www.ijemr.net ISSN (ONLINE): 2250-0758, ISSN (PRINT): 2394-6962

Volume-7, Issue-3, May-June 2017

International Journal of Engineering and Management Research

Page Number: 210-215

R3BP with Stokes Drag Effect when the Smaller Primary is a Finite Straight Segment and Bigger One is an Oblate Spheroid

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ABSTRACT

In this paper, we study the effect of Stokes drag in the Restricted Three Body Problem (R3BP). It examines the existence and stability of non-collinear libration points $L_{4,5}$ in the restricted three body problem. The bigger primary is taken as oblate spheroid and smaller one as a finite straight segment. The linear stability of non-collinear libration points is also discussed. The non-collinear libration points are found to be unstable.

Keywords- Restricted three body problem, Libration points, Straight segment, Linear stability, Stokes drag, Oblate Spheroid

I. INTRODUCTION

The restricted three body problem is one of the most important problems in celestial mechanics. It is the simplest form of the general three body problem, in which a particle of infinitesimal mass moves in the gravitational field of two massive bodies (called the primaries). Lagrange showed that in this frame five libration points do exist, out of which three are collinear with the primaries and two forms equilateral triangle with the primaries (Szebehely [15]). Aggarwal et al. [1] discussed the non-linear stability of the triangular libration point L_4 of the R3BP under the presence of the third and fourth order resonances by taking bigger primary as an oblate body and the smaller one as a triaxial body and both are source of radiation. They found that L_4 is always unstable. The equilibrium solutions and linear stability of m_3 and m_4 considering one of the primaries as an oblate spheroid have been examined by Aggarwal and Kaur [2]. Jain and Aggarwal [3] determined the existence and stability of libration points in the restricted problem under the effect of Poynting Robertson Light Drag and conclude that both the non-collinear libration points are unstable. Jain and Aggarwal [4] have performed an analysis in the R3BP with Stokes drag effect by taking both primaries as the point masses and found that non-collinear stationary solutions are linearly unstable. By considering smaller primary as an oblate spheroid, the existence and stability of the non-collinear libration points with Stokes drag effect have been examined by Jain and Aggarwal [5]. They found that the non-collinear libration points are unstable. In a series of paper, Kumar et al. [6, 7, 8] discussed a lot of work about R3BP. Kumar et al. [9] studied existence and stability of libration points in the R3BP under the combined effects of finite straight segment and oblateness. They found that, there exist five libration points, out of which three are collinear and two are non-collinear with the primaries. The collinear libration points are unstable for all values of mass parameter μ, and the non-collinear libration points are stable if $\mu < \mu_c$, where $\mu_c = 0.038521\text{-}0.007356l^2\text{-}0.285002A$. Liou et al. [10] have examined the effects of radiation pressure, Poynting-Robertson drag and Solar wind drag on dust grains trapped in mean motion resonances with the Sun and Jupiter in the R3BP. They concluded that all dust grain orbits are unstable in time when P-R and solar wind drag are included in the Sun-Jupiter-dust system. Mishra et al. [11] examined the stability of triangular equilibrium points in photogravitational elliptic restricted three body problem with Poynting-Robertson drag by considering the smaller primary as an oblate spheroid and bigger primary as radiating. They concluded that the triangular equilibrium points remain unstable. The location and stability of equilibrium points in the planar circular restricted three body problem when the third body is acted on by a variety of drag forces have been investigated by Muray [12]. He found that L_4 and L_5 are asymptotically stable. Furthermore, the motion of a particle under the gravitational field of a massive straight segment has been studied by Riaguas et al. [13]. This model is used as an approximation to the gravitational field of irregular shaped bodies such as asteroids, nuclei and planets moons. Riaguas et al. [14] have studied the non-linear stability of the equilibria in the gravity field of the finite straight segment and determined the orbital stability of the equilibria, for all values of the parameter *k*. They concluded that there are no non-collinear equilibrium solutions of the system.

