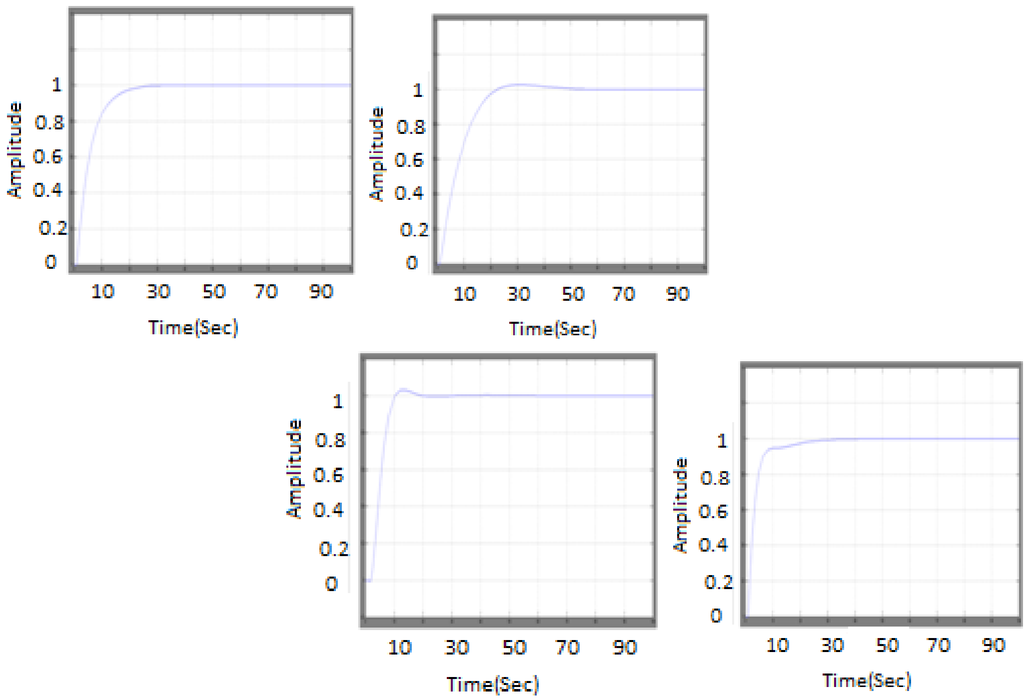

Figure 5. Output responses of scope 1,2,3 & 4

From Fig-5, it can be observed that the system outputs y_1,y_2 ,y_3 and y_4 reach at its steady state value after a time interval.

IV. SYSTEM RESPONSE WITH ONE INPUT AT A TIME

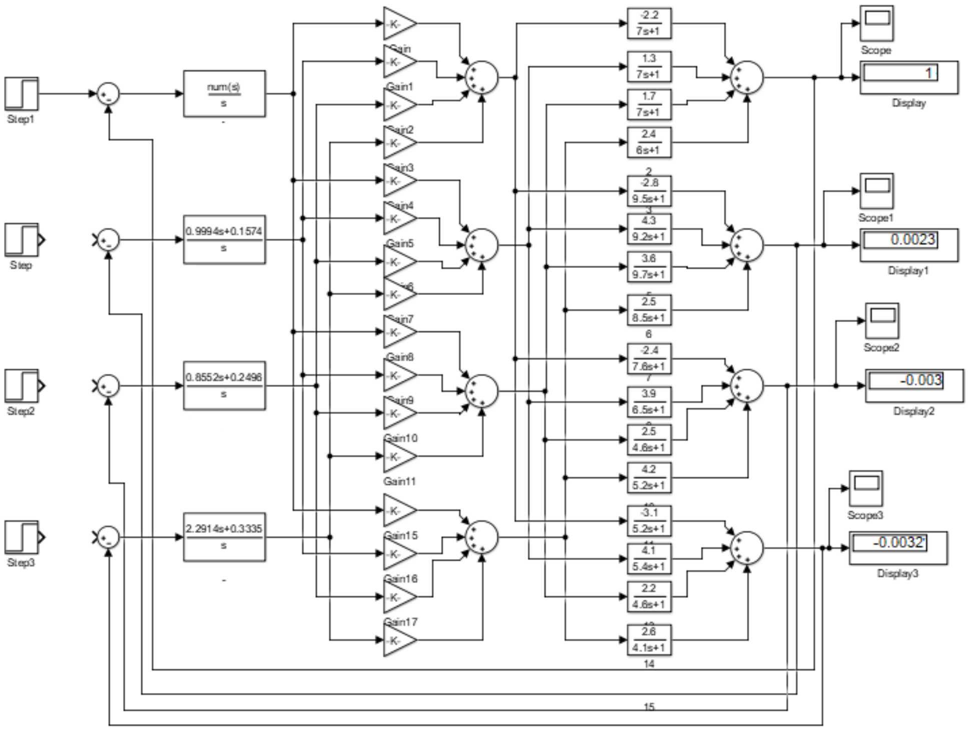

Figure 6. Simulink diagram for the closed loop system with input-1.

