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Performance Analysis of MUSIC and MVDR DOA Estimation Algorithm

Bhupenmewada[1], Prof. Kamal Niwaria[2] and Dr. Manish Jain[3]
[1]M-TechScholar, Department of Electronics & Communication Engineering, RKDF Institute of Science and Technology, Bhopal, INDIA
[2]Assistant Professor, Department of Electronics & Communication Engineering, RKDF Institute of Science and Technology, Bhopal, INDIA
[3]Associate Professor, Department of Electronics & Communication Engineering, RKDF Institute of Science and Technology, Bhopal, INDIA

[2]Corresponding Author: kamalniwaria@gmail.com

ABSTRACT

The communication systems use concept of smart antennas, which is based on digital signal processing algorithms. In this way, the smart antennas system becomes capable to locate and track signals by the both: users, interferers and dynamically adapts the antenna pattern to enhance the reception in Signal-of-Interest direction and minimizing interference in Signal-of-Not-Interest (SONI) direction. Hence, Space Division Multiple Access system (SDMA), which uses smart antennas, is being used more often in wireless communications, because it shows improvement in channel capacity and co-channel interference. However, performance of smart antenna system greatly depends on efficiency of digital signal processing algorithms. The algorithm uses the Direction of Arrival (DOA) algorithms to estimate the number of incidents plane waves on the antenna array and their angle of incidence. In this paper the performance of Direction-of-Arrival (DOA) algorithms MUSIC and MVDR are investigated. The simulation results shows that, the advantages in performance of one algorithm over another vary with the conditions and is significantly influenced by both of the environment as well as the system. Thus, careful consideration is imperative to the conditions and system parameters specific to the planned deployment. The algorithms have been simulated in MATLAB 7.4 version.

Keywords-- SDMA, DOA, MUSIC and MVDR

I. INTRODUCTION

In resent years the high demand on the usage of the wireless communication system has put more emphasize on the requirement of higher system capacities. The system capacity can be improved by either enlarging its frequency bandwidth or adding new range of frequency spectrum to wireless services. But because of obvious reasons, since the electromagnetic spectrum is a limited resource, it is not easy to get new spectrum allocation without the international coordination on the global level. So, one alternative approach is to use existing spectrum more efficiently. Efficient source and channel coding as well as reduction in transmission power or transmission bandwidth or both are possible solutions to the challenging issue. With the advances in digital techniques, the frequency efficiency can be improved by multiple access technique (MAT), which gives mobile users access to scarce resource (base station) and hence improves the system's capacity [1]. The existing Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA) can be enlarged by adding a new parameter 'space' or 'angle' [2], which results in MAT known as 'Space Division Multiple Access' (SDMA). At the receiver's side, the transmitted signal is received with its multipath components plus interferers' signal, as well as with present noise. Thus, detection of the desired signal is a challenging task. In this context smart antennas emerged as one of the most expected technologies, which are adapted to the demanding high-bit rate or high quality in broadband commercial wireless communication such as mobile internet or multimedia services [3], [4].The Smart Antenna System (SAS) employs the antenna elements and the digital signal processing which enables it to form a beam to a desired direction taking into account the multipath signal components. In this way, Signal-to-Interference-and-Noise Ratio (SINR) improves by producing nulls towards the interferers Signal-of–No-Interest (SONI) [5].The performance of SAS greatly depends on the performance on DOA estimation.

In first section of paper, the performance of two algorithms for direction of arrival i.e. MUSIC and MDR respectively are investigated. The performance of these algorithms is analyzed by considering parameters like number of array elements, user space distribution, number of snapshots, signal to noise ratio, Mean Square Error (MSE), which results in optimum array design in SAS. In second section the simulation result, conclusion and future work is presented. The algorithms have been simulated in MATLAB 7.4 version.

II. DOA ESTIMATION ALGORITHMS

The algorithms based on DOA are classified as non-subspace or quadratic type and subspace type [6]. The Barltett and Capon (Minimum Variance Distortionless Response) [6] are quadratic type algorithms. Both the methods are highly dependent on physical size of array aperture, which results in poor resolution and accuracy, [5], [7], [9], [11], [12], [13]. Subspace based DOA estimation method is based on the Eigen decomposition [8]. The subspace based DOA estimation algorithm MUSIC provides high resolution, and is more accurate and not limited to physical size of array aperture [2] [7]. The various DOA algorithm performance is analyzed based on number of snapshots, number of users, user space distribution, number of array elements, SNR and MSE.

