ABSTRACT
In this paper, we study the notion of intuitionistic fuzzy gi compact spaces and study some of their properties.

KEYWORDS—Intuitionistic Fuzzy gi Compact Space, Intuitionistic Fuzzy Open and Closed Set, Fuzzy Set

I. INTRODUCTION AND PRELIMINARIES

Topology is the mathematical study of shapes and spaces. Topology developed as a field of study out of geometry and set theory, through analysis of such concepts as space, dimension and transformation. In 1970, Levine[10] initiated the study of so called generalized closed sets. The notion has been studied extensively in recent years by many topologists because generalized closed sets play not only important role in generalization of closed sets but also they have suggested some new separation axioms, some of them have been found to be useful in computer science, digital topology and quantum physics.


The notion of intuitionistic fuzzy sets was introduced by Atanassov[1] as a generalization of fuzzy sets. In 1997, Coker[6] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we introduce the notions of gi closed sets and gi open sets in intuitionistic fuzzy topological spaces and study some their properties in intuitionistic fuzzy topological spaces.


Throughout this paper, (X, τ ) or simply X denote the intuitionistic fuzzy topological spaces (IFTS in short). For a subset A of X , the closure, the interior and the complement of A are denoted by cl(A) , int(A) and AC respectively. We recall some basic definitions that are used in the sequel.

Definition 1.1.
[1] Let A and B be IFSs of the form A = {<x, µA(x), νA(x)> / x ∈ X } and B = {<x, µB(x), νB(x)> / x ∈ X }. Then
1. A ⊆ B if and only if µA(x) ≤ µB(x) and νA(x) ≥ νB(x) for all x ∈ X ,
3. A C = {<x, νA(x), µA(x)> / x ∈ X } ,
4. A ∩ B = {<x, µA(x) ∧ µB(x), νA(x) ∨ νB(x)> / x ∈ X } ,
5. A ∪ B = {<x, µA(x) ∨ µB(x), νA(x) ∧ νB(x)> / x ∈ X } .

For the sake of simplicity, we shall use the notation A = { x, µA, νA } instead of A = { { x, µA(x), νA(x)> / x ∈ X } . Also for the sake of simplicity, we shall use the notation A = { x, (µA, µB), (νA, νB)} instead of A = { x, (A/µA, B/µB), (A/νA, B/νB}) .The intuitionistic fuzzy sets 0~ = {<x, 0, 1> / x ∈ X } and 1~ = {<x, 1, 0> / x ∈ X } are respectively the empty set and the whole set in X .

Definition 1.2.
[4] An IFS A = {x, µA, νA} in an IFTS (X, τ ) is said to be an
1. intuitionistic fuzzy semi-closed set (IFSCS in short) if cl(int(A)) ⊆ A,
2. intuitionistic fuzzy semi-open set (IFSOS in short) if A ⊆ cl(int(A)),
3. intuitionistic fuzzy α-closed set (IF α CS in short) if cl(int(int(A))) ⊆ A,
4. intuitionistic fuzzy α-open set (IF α OS in short) if A ⊆ cl(int(int(A))),
5. intuitionistic fuzzy regular closed set (IFRCS in short) if cl(int(A)) = A,
6. intuitionistic fuzzy regular open set (IFROS in short) if A = int(cl(A)) ,
7. intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X .The complement of an IFGCS is an IFGOS [8].
8. intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFROS in X . The complement of an IFRGCS
is an IFROS [7].

9. intuitionistic fuzzy generalized \(\alpha\)-closed set (IFG \(\alpha\) CS in short) if \(\text{acl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IF \(\alpha\) OS in \(X\). The complement of an IF \(\alpha\) CS is an IFG \(\alpha\) OS [6].

10. intuitionistic fuzzy \(\alpha\)-generalized closed set (IFG\(\alpha\)CS) [6] if \(\text{acl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFOS in \(X\).

11. intuitionistic fuzzy \(W\)-closed set (IFWCS in short) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFCS in \(X\). The complement of an IFWCS is an IFWOS [5].

**Definition 1.3.**

[1] Let \(X\) be a non-empty set. An intuitionistic fuzzy set (IFS in short) \(A\) in \(X\) is an object having the form \(A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}\) where the functions \(\mu_A : X \rightarrow [0, 1]\) and \(\nu_A : X \rightarrow [0, 1]\) denote the degree of membership (namely \(\mu_A(x)\)) and the degree of non-membership (namely \(\nu_A(x)\)) of each element \(x\) in \(X\) to the set \(A\) respectively and \(0 \leq \mu_A(x) + \nu_A(x) \leq 1\) for each \(x \in X\). Denote by IFS(\(X\)), the set of all intuitionistic fuzzy sets in \(X\).

**Definition 1.4.**

[3] An intuitionistic fuzzy topology (IFT in short) on \(X\) is a family \(\tau\) of IFSs in \(X\) satisfying the following axioms:

1. \(\emptyset, X \in \tau\),
2. \(G_1 \cap G_2 \in \tau\) for any \(G_1, G_2 \in \tau\),
3. \(\bigcup G_i \in \tau\) for any family \(\{G_i / i \in J\} \subseteq \tau\).