II. EQUATIONS OF MOTION

Let m_1 be the mass of oblate spheroid and m_2 be the mass of a finite straight segment (called primaries), both are moving with angular velocity *n* (say) in circular orbits about their common centre of mass *O*. There is an infinitesimal mass m_3 which is moving in the plane of motion of m_1 and m_2 ($m_1 \geq m_2$). *O*(*XYZ*) and *O*(*xyz*) are inertial and synodic coordinate system respectively. The line joining m_1 and m_2 is taken as *X*-axis and *O* their centre of mass as origin and the line passing through *O* and perpendicular to *OX* and lying in the plane of motion of m_1 and m_2 is taken as *Y*-axis. *O*(*xyz*) initially coincident with the inertial coordinate system *O*(*XYZ*). The synodic axes are rotating with angular velocity *n* (say) about *Z*-axis (the *z*-axis is coincident with *Z*-axis) (Fig. 1).

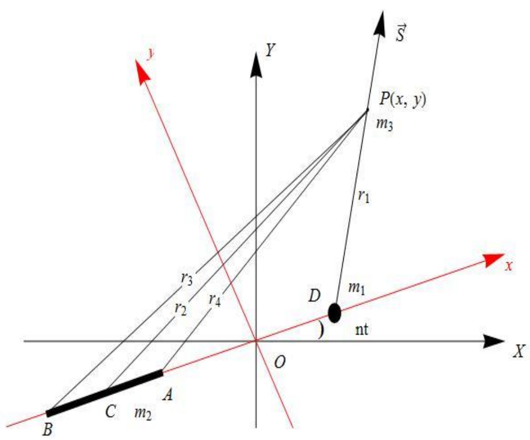

Fig.1.The configuration of R3BP with Stokes drag

The equations of motion of m_3 in the dimensionless synodic coordinate system are

$$\ddot{x} - 2n\dot{y} = \Omega_x - k(\dot{x} - y + \alpha S'_y),$$
$$\ddot{y} + 2n\dot{x} = \Omega_y - k(\dot{y} + x - \alpha S'_x), \qquad (1)$$

where

$$\Omega = \frac{1}{2}n^2\left(x^2 + y^2\right) + \frac{(1-\mu)}{r_1}\left(1 + \frac{A}{2r_1^2}\right) + \frac{\mu}{2l}\log\frac{(r_3 + r_4 + 2l)}{(r_3 + r_4 - 2l)},$$

$$\Omega_x = n^2 x - \frac{(1-\mu)(x-\mu)}{r_1^3}\left(1 + \frac{3A}{2r_1^2}\right) - \frac{2\mu(x-\mu+1)}{r_3 r_4 (r_3 + r_4)},$$

$$\Omega_y = n^2 y - \frac{(1-\mu)y}{r_1^3}\left(1 + \frac{3A}{2r_1^2}\right) - \frac{2\mu y(r_3 + r_4)}{r_3 r_4 \left\{(r_3 + r_4)^2 - 4l^2\right\}},$$

$$\mu = \frac{m_2}{m_1 + m_2} \leq \frac{1}{2}, \Rightarrow m_2 = \mu, m_1 = (1-\mu),$$

$$r_1^2 = (x-\mu)^2 + y^2, r_2^2 = (x-\mu+1)^2 + y^2,$$

$$r_3^2 = \{x - (\mu - 1 - l)\}^2 + y^2,\ r_4^2 = \{x - (\mu - 1 + l)\}^2 + y^2,$$

$$A = \frac{a^2 - c^2}{5}, n^2 = (1 + l^2 + \frac{3}{2}A), l << 1, 0 < A << 1,$$

$2l =$ dimensionless length of the straight segment *AB*,

$\vec{S} =$ Stokes drag Force acting on m_3 due to m_1 along m_1m_3.

The components of Stokes drag along the synodic axes are $S_x = k(\dot{x} - y + \alpha S'_y)$ and $S_y = k(\dot{y} + x - \alpha S'_x)$, where $k \in (0,1)$ is the dissipative constant depending on several physical parameters like the viscosity of the gas, the radius and mass of the particle. $S' = S'(r) = r^{-3/2}$, is the Keplerian angular velocity at distance $r = \sqrt{x^2 + y^2}$ from the origin *O* and $\alpha \in (0,1)$ is the ratio between the gas and Keplerian velocities. $\vec{r} = \overline{OP} = x\hat{i} + y\hat{j}, \vec{\omega} = nK =$ angular velocity of the axes *O(x, y)*=constant. The Stokes drag effect is of the order of $k = 10^{-5}, \alpha = 0.05$ (generally $k \in (0,1)$ and $\alpha \in (0,1)$ as stated above).