III. MUSIC

MUSIC means Multiple Signal Classification. It is one of the high resolution subspace DOA algorithms, which gives the estimation of number of signals arrived, hence their direction of arrival [6]. MUSIC deals with the decomposition of covariance matrix into two orthogonal matrices, i.e., signal-subspace and noise-subspace. Estimation of DOA is performed from one of these subspaces, assuming that noise in each channel is highly uncorrelated. This makes the covariance matrix diagonal.

Consider the model equation when M signals incident on array and corrupted by noise:

$$x = a_0 + \sum_{m=1}^{M}(\alpha_m \, s(\emptyset_m) + n) \qquad (1)$$

Using equation … (1)

$$x = S\alpha + n \qquad (2)$$

$$S = [s(\emptyset_1)\, s(\emptyset_2) \dots\dots\dots, s(\emptyset_M)] \qquad (3)$$

$$\alpha = [\alpha_1, \alpha_2 \dots\dots\dots \alpha_M]^T \qquad (4)$$

The matrix S is an N ×M order matrix of the M steering vectors. Assuming that the different signals are uncorrelated, the correlation matrix of x can be given by:

$$R = E\,[xx^H] \qquad (5)$$

$$= E\,[S\alpha\alpha^H S^H] + E\,[nn^H],$$

$$= SAS^H + \sigma^2 I, \qquad (6)$$

$$= R_s + \sigma^2 I, \qquad (7)$$

$$R_s = SAS^H \qquad (8)$$

$$\mathbf{A} = \begin{bmatrix} E\,[|\alpha_1|^2] & 0 & \dots & 0 \\ 0 & E\,[|\alpha_1|^2] & \dots & 0 \\ 0 & 0 & \dots & E\,[|\alpha_M|^2] \end{bmatrix} \qquad (9)$$

The signal covariance matrix, R_s, is clearly a N × N order matrix with rank M. Therefore it has $N-M$ eigenvectors corresponding to the zero Eigen value. Let q_m be such an eigenvector. So,

$$R_s q_m = SAS^H q_m = 0, \qquad (10)$$

$$\Rightarrow q_m^H SAS^H q_m = 0, \qquad (11)$$

$$\Rightarrow S^H q_m = 0 \qquad (12)$$

The validity of final equation holds as the matrix A is clearly positive definite. Equation (12) implies that all $N-M$ eigenvectors (q_m) of R_s corresponding to the zero eigenvalue are orthogonal to all M signal steering vectors. This is the very basis for MUSIC. Let Q_N be the N × (N −M) matrix of these eigenvectors. MUSIC plots the pseudo-spectrum can be given by;

$$\boldsymbol{P}_{\mathrm{MUSIC}}(\emptyset) = \frac{1}{\sum_{m=1}^{N-M} |S^H(\emptyset) q_m \alpha_1|^2}$$

$$= \frac{1}{S^H(\emptyset) Q_n Q_n^H s(\emptyset)}$$

$$= \frac{1}{\|Q_n^H s(\emptyset)\|^2} \qquad (13)$$

Since the eigenvectors making up Q_n are orthogonal to the signal steering vectors, the denominator becomes zero when $\emptyset$ is a signal direction. Therefore, the estimated signal directions are the M largest peaks in the pseudo-spectrum. However, in any practical situation, the signal covariance matrix R_s would not be available. The most we can expect is to be able to estimate R the signal covariance matrix. The key is that the eigenvectors in Q_n can be estimated from the eigenvectors of R.

