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Fair Restrained Dominating Set in the Corona of Graphs

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ABSTRACT

In this paper, we give the characterization of a fair restrained dominating set in the corona of two nontrivial connected graphs and give some important results.

Keywords— Fair Dominating Set, Restrained Dominating Set, Fair Restrained Dominating Set, Corona, Cartesian Product

I. INTRODUCTION

A graph G is a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set called the vertex-set of G and $E(G)$ is a set of unordered pairs $\{u, v\}$ (or simply uv) of distinct elements from $V(G)$ called the edge-set of G. The elements of $V(G)$ are called vertices and the cardinality $|V(G)|$ of $V(G)$ is the order of G. The elements of $E(G)$ are called edges and the cardinality $|E(G)|$ of $E(G)$ is the size of G. If $|V(G)| = 1$, then G is called a trivial graph. If $E(G) = \emptyset$, then G is called an empty graph. The open neighborhood of a vertex $v \in V(G)$ is the set $N_G(v) = \{u \in V(G): uv \in E(G)\}$. The elements of $N_G(v)$ are called neighbors of v. The closed neighborhood of $v \in V(G)$ is the set $N_G[v] = N_G(v) \cup \{v\}$. If $X \subseteq V(G)$, the open neighborhood of X in G is the set $N_G(X) = \cup_{v \in X} N_G(v)$. The closed neighborhood of X in G is the set $N_G[X] = \cup_{v \in X} N_G[v] = N_G(X) \cup X$. When no confusion arises, $N_G\,[x]$ [resp. $N_G(x)$] will be denoted by $N[x]$ [resp. $N(x)$]. For the general terminology in graph theory, readers may refer to [1].

Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [2]. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$, i.e. , $N[S] = V(G)$. The domination number $\gamma(G)$ of G is the smallest cardinality of a dominating set of G. Related studies on domination in graphs were found in the papers [3,4,5,6,7,8,9,10,11,12].

In 2011, Caro, Hansberg and Henning [13] introduced fair domination and k-fair domination in graphs. A dominating subset S of $V(G)$ is a fair dominating set in G if all the vertices not in S are dominated by the same number of vertices from S, that is, $|N(u) \cap S| = |N(v) \cap S|$ for every two distinct vertices u and v from $V(G) \setminus S$ and a subset S of $V(G)$ is a k-fair dominating set in G if for every vertex $v \in V(G) \setminus S$, $|N(v) \cap S| = k$. The minimum cardinality of a fair dominating set of G, denoted by $\gamma_{fd}(G)$, is called the fair domination number of G. A fair dominating set of cardinality $\gamma_{fd}(G)$ is called γ_{fd} -set. Related studies on fair domination in graphs were found in the papers [14,15,16,17,18].

The restrained domination in graphs was introduced by Telle and Proskurowski [19] indirectly as a vertex partitioning problem. Accordingly, a set $S \subseteq V(G)$ is a restrained dominating set if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G) \setminus S$. Alternately, a subset S of $V(G)$ is a restrained dominating set if $N[S] = V(G)$ and $\langle V(G) \setminus S \rangle$ is a subgraph without isolated vertices. The minimum cardinality of a restrained dominating set of G, denoted by $\gamma_r(G)$, is called the restrained domination number of G. A restrained dominating set of cardinality $\gamma_r(G)$ is called γ_r -set. Restrained domination in graphs was also found in the papers [20,21,22,23,24,25,26,27].

In this paper, we extend the study of fair restrained dominating set by giving the characterization of a fair restrained dominating set in the corona of two nontrivial connected graphs and give some important results. A fair dominating set $S \subseteq V(G)$ is a fair restrained dominating set if every vertex not in S is adjacent to a vertex in S and to a vertex in $V(G) \setminus S$. The minimum cardinality of a fair restrained dominating set of G, denoted by $\gamma_{frd}(G)$, is called the fair restrained domination number of G. A fair restrained dominating set of cardinality $\gamma_{frd}(G)$ is called γ_{frd}-set.

II. RESULTS

Remark 2.1 *[13] If $G \neq \overline{K}_n$, then $\gamma_{fd}(G) = min\{\gamma_{kfd}(G)\}$, where the minimum is taken over all integers k where $1 \leq k \leq |V(G)| - 1$.*

Consider the graph $G \cong P_1 + \overline{K}_n$, where $n \geq 2$ and $V(P_1) = \{x\}$ (See Figure 1). The set $S = V(P_1)$ is a mini-

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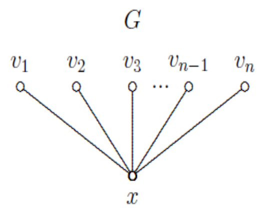

Figure 1: $G \cong P_1 + \overline{K_n}$, where $n \geq 2$ and $V(P_1) = \{x\}$

mum fair dominating set of G but not a restrained dominating set. Clearly, for any subset S of $V(G)$, S is not a restrained dominating set of G. Hence, S is not a fair restrained dominating set of G.

Since $\gamma_{frd}(G)$ does not always exists in a connected nontrivial graph G, we denote $\mathcal{FR}(G)$ a family of all graphs with fair restrained dominating set. Thus, for the purpose of this study, it is assumed that all connected nontrivial graphs considered belong to the family $\mathcal{FR}(G)$.