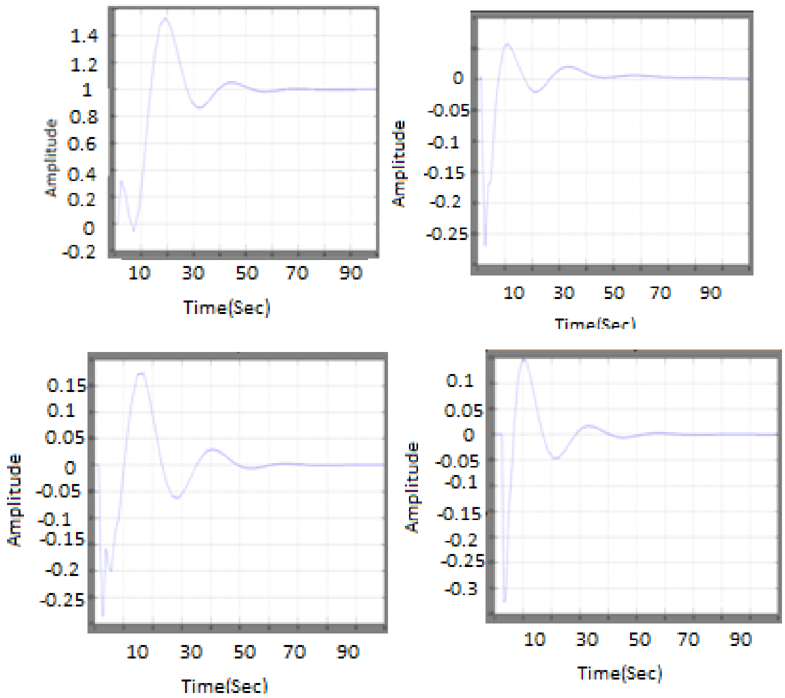

Figure 7. Output responses of scope 1,2,3 & 4

From Fig 6 and 7 it can be seen that when only one reference has taken and the output response in the corresponding scope is stable whereas the response in the alternate scope is having a very less value (≈0) which means that that a good degree of decoupling has been achieved.

V. CONCLUSION

The decoupler design becomes easy for the AMM method by making use of the zero-frequency decoupling technique. The controller design method has been successfully applied to highly interacting MIMO systems taken from the open literature. The various examples convincingly illustrate the applicability and suitability of the proposed controller design methods to real life process control problems. Once the desired time and frequency domain specifications are translated into mathematically useful realizable desired transfer function models, the proposed controller design methods may be used to easily design non-interacting MIMO rational cascade controllers, the parameters of which can be easily determined by solving a set of linear algebraic equations. The methods are conceptually appealing and do not involve any optimization technique for finding the unknown controller parameters.

REFERENCES

[1] Manoj Samal & Anjan Kumar Sahoo. (2016 Oct). Design of a controller for MIMO system with time delay by using zero frequency decoupler. *IEEE International Conference, SCOPE, 5*, 67-70.

[2] A.N. Gündea, A.N. Metea, & A. Palazo§lu. (2009). Reliable decentralized PID controller synthesis for two-channel MIMO processes. *Automatica, 45*, 353-363.

[3] Pontus Nordfeldt & Tore Hagglund. (2006). Decoupler and PID controller design of TITO systems. *Journal of Process Control 16*, 923–936.

[4] Branislav T. Jevtovica, & Miroslav R. Matausekb. (2010). PID controller design of TITO system based on ideal decoupler. *Journal of Process Control*, 869–876.

[5] Zeng-Rong Hu, Dong-Hai Li, Jing Wang, & Feng Xue. (2011). Analytical design of PID decoupling control for TITO processes with time delays. *Journal of Computers, 6*(6), 1064-1070.

[6] Qiang Xiong, Wen-Jian Cai, & Ming He. (2006). A practical decentralized PID auto-tuning method for TITO systems under closed loop. *International Journal of Innovative Computing, Information and Control, 2*(2), 305-322.

[7] S. Dolpullop & S. Chitwong. (2014). Designing decoupler and PID controller for TITO System. *Advanced Materials Research, 931-932*, 1270-1274.