For any eigenvector $q_m \in Q$

$$R_s q_m = \lambda q_m$$

$$R q_m = R_s q_m \sigma^2 I q_m,$$

$$= (\lambda_m + \sigma^2 q_m), \qquad (14)$$

i.e. any eigenvector of R_s is also an eigenvector of R with corresponding Eigen value $\lambda + \sigma^2$. Let $R_s = Q\Lambda Q^H$. Therefore, R=Q $[\Lambda + \sigma^2 I]Q^H$

$$= Q \begin{matrix} \lambda_m + \sigma^2 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \lambda_m + \sigma^2 & \cdots & 0 & \cdots 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_m + \sigma^2 \ddots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & \sigma^2 \end{matrix}$$

Based on this Eigen decomposition, the eigenvector matrix Q can be portioned into a signal matrix Q_s with M columns, corresponding to the M signal Eigen values, and a matrix Q_n, with (N − M) columns, corresponding the noise Eigen values (σ^2). Note that Q_n, the N × (N − M) matrix of eigenvectors corresponding to the noise Eigen value(σ^2), is exactly the same as the matrix of eigenvectors of Rs corresponding to the zero-Eigen value. This is the matrix used in Eqn. (13). Q_s Defines the signal subspace, while Q_n, defines the noise subspace.There are few important observations to be made:

The m^{th} signal Eigen value is given by

$\lambda_m + \sigma^2 = N|\alpha_m|^2 + \sigma^2$

The smallest Eigen values of R are the noise Eigen values and are all equal to σ^2, i.e., one way of distinguishing between the signal and noise Eigen values (equivalently the signal and noise subspaces) is to determine the number of small Eigen values that are equal.

By orthogonality of Q, $Q_s \perp Q_n$

Using the final two observations, we see that all noise eigenvectors are orthogonal to the signal steering vectors. This is the basis for MUSIC. Consider the following function of:

$$\boldsymbol{P}_{\text{MUSIC}}(\emptyset) = \frac{1}{\sum_{m=M+1}^{N} |q_n^H s(\emptyset)|^2} = \frac{1}{S^H(\emptyset) Q_n Q_n^H s(\emptyset)} \quad (15)$$

Where q_m is one of the $(N - M)$ noise eigenvectors. If $\emptyset$ is equal to DOA one of the signals, $s(\emptyset) \perp q_m$ and the denominator is identically zero. MUSIC, therefore, identifies as the directions of arrival, the peaks of the function $\boldsymbol{P}_{\text{MUSIC}}(\emptyset)$.

IV. MINIMUM VARIANCE DISTORTION LESS RESPONSE

MVDR means Minimum Variance Distortionless Response. In Figure-1 a uniform linear array (ULA) of N equally spaced sensors is shown. A number of plane waves from M narrowband sources impinging from different angles θ_i, i = 1, 2… M. At a particular instant of time t, t=1, 2… K, where K is the total number of snapshots taken, the array output will consist of the signal, and in addition to that noise components

The signal vector x (t) can be defined as different angles θ_i, i = 1, 2… …M.

$$x(t) = \sum_{m=1}^{M} a(\theta_m) s_m(t) \quad (16)$$

Where s (t) is an M×1 vector of source waveforms and for a particular source at direction θ from the array bore sight; a(θ) is an N×1 vector referred to as the array response to that source or array steering vector for that direction. It is given by:

$$a(\theta) = \left[e^{-j\emptyset} \dots \dots \dots . e^{-j(N-1)\emptyset}\right]^T \quad (17)$$

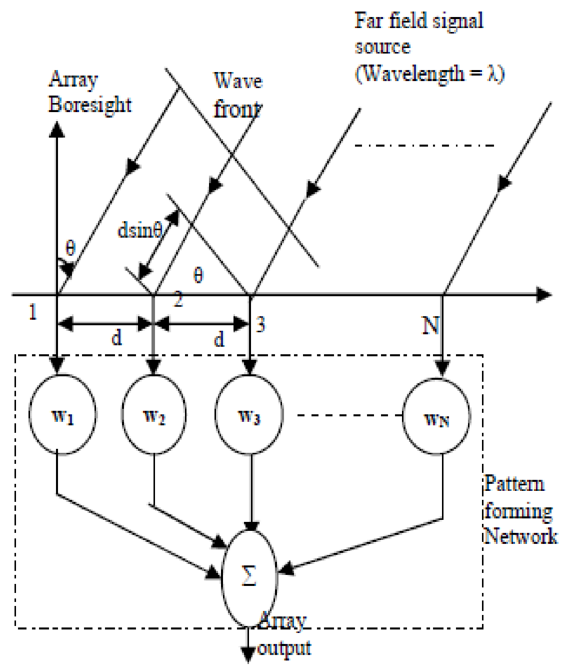

Fig.-1. Uniform Linear Array with M- Element

Where T is the transposition operator and Φ represents the electrical phase shift from element to element along the array. This can be defined by:

$$\emptyset = (2\pi/\lambda)\, d\cos\theta$$

Where d is the element spacing and λ is the wavelength of the received signal. The signal vector x (t) of size N×1 can be written as:

$$x(t) = A.s(t)$$

Where, $A = [a(\theta_1) \dots a(\theta_M)]$ is an N×M matrix of steering vectors. The array output consists of the signal plus noise components, and it can be defined as:

$$u(t) = x(t) + n(t)$$

Where $x(t)$ and $n(t)$ are signal and noise components.