III. LIBRATION POINTS

The libration points are the solution of the equations

$$n^2 x - \frac{(1-\mu)(x-\mu)}{r_1^3}\left(1 + \frac{A}{2r_1^2}\right) - \frac{2\mu\{x - (\mu - 1)\}}{r_3 r_4 (r_3 + r_4)} + k\left[y + \frac{3}{2}\alpha\left(x^2 + y^2\right)^{-7/4} y\right] = 0, \qquad (2)$$

$$n^2 y - \frac{(1-\mu)y}{r_1^3}\left(1 + \frac{A}{2r_1^2}\right) - \frac{2\mu y(r_3 + r_4)}{r_3 r_4 \left\{(r_3 + r_4)^2 - 4l^2\right\}} - k\left[x + \frac{3}{2}\alpha\left(x^2 + y^2\right)^{-7/4} x\right] = 0. \qquad (3)$$

If we take *k=0*, then results agree with Kumar et al. [9]. The non-collinear libration points when smaller primary is a finite straight segment are given by Kumar et al. [9]

$$x_0 = \left(\mu - \frac{1}{2}\right) + \frac{(\mu+3)}{24(\mu-1)} l^2 + \frac{1}{2} A,$$

$$y_0 = \pm\left(\frac{\sqrt{3}}{2} + \frac{(19-23\mu)}{24\sqrt{3}(\mu-1)} l^2 - \frac{1}{2\sqrt{3}} A\right). \qquad (4)$$

Now, we assume that the solution of the Eqs. (2) and (3) when $k \neq 0$ and $y \neq 0$

$$x' = x_0 + \alpha_1,\; y' = y_0 + \alpha_2,\; \alpha_1, \alpha_2 << 1. \qquad (5)$$

Putting these values of x', y' in Eqs. (2) and (3) and applying Taylor's series, we get

$$\alpha_1\left[1 + (1-\mu)a_1 + \mu\left\{\frac{2(x_0-\mu+1)^2}{a_2 r'_3 r'_4} a_3 - \frac{2}{a_2 r'_3 r'_4}\right\}\right]$$
$$+\alpha_2\left[\frac{3(x_0-\mu)(1-\mu)y_0}{r'^{5/2}_1} + \mu\left\{\frac{2(x_0-\mu+1)y_0}{a_2 r'_3 r'_4} a_3\right\}\right]$$
$$+A\left(\frac{3y_0 b_1 \mu \alpha_2}{r'^{7/2}_1} + \frac{9}{2}\frac{b_1^2 \mu \alpha_1}{r'^{7/2}_1} + \frac{3\mu b_1^2 \alpha_1}{r'^{7/2}_1} + \frac{3}{2}\alpha_1\right.$$
$$\left.+\frac{9}{2}\frac{\mu y_0 b_1 \alpha_2}{r'^{7/2}_1} - \frac{3}{2}\frac{\mu b_1}{r'^{7/2}_1} - \frac{3\mu\alpha_1}{2r'^{7/2}_1}\right)$$
$$+k\left[y_0 + \frac{3}{2}\alpha\left(x_0^2 + y_0^2\right)^{-7/4} y_0\right] = 0, \qquad (6)$$

