Intuitionistic Fuzzy $\tilde{g}$ Connected Spaces

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ABSTRACT
In this paper, we study the notion of intuitionistic fuzzy $\tilde{g}$ connected spaces and study some of their properties.

Keywords-- Intuitionistic Fuzzy $\tilde{g}$ Connected Space, Intuitionistic Fuzzy Compact Space, Hypothesis

I. INTRODUCTION
Connectedness plays an important role in spacial kind of objects. Yong Chan and Abbas [8] have investigated connectedness in intuitionistic fuzzy topological spaces. In 1997, Coker [5] introduced intuitionistic fuzzy $C_5$ connectedness in intuitionistic fuzzy topological spaces and in 2000, Turnali and coker [7] introduced GO connectedness in intuitionistic fuzzy topological spaces. There after several author have extended the concept of connectedness in intuitionistic fuzzy topological spaces. In this chapter, we study $\tilde{g}$ connectedness and investigate some of their properties in intuitionistic fuzzy topological spaces.

II. INTUITIONISTIC FUZZY $\tilde{g}$ CONNECTED SPACES

**Definition 2.1.**
An IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy $\tilde{g}$ connected space (IF$\tilde{g}$ connected spaces in short) if the only IFSSs which are both an IF$\tilde{g}$ OS and an IF$\tilde{g}$ CS are $0\sim, 1\sim$.

**Theorem 2.2.**
Every IF$\tilde{g}$ connected space is IF $C_5$ connected space, but not conversely.

**Proof:** Let $(X, \tau)$ be an intuitionistic fuzzy $\tilde{g}$ connected space and suppose that $(X, \tau)$ is not intuitionistic fuzzy $C_5$ connected. Then there exists a proper intuitionistic fuzzy set $A$ which is both an intuitionistic fuzzy open and intuitionistic fuzzy closed. Since every intuitionistic fuzzy open set (resp. intuitionistic fuzzy closed set) is intuitionistic $\tilde{g}$ open (resp. intuitionistic fuzzy $\tilde{g}$ closed), $X$ is not intuitionistic fuzzy $\tilde{g}$ connected, a contradiction. Therefore $(X, \tau)$ must be an intuitionistic C5 connected space.

**Example 2.3.**
Let $X = \{a, b\}$ and $G = \langle (0.5, 0.6), (0.5, 0.4) \rangle$. Then $\tau = \{0\sim, G, 1\sim\}$ is IFT on $X$. Then $X$ is an IF $C_5$ connected space but not IF$\tilde{g}$ connected, since the IF$\tilde{g}$ $A = \langle (0.4, 0.4), (0.6, 0.6) \rangle$ in $X$ is both an IF$\tilde{g}$ CS and IF$\tilde{g}$ OS in $X$.

**Theorem 2.4.**
An IFTS $(X, \tau)$ is IF$\tilde{g}$ connected if and only if there exists no non zero IF$\tilde{g}$ OSs $A$ and $B$ in $X$ such that $A = Bc$.

**Proof:** Necessity: Suppose that $A$ and $B$ are IF$\tilde{g}$ OSs such that $A = 0\sim = B \neq Bc$. Since $A = Bc$, $B$ is an IF$\tilde{g}$ OS which implies that $Bc = A$ is an IF$\tilde{g}$ CS and $B = 0\sim$ this implies that $Bc = 1\sim$. Hence there exists a proper IF$\tilde{g}$ $A (A = 0\sim, A = 1\sim)$ such that $A$ is both an IF$\tilde{g}$ OS and IF$\tilde{g}$ CS. But this is contradiction to the fact that $X$ is intuitionistic fuzzy $\tilde{g}$ connected.

**Sufficiency:** Let $(X, \tau)$ be an IFTS and $A$ is both an IF$\tilde{g}$ OS and IF$\tilde{g}$ CS in $X$ such that $A = 0\sim = Bc$. Now take $B = Ac$. In this case $B$ is intuitionistic fuzzy $\tilde{g}$ open and $A = 1\sim$. This implies that $B = Ac = 0\sim$ which is a contradiction. Hence there is no proper intuitionistic fuzzy set of $X$ which is both an IF$\tilde{g}$ OS and IF$\tilde{g}$ CS. Therefore $(X, \tau)$ is intuitionistic fuzzy $\tilde{g}$ connected.

**Theorem 2.5.**
An IFTS $(X, \tau)$ is intuitionistic fuzzy $\tilde{g}$ connected if and only if there exists no non zero IF$\tilde{g}$ OSs $A$ and $B$ in $X$ such that $A = Ac$, $B = (cl(A))c, A = (cl(B))c$. Since $(cl(A))c$ and $(cl(B))c$ are IF$\tilde{g}$ OSs in $X$. This implies that $(X, \tau)$ is not IF$\tilde{g}$ connected, which is a contradiction.