In this case the pair \((X, \tau)\) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in \(\tau\) is known as an intuitionistic fuzzy open set (IFOS in short) in \(X\). The complement \(A^C\) of an IFOS \(A\) in an IFTS \((X, \tau)\) is called an intuitionistic fuzzy closed set (IFWCS in short) if \(A^C\) is an IFWOS in \(X\).

**Definition 3.1.**

A collection \(\{A_i / i \in \Lambda\}\) of IFG\(\alpha\)OSs in IFTS \((X, \tau)\) is called intuitionistic fuzzy \(\alpha\) compact set (IFG\(\alpha\)CS) in IFTS if \(\bigcup A_i \subseteq X\) and \(\bigcap_{i \in \Lambda} A_i = \emptyset\).

**Definition 3.2.**

An IFTS \((X, \tau)\) is said to be intuitionistic fuzzy \(\alpha\) compact (IFG\(\alpha\) compact space) if \(\bigcup A_i \subseteq X\) and \(\bigcap_{i \in \Lambda} A_i = \emptyset\).

**Definition 3.3.**

An IFTS \((X, \tau)\) is said to be intuitionistic fuzzy \(\alpha\) compact set (IFG\(\alpha\)CS) in IFTS if \(\bigcup A_i \subseteq X\) and \(\bigcap_{i \in \Lambda} A_i = \emptyset\).

**Definition 3.4.**

A subset \(B\) of IFTS \((X, \tau)\) is said to be intuitionistic fuzzy \(\alpha\) compact if \(B\) is an IFG\(\alpha\) compact space.

**Theorem 3.5.**

A IFG\(\alpha\) closed subset of IFG\(\alpha\) compact space is IFG\(\alpha\) compact relative to \(X\).

**Proof:** Let \(A\) be an intuitionistic fuzzy \(\alpha\) closed subset of IFG\(\alpha\) compact space \((X, \tau)\). Then \(A^C\) is IFG\(\alpha\) open in \(X\). Let \(M\) be a cover of \(A\) by IFG\(\alpha\) open sets in \(X\). Then the family \(\{M, A^C\}\) is intuitionistic fuzzy \(\alpha\) open cover of \(X\). Since \(X\) is intuitionistic fuzzy \(\alpha\) compact, it has a finite sub cover say \(\{G_1, G_2, G_3, \ldots, G_n\}\).
If this sub cover contains $A_c$, we discard it. Otherwise leave the sub cover as it is. Thus we obtained a finite intuitionistic fuzzy open sub cover of $A$. Therefore $A$ is intuitionistic fuzzy compact relative to $X$.

**Theorem 3.6.**

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF\$g\$ irresolute mapping and if $B$ is IF\$g\$ compact relative to $X$, then image $f(B)$ is IF\$g\$ compact relative to $Y$.

**Proof:** Let $\{A_i : i \in \Lambda\}$ be an IF\$g\$ open set of $Y$ such that $f(B) \subseteq \bigcup \{A_i : i \in \Lambda\}$. By using the assumption, there exists a finite subset $\Lambda_0$ such that $B \subseteq \bigcup \{f^{-1}(A_i) : i \in \Lambda_0\}$. Therefore $f(B) \subseteq \bigcup \{A_i : i \in \Lambda_0\}$ which shows that $f(B)$ is IF\$g\$ compact relative to $Y$.

**Theorem 3.7.**

An IF\$g\$ irresolute image of a IF\$g\$ compact space is IF\$g\$ compact space.

**Proof:** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF\$g\$ irresolute map from a IF\$g\$ compact space $(X, \tau)$ onto a IFTS $(Y, \sigma)$. Let $\{A_i : i \in \Lambda\}$ be an Intuitionistic fuzzy open cover of $X$. Since $X$ is IF\$g\$ compact, it has finite intuitionistic fuzzy sub cover say $\{f^{-1}(A_1), f^{-1}(A_2), ....f^{-1}(A_n)\}$. Since $f$ is onto $\{A_1, A_2, .....A_n\}$ is an IF\$g\$ open cover of $Y$ and so $(Y, \sigma)$ is IF\$g\$ compact.

**Theorem 3.8.**

An IF\$g\$ continuous image of a IF\$g\$ compact space is intuitionistic fuzzy compact space.

**Proof:** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF\$g\$ continuous map from a IF\$g\$ compact space $(X, \tau)$ onto a IFTS $(Y, \sigma)$. Let $\{A_i : i \in \Lambda\}$ be an Intuitionistic fuzzy open cover of $Y$, then $\{f^{-1}(A_i) : i \in \Lambda\}$ is a IF\$g\$ open cover of $X$. Since $X$ is IF\$g\$ compact, it has finite intuitionistic fuzzy sub cover say $\{f^{-1}(A_1), f^{-1}(A_2), ....f^{-1}(A_n)\}$. Since $f$ is onto $\{A_1, A_2, .....A_n\}$ is an intuitionistic fuzzy open cover of $Y$ and so $(Y, \sigma)$ is intuitionistic fuzzy compact.

**REFERENCES**