and

$$\alpha_1\left[(1-\mu)\left\{\frac{3y_0(x_0-\mu)}{r'^{5/2}_1}\right\} + \mu\left\{\frac{-2y_0 a_4}{r'_3 r'_4 a_5} + \frac{4y_0 a_2^2}{r'_3 r'_4 a_5} a_4\right.\right.$$
$$\left.\left.+\frac{2y_0\{x_0-(\mu-1+l)\}a_2}{r'_3 (r'_4)^{3/2} a_5} + \frac{2y_0\{x_0-(\mu-1-l)\}a_2}{(r'_3)^{3/2} r'_4 a_5}\right\}\right]$$
$$+\alpha_2\left[1 + (1-\mu)a_6 + \mu\left\{\frac{2y_0^2 a_2}{(r'_3)^{3/2} r'_4 a_5} + \frac{2y_0^2 a_2}{r'_3 (r'_4)^{3/2} a_5}\right.\right.$$
$$\left.\left.-\frac{2y_0 a_7}{r'_3 r'_4 a_5} - \frac{2a_2}{r'_3 r'_4 a_5} + \frac{4y_0 a_2^2 a_7}{r'_3 r'_4 a_5}\right\}\right]$$
$$+A\left(\frac{3y_0 b_1 \mu \alpha_1}{r'^{7/2}_1} + \frac{9}{2}\frac{y_0 b_1^2 \mu \alpha_1}{r'^{7/2}_1} + \frac{3\mu y_0^2 \alpha_2}{r'^{7/2}_1} + \frac{3}{2}\alpha_2\right.$$
$$\left.+\frac{9}{2}\frac{\mu y_0^2 \alpha_2}{r'^{7/2}_1} - \frac{3}{2}\frac{\mu\alpha_2}{r'^{5/2}_1} - \frac{3\mu y_0}{2r'^{7/2}_1}\right)$$
$$-k\left[x_0 + \frac{3}{2}\alpha\left(x_0^2 + y_0^2\right)^{-7/4} x_0\right] = 0, \qquad (7)$$

where

$$r'_1 = \sqrt{(x_0-\mu)^2 + y_0^2},\; r'_3 = \sqrt{\{x_0-(\mu-1-l)\}^2 + y_0^2},$$
$$r'_4 = \sqrt{\{x_0-(\mu-1+l)\}^2 + y_0^2}.$$

$$a_1 = \frac{3(x_0-\mu)^2}{r'^{5/2}_1} - \frac{1}{r'^{3/2}_1},\; a_2 = r'_3 + r'_4,$$

$$a_3 = \frac{1}{r'_3 r'_4} + \frac{1}{r'^{1/2}_4} + \frac{1}{r'^{1/2}_1},$$

$$a_4 = \frac{x_0-(\mu-1+l)}{r'_4} + \frac{x_0-(\mu-1-l)}{r'_3},$$

$$a_5 = a_2^2 - 4l^2,\; a_6 = \frac{3y_0^2}{r'^{5/2}_1} - \frac{1}{r'^{3/2}_1},$$

$$a_7 = \frac{y_0}{r'_4} + \frac{y_0}{r'3},\; b_1 = x_0 - \mu + 1.$$

Substituting the values of x_0 and y_0 in Eqs. (6) and (7), we get

$$\alpha_1 = \frac{1}{48\sqrt{3}}(1-60\alpha)k - \frac{1}{2\sqrt{3}A}k - \frac{\sqrt{3}k\alpha}{2A}$$
$$+\frac{1}{144\sqrt{3}}(189\alpha - 16)k l^2,$$

$$\alpha_2 = \left(\frac{1}{144} - \frac{5}{12}\alpha\right)k + \frac{k}{6A} + \frac{\alpha k}{2A} + \frac{\mu k}{6A}$$
$$+\left(-\frac{1}{27} - \frac{11}{16}\alpha\right)k l^2.$$

Thus, the location of non-collinear libration points $L_4(x', y')$ and $L_5(x', y')$ in our case are

$$x' = \left(\mu - \frac{1}{2}\right) + \frac{(\mu+3)}{24(\mu-1)} l^2 + \frac{1}{2A} + \frac{1}{48\sqrt{3}}(1-60\alpha)k$$
$$+\frac{1}{144\sqrt{3}}(189\alpha-16)k l^2 - \frac{1}{2\sqrt{3}A}k - \frac{\sqrt{3}k\alpha}{2A},$$

$$y' = \pm\left(\frac{\sqrt{3}}{2} + \frac{(19-23\mu)}{24\sqrt{3}(\mu-1)} l^2 - \frac{1}{2\sqrt{3}} A\right) + \left(\frac{1}{144} - \frac{5}{12}\alpha\right)k$$
$$+\left(-\frac{1}{27} - \frac{11}{16}\alpha\right)k l^2 + \frac{k}{6A} + \frac{\alpha k}{2A} + \frac{\mu k}{6A}.$$