If there are D signals incident on the array, the received input data vector at an M-element array can be expressed as a linear combination of the D incident waveforms and noise.

$$u = \sum_{n=1}^{D} a(\emptyset_1) S_1 + n = AS + n \quad (18)$$

Where A is the matrix of steering vectors,

$$A = [a(\emptyset_1)\, a(\emptyset_2) \dots \dots \dots a(\emptyset_D)]$$

Where $S = [S_1 \dots \dots S_D]$ is the signal vector, and $n = n_1 \dots \dots n_M]$ is a noise vector with components of varianceσ_n^2.

Now equation -18 can be written in the form of matrix of size N×K as:

$$U = A.S + N$$

Where S=[s (1) … s (K)] is an M×K matrix of source waveforms and N= [n (1)………. N (K)] is an N×K matrix of sensor noise. The spatial correlation matrix R of the observed signal vector u (t) can be defined as:

$$R = E[u(t).u(t)^H] \quad (19)$$

Where E and H are the expectation and conjugate transpose operators, respectively. The spatial correlation matrix R can now be express can now be expressed as:

$$R = E[A.s(t).s(t)^H.A^H] + E[n(t).n(t)^H] (20)$$

The peaks in the MVDR angular spectrum occur whenever the steering vector is orthogonal to the noise subspace. This technique minimizes the contribution of the undesired interferences by minimizing the output power while maintaining the gain along the look direction to be constant, usually unity. That is

$$min\ E\ [|y(\theta)|^2] = \min w^H R w, w^H A = 1 \quad (21)$$

Using Lagrange multiplier, the weight vector can be given by,

$$W = \frac{R^{-1}A}{R^{-1}A^H A} \quad (22)$$

The output power of the array as a function of the DOA estimation, using MVDR beamforming method, is given by MVDR spatial spectrum as,

$$P_{MVDR}(\theta) = \frac{R^{-1}A}{R^{-1}A^H A} \quad (23)$$

The angles of arrival are estimated by detecting the peaks in the angular spectrum.

V. SIMULATION RESULT

The MUSIC and MVDR techniques for DOA estimations are simulated using MATLAB. Performance of the algorithm has been analyzed by considering function of array elements, of SNR and as a function of snapshots. The simulation has been run for signals coming from different angles at 15°, 20°, 25°, 30° and 35° for 200 snapshots, power of incoming signal 15 dB, SNR of 50dB, and 8 array elements.

MUSIC algorithm estimates the number of incident signals on the array and their directions of arrival. The first step is the calculation of the covariance matrix. The next step involves calculation of the Eigen values and Eigen vectors. On knowing the minimum Eigen value the multiplicity of this Eigen value is obtained. The next step is to obtain the noise eigenvector matrix of these minimum Eigen values and form the Noise Eigen Vector. Using this Noise Eigen vector the direction of arrival can be plotted in a single direction or multiple dimensions.