**Sufficiency:** Let $A$ is both an IF$\tilde{g}$ OS and IF$\tilde{g}$ CS such that $A = 0\sim = Ac$. Taking $B = Ac$, we obtain a contradiction to our hypothesis. Hence $(X, \tau)$ is an IF$\tilde{g}$ connected space.

**Theorem 2.6.**
Let $(X, \tau)$ be an intuitionistic fuzzy $\tilde{gt} 1/2$ space, then the following conditions are equivalent:

i) $X$ is intuitionistic fuzzy $\tilde{g}$ connected, ii) $X$ is intuitionistic fuzzy $C_5$ connected.
Proof: (i) $\Rightarrow$ (ii) is obvious by the Theorem 2.2. (ii) $\Rightarrow$ (i) Let $(X, \tau)$ be an IF C5 connected space. Suppose $(X, \tau)$ is not intuitionistic fuzzy connected, then there exists a proper IFS $A$ in $(X, \tau)$ such that $A$ is both an IFg OS and an IFg CS. But since $X$ is an IFg T1/2 space, $A$ is an IFOS and IFCS which implies that $X$ is not IF C5 connected, a contradiction. Therefore $(X, \tau)$ must be IFg connected.

Theorem 2.7. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFg continuous surjection and $(X, \tau)$ is an IFg connected space, then $(Y, \sigma)$ is an IF C5 connected space.

Proof: Let $(X, \tau)$ be an IFg connected space. Suppose $(Y, \sigma)$ is not IF C5 connected, then there exists a proper IFS $A$ which is both an IFOS and an IFCS in $(Y, \sigma)$. Since $f$ is an IFg continuous mapping, $f^{-1}(A)$ is both an IFg OS and an IFg CS in $(X, \tau)$. But this is a contradiction to our hypothesis. Hence $(Y, \sigma)$ must be an IF C5 connected space.

Theorem 2.8. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFg irresolute surjection and $(X, \tau)$ is an IFg connected space, then $(Y, \sigma)$ is also an IFg connected space. Proof: Suppose $(Y, \sigma)$ is not an IFg connected space, then there exists a proper IFS $A$ such that $A$ is both an IF OS and an IFg CS in $(Y, \sigma)$. Since $f$ is an IFg irresolute surjection, $f^{-1}(A)$ is both an IFg OS and an IFg CS in $(X, \tau)$. But this is a contradiction to our hypothesis. Hence $(Y, \sigma)$ must an be an IFg connected space.

Definition 2.9. An IFTS $(X, \tau)$ is IFg connected between two IFSs $A$ and $B$ if there is no IFg OS $E$ in $(X, \tau)$ such that $A \subseteq E$ and $E \subseteq B$.

Theorem 2.10. If an IFTS $(X, \tau)$ is IFg connected between two IFSs $A$ and $B$, then it is IF C5 connected between $A$ and $B$ but the converse may not be true in general.

Proof: Suppose $(X, \tau)$ is not IF C5 connected between $A$ and $B$, then there exists an IFOS $E$ in $(X, \tau)$ such that $A \subseteq E$ and $E \subseteq B$. Since every IFOS is an IFg OS, there exists an IFg OS $E$ in $(X, \tau)$ such that $A \subseteq E$ and $E \subseteq B$. This implies that $(X, \tau)$ is not IFg connected between $A$ and $B$, contradiction to our hypothesis. Therefore $(X, \tau)$ must be IF C5 connected between $A$ and $B$.

Theorem 2.11. If an IFTS $(X, \tau)$ is IFg connected between two IFSs $A$ and $B$ and $A \subseteq A_1, B \subseteq B_1$, then $(X, \tau)$ is IFg connected between $A_1$ and $B_1$.

Proof: Suppose that $(X, \tau)$ is not IFg connected between $A_1$ and $B_1$, then $b$.

Theorem 2.12. Let $(X, \tau)$ be an IFTS and $A$ and $B$ be IFTSs in $(X, \tau)$. If $A \subseteq B$, then $(X, \tau)$ is IFg connected between $A$ and $B$.

Proof: Suppose that $(X, \tau)$ is not IFg connected between $A$ and $B$. Then there exists an IFg OS $E$ in $(X, \tau)$ such that $A \subseteq E$ and $E \subseteq B$. This implies that $A \subseteq E$. But this is a contradiction to our hypothesis. Therefore $(X, \tau)$ must be IFg connected between $A$ and $B$.

REFERENCES