IV. STABILITY OF NON-COLLINEAR LIBRATION POINTS

Following the procedure of Jain and Aggarwal [5], we get the characteristic equation

$$\lambda^4 - \left(k_{x',\dot{x}'} + k_{y',\dot{y}'}\right)\lambda^3 + [2(e-n^2) - f - h - k_{x',x'} + 2n$$
$$-k_{y',y'} + p + i + 2\left(k_{x',\dot{y}'} - k_{y',\dot{x}'}\right) + p_3 + p_1 + 4$$
$$-\left(k_{x',\dot{y}'}k_{y',\dot{x}'} + k_{x',\dot{x}'}k_{y',\dot{y}'}\right)]\lambda^2 + \left[\left(n^2 - e + f\right)k_{x',\dot{x}'}\right.$$
$$+(h-e-i)k_{y',\dot{y}'} + 2n\left(k_{x',y'} + k_{y',x'}\right) + n^2 k_{y',\dot{y}'} - pk_{x',\dot{x}'}$$
$$+ j\left(2n - k_{y',\dot{x}'}\right) + m\left(2n + k_{x',\dot{y}'}\right) + p_2\left(k_{x',\dot{y}'} + k_{y',\dot{x}'}\right) - p_3 k_{x',\dot{x}'}$$
$$\left.+\left(k_{x',\dot{x}'} + k_{y',y'} + k_{x',x'}k_{y',\dot{y}'}\right) - \left(k_{x',\dot{y}'}k_{y',x'} + k_{y',\dot{x}'} + k_{x',y'}\right)\right]\lambda$$
$$+\left[\left(e-h-n^2\right)\left(e-f-n^2\right) - g^2 + \left(n^2-e+f\right)k_{x',x'}\right.$$
$$+\left(n^2-e+h\right)k_{y',y'} - g\left(k_{x',y'}+k_{y',x'}\right) + p\left(e-h+i-n^2\right)$$
$$+i\left(e-f-n^2\right) - pk_{x',x'} - ik_{y',y'} + m\left(g+j+k_{x',y'}\right)$$
$$+j\left(k_{y',x'} - g\right) + p_3\left(e-h+p_1-n^2\right) + p_1\left(e-f-n^2\right)$$
$$+p_2\left(k_{x',y'}+k_{y',x'}\right) - p_3k_{x',x'} - p_1k_{y',y'}$$
$$\left.+\left(k_{x',x'}k_{y',y'} - k_{x',y'}k_{y',x'}\right)\right] = 0,$$

where the values of *e, f, g, h, i, j, m, p* and k_i*'s* are given in appendix.

In general form, the above equation can be written as

$$\lambda^4 + \sigma_3\lambda^3 + (\sigma_{20}+\sigma_2)\lambda^2 + \sigma_1\lambda + (\sigma_{00}+\sigma_0) = 0,$$

where $\sigma_{00} = \left(e-h-n^2\right)\left(e-f-n^2\right) - g^2,$

$$\sigma_0 = \left(n^2-e+f\right)k_{x',x'} + \left(n^2-e+h\right)k_{y',y'}$$
$$-g\left(k_{x',y'}+k_{y',x'}\right) + p\left(e-h+i-n^2\right)$$
$$+i\left(e-f-n^2\right) - pk_{x',x'} - ik_{y',y'} + m\left(g+j+k_{x',y'}\right)$$
$$+j\left(k_{y',x'}-g\right),$$
$$\sigma_1 = \left(n^2-e+f\right)k_{x',\dot{x}'} + (h-e-i)k_{y',\dot{y}'}$$
$$+2n\left(k_{x',y'}+k_{y',x'}\right) + n^2k_{y',\dot{y}'} - pk_{x',\dot{x}'} + j\left(2n-k_{y',\dot{x}'}\right)$$
$$+m\left(2n+k_{x',\dot{y}'}\right),$$
$$\sigma_{20} = 2(e-n^2) - f - h,$$
$$\sigma_2 = -k_{x',x'} + 2n - k_{y',y'} + p + i,$$
$$\sigma_3 = -\left(k_{x',\dot{x}'} + k_{y',\dot{y}'}\right),$$