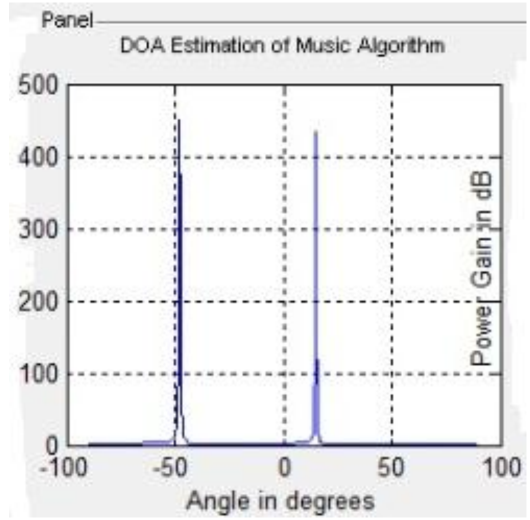

Fig.2 -DOA estimation at angle-15°

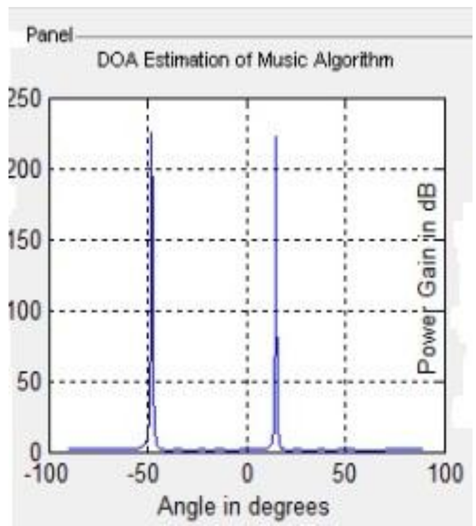

Fig.3 -DOA estimation at angle-20°

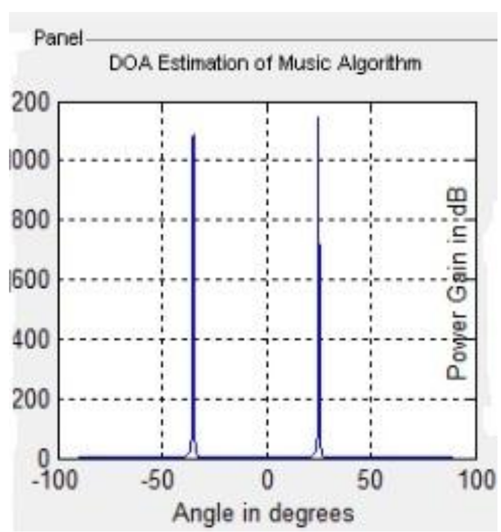

Fig.4 -DOA estimation at angle-25°

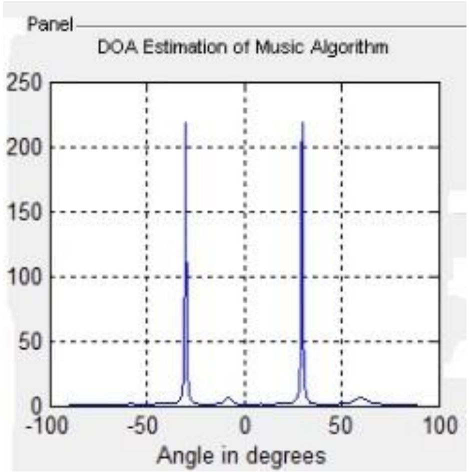

Fig.5 -DOA estimation at angle-30°

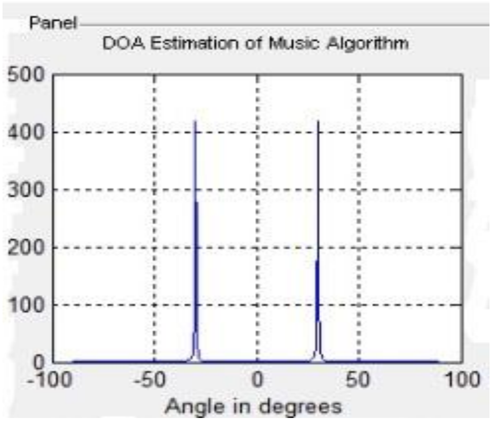

Fig.6 -DOA estimation at angle-35°

It has been observed that in MVDR spectrum there is a sharp peak in an angular spectrum and a lower noise floor compared to the MUSIC algorithm. In MUSIC spectrum sharper peak indicates the location of desired user where as in MVDR power plot maximum power at an angle indicates the location of desired user. Fig-2-11 shows the comparison of both the algorithms.