Here σ_{00}, σ_{20} and $\sigma_i\,(i=0,1,2,3)$ can be derived by evaluating *e, f, g, h, i, j, m* and *p* defined in the appendix. The value of the coefficient in the zero drag case is denoted by adding additional subscript 0. Now, we have

$$\sigma_{00} = 3\mu + \frac{15}{2}\mu l^2 + \left(\frac{-7}{96\sqrt{3}} - \frac{173\alpha}{16\sqrt{3}}\right)l^2k$$
$$+\left(\frac{43}{288\sqrt{3}} + \frac{1271\alpha}{48\sqrt{3}}\right)\mu l^2k + \frac{9}{2}A - 9\mu A + \frac{3\sqrt{3}}{4}k,$$
$$\sigma_{20} = -3 + \mu + \left(-3+\frac{3}{2}\mu\right)l^2 + \left(\frac{-7}{288\sqrt{3}} - \frac{173\alpha}{48\sqrt{3}}\right)l^2k$$
$$+\left(\frac{5}{288\sqrt{3}} + \frac{193\alpha}{48\sqrt{3}}\right)\mu l^2k - \frac{\sqrt{3}}{A}k + \frac{2}{\sqrt{3}}k,$$
$$\sigma_0 = \frac{33}{16}\mu + \frac{-9}{8}\mu l + \frac{63}{16}\mu l^2 + \left(\frac{-3\sqrt{3}}{2} - \frac{63\sqrt{3}\alpha}{32}\right)k$$
$$+\left(\frac{313\sqrt{3}}{128} + \frac{1113\sqrt{3}\alpha}{128}\right)\mu k + \left(\frac{-169}{128\sqrt{3}} - \frac{13\sqrt{3}\alpha}{8}\right)\mu lk$$
$$+\left(\frac{-25}{4\sqrt{3}} - \frac{469\sqrt{3}\alpha}{128}\right)l^2k + \left(\frac{25945}{2304\sqrt{3}} + \frac{88763\alpha}{1537\sqrt{3}}\right)\mu l^2k$$
$$+\frac{27}{16}A + \frac{9}{4}\mu A + \frac{3\sqrt{3}}{8}k,$$
$$\sigma_1 = \sqrt{3}\mu + \sqrt{3}\mu l + \frac{5}{12\sqrt{3}}\mu l^2 + \left(-1+\frac{21\alpha}{4}\right)k$$
$$+\left(\frac{115}{144} + \frac{1337\alpha}{48}\right)\mu k + \left(\frac{-19}{144} + \frac{95\alpha}{12}\right)\mu lk$$
$$+\left(1+\frac{21}{8}\alpha\right)l^2k + \left(\frac{-35}{36} + \frac{22913\alpha}{576}\right)\mu l^2k$$
$$-\frac{3}{2}Ak + \frac{33}{8}\mu Ak,$$
$$\sigma_2 = -\frac{1}{4}\mu - \frac{1}{6}\mu l^2 + \frac{21\sqrt{3}\alpha}{8}k + \left(\frac{1}{48\sqrt{3}} + \frac{149\alpha}{32\sqrt{3}}\right)\mu k$$
$$+\frac{49}{4\sqrt{3}}\alpha l^2k + \left(\frac{-341}{3456\sqrt{3}} + \frac{30469\alpha}{1152\sqrt{3}}\right)\mu l^2k - \frac{3}{4}A - \frac{15}{4}\mu A,$$
$$\sigma_3 = 2k.$$

To check the stability of non-collinear libration points, we have followed the procedure of Jain and Aggarwal [5]. We have found that in our case $\sigma_1 = -k$ and $\sigma_3 = -2k$ and therefore, $\sigma_1 > \sigma_3$ and hence, $L_{4,5}$ is not asymptotically stable. Further, one of the roots of λ has positive real root. Hence $L_{4,5}$ is not stable. Hence we conclude that $L_{4,5}$ are linearly unstable.

V. CONCLUSION

The existence and stability of the non-collinear libration points in the restricted three body problem under the combined effects of Stokes drag, straight segment and oblateness has been investigated. There exist two non-collinear libration points with the primaries. Using the terminology of Murray [12], we have examined the stability of non-collinear libration points and found that in our case $\sigma_1 = -k$ and $\sigma_3 = -2k$ which proves that $L_{4,5}$ are linearly unstable.

If we take *k=0* and *l=0*, *A=0*, our results confirm with the classical restricted three body problem (Szebehely [15]). If *l=0* and $k \neq 0, A \neq 0$, then the results confirm with Jain and Aggarwal [5]. If we consider *k=0*, and $l \neq 0, A \neq 0$, then the results agree with Kumar et al. [9].

Appendix

$$e=\frac{1-\mu}{r'^3_1},\ f=\frac{3(1-\mu)}{r'^5_1}y'^2,$$

$$g=\frac{3(1-\mu)(x'-\mu)}{r'^5_1}y',\ h=\frac{3(1-\mu)(x'-\mu)^2}{r'^5_1},$$

$$i=\frac{2\mu}{r'_3 r'_4 a_2}\left\{1-\frac{a_8}{a_2}-a_9-a_{10}\right\},$$

$$j=\frac{2\mu(x'-\mu+1)y'}{r'_3 r'_4 a_2}\left\{\frac{1}{r'^2_3}+\frac{1}{r'^2_4}+\frac{1}{r'_3 r'_4}\right\},$$

$$m=\frac{2\mu y'}{r'_3 r'_4 a_5}\left(\frac{a_{11}}{r'_4}+\frac{a_{12}}{r'_3}\right)\left(1-\frac{2a_2^2}{a_5}\right)$$

$$-\frac{2\mu y' a_2}{r'_3 r'_4 a_5}\left(\frac{a_{11}}{r'^2_4}+\frac{a_{12}}{r'^2_3}\right),$$

$$p=\frac{2\mu a_2}{r'_3 r'_4 a_5}-\frac{2\mu y'^2 a_2}{r'_3 r'_4 a_5}\left(\frac{1}{r'^2_3}+\frac{1}{r'^2_4}\right)$$

$$+\frac{2\mu y'}{r'_3 r'_4 a_5}\left(\frac{y'}{r'_3}+\frac{y'}{r'_4}\right)\left\{1-\frac{2a_2}{a_5}\right\},$$

$$p_1=3A\left(\frac{\mu}{2r'^5_1}-\frac{3\mu(x'-\mu+1)^2}{2r'^7_1}-\frac{(x'-\mu+1)^2}{r'^7_1}\right),$$

$$p_2=3A\left(\frac{3\mu(x'-\mu+1)}{2r'^7_1}+\frac{\mu(x'-\mu+1)}{r'^7_1}\right)y',$$

$$p_3=3A\left(\frac{\mu}{2r'^5_1}-\frac{3\mu y'^2}{2r'^7_1}-\frac{\mu y'^2}{r'^7_1}\right),$$

$$a_8=\frac{(x'-\mu+1)\{x'-(\mu-1+l)\}}{r'_4}+\frac{x'-(\mu-1-l)}{r'_3},$$

$$a_9=\frac{(x'-\mu+1)\{x'-(\mu-1+l)\}}{r'^2_4},$$

$$a_{10}=\frac{(x'-\mu+1)\{x'-(\mu-1-l)\}}{r'^2_3},$$

$$a_{11}=\{x'-(\mu-1+l)\},\ a_{12}=\{x'-(\mu-1-l)\},$$

$$k_{x',x'}=-\frac{21}{4}\alpha\left(x'^2+y'^2\right)^{-11/4}x'y'k,\ k_{x',\dot{x}'}=-k,$$

$$k_{x',y'}=k-\frac{21}{4}\alpha\left(x'^2+y'^2\right)^{-11/4}y'^2k,\ k_{x',\dot{y}'}=0,$$

$$k_{y',x'}=-k-\frac{21}{4}\alpha\left(x'^2+y'^2\right)^{-11/4}x'^2k,\ k_{y',\dot{x}'}=0,$$

$$k_{y',y'}=-\frac{21}{4}\alpha\left(x'^2+y'^2\right)^{-11/4}x'y'k,\ k_{y',\dot{y}'}=-k.$$

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