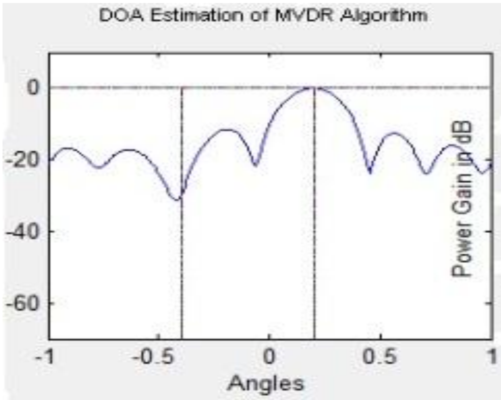

Fig.7 -DOA estimation at angle-15°

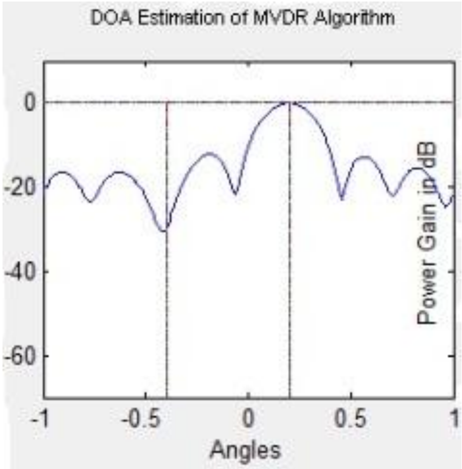

Fig.8 -DOA estimation at angle-20°

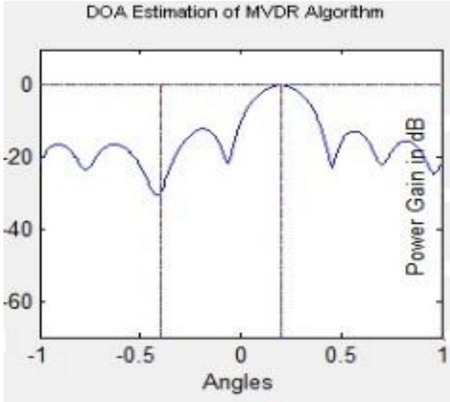

Fig.9 -DOA estimation at angle-25°

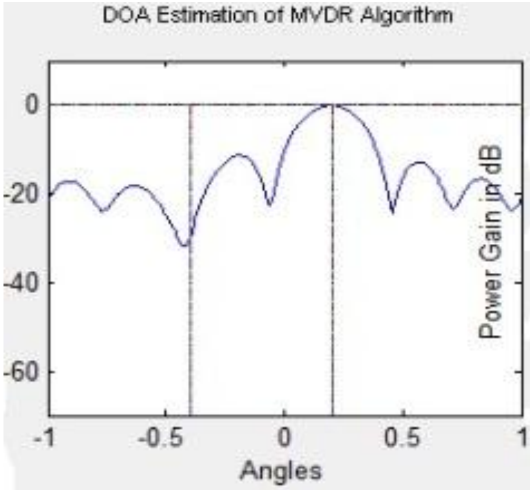

Fig.10 -DOA estimation at angle-30°

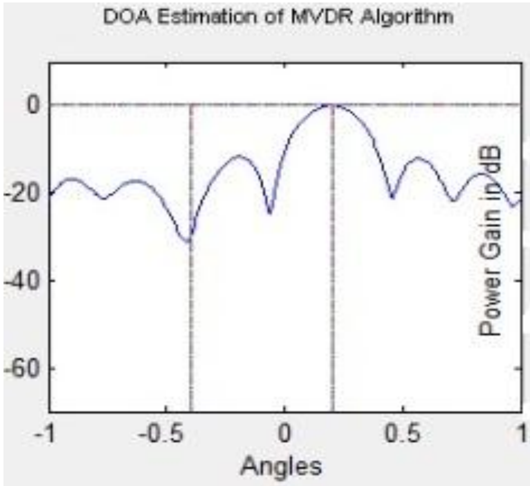

Fig.11 -DOA estimation at angle-35°

VI. CONCLUSION AND FUTURE WORK

This paper presents results of direction of arrival estimation using MUSIC and MVDR algorithms. The simulation results show that performance of MUSIC and MVDR improves on increasing the numbers of elements in array. These improvements are analyzed in the form of sharper peaks in spectrums and smaller errors in angle detection. Results indicate that as the number of snapshots increased, a decrement in MSE is observed. It results in accurate detection of closely spaced signals. . The simulation results show that, the advantage of performance of one algorithm over another varies with the conditions and is significantly influenced by both of the environment as well as the system. Thus, careful consideration is imperative to the conditions and system parameters specific to the planned deployment. In this context, this study proposes a new possibility of user separation through SDMA and can be widely used in the design of smart antenna system.

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