Remark 2.2 *Let G be a connected graph. Then for any positive integer n, $P_1 + \overline{K}_n \notin \mathcal{FR}(G)$.*

From the definition of a fair restrained domination number $\gamma_{frd}(G)$ of G, the following result is immediate.

Remark 2.3 *[14] Let G be any connected graph of order $n \geq 3$. Then*
(i) $1 \leq \gamma_{frd}(G) \leq n - 2$, and
(ii) $\gamma(G) \leq \gamma_r(G) \leq \gamma_{frd}(G)$.

We need a corollary for the following known theorems.

Theorem 2.4 *[14] Let G be a connected graph of order $n \geq 3$. Then $\gamma_{frd}(G) = 1$ if and only if $G \cong K_1 + H$ where H is a graph without isolated vertices.*

Theorem 2.5 *[14] Let G and H be nontrivial connected graphs. Then a nonempty proper subset S of $V(G + H)$ is a fair restrained dominating set of $G + H$ if and only if one of the following statement is satisfied:*
(i) $S = V(G)$ or S is a k-fair dominating set of G where $k = |S|$.
(ii) $S = V(H)$ or S is a k-fair dominating set of H where $k = |S|$.
(iii) $S = S_G \cup S_H$ where $S_G \subset V(G)$ is an r-fair dominating set of G, and $S_H \subset V(H)$ is an s-fair dominating set of H, and $r - s = |S_G| - |S_H|$.

The next result is an immediate consequence of Theorem 2.4 and Theorem 2.5.

Corollary 2.6 *Let $G \cong K_1 + H$ where H is a nontrivial connected graph of order $m \geq 3$. Then a nonempty subset S of $V(G)$ is a fair restrained dominating set if and only if one of the following is satisfied:*
(i) $S = V(K_1)$.
(ii) $S = V(K_1) \cup S_v$ where S_v is fair restrained dominating set of H.
(iii) $S = S_v$ where S_v is an $|S_v|$-fair dominating set of H.

Proof: Suppose that a nonempty subset S of $V(G)$ is a fair restrained dominating set of $G \cong K_1 + H$ where H is a nontrivial connected graph. Consider the following cases:

Case 1. Let $|S| = 1$. Then $\gamma_{frd}(G) = 1$ by Theorem 2.4. If $S = V(K_1)$ then we are done with the proof of statement (i). Suppose that $S \neq V(K_1)$. Then $S \subset V(H)$. Since S is a dominating of order 1, it follows that S is a 1-fair dominating set of H. Set $S = S_v$ where $|S_v|$-fair dominating of H. This proves statement (iii) where $|S_v| = 1$.

Case 2. Let $|S| \neq 1$. If $V(K_1) \subset S$, then let $S = V(K_1) \cup S_v$ where $S_v \subset V(H)$. Suppose that S_v is not a dominating set of H. Let $H \cong P_3 = [x_1, x_2, x_3]$ with $S_v = \{x_1\}$. Clearly, $S = V(K_1) \cup S_v$ is a restrained dominating set of G but not a fair dominating set of G since

$$|N_G(x_2) \cap S| = |S| = 2 \neq 1 = |V(K_1)| = |N_G(x_3) \cap S|.$$

This contradict to our assumption that S is fair restrained dominating set of S. Thus, S_v must be a dominating set of H. If S_v is not a fair dominating set of H, then $S = V(K_1) \cup S_v$ is not a fair dominating set of G is clear. By contrapositive, S_v is a fair dominating set of H. Similarly, S_v must be a restrained dominating set of H. Hence, S_v is a fair restrained dominating set of H. This proves statement (ii). If $V(K_1) \nsubseteq S$, then $S \subset V(H)$. Thus, S is a k-fair dominating set of H where $k = |S|$ by Theorem 2.5(ii). Hence, $S = S_v$ where S_v is an $|S_v|$-fair dominating set of H proving statement (iii).

For the converse, suppose that statement (i) is satisfied. Then $S = V(K_1)$ is a minimum fair restrained dominating set of $G \cong K_1 + H$ by Theorem 2.4. Hence, S is a fair restrained dominating set of G. Suppose that statement (ii) is satisfied. Then $S = V(K_1) \cup S_v$ where S_v is fair dominating set of H. Clearly, S is a dominating set of G. Since S_v is fair dominating set of H, $|N_H(x) \cap S_v| = |N_H(y) \cap S_v)|$ for every distinct vertices x and y in $V(H) \setminus S_v$. Thus, for every distinct vertices x and y in $V(G) \setminus S$,

$$\begin{aligned} |N_G(x) \cap S| &= |\big(V(K_1) \cup N_H(x)\big) \cap (V(K_1) \cup S_v)| \\ &= |V(K_1) \cup (N_H(x) \cap S_v)| \\ &= |V(K_1) \cup (N_H(y) \cap S_v)| \\ &= |\big(V(K_1) \cup N_H(y)\big) \cap \ (V(K_1) \cup S_v)| \\ &= |N_G(y) \cap \ S|. \end{aligned}$$

Thus, S is a fair dominating set of G. Let $x \in V(H) \setminus S_v$. Since S_v is a restrained dominating set of H and $|V(H)| \geq 3$, there exists $y \in V(H) \setminus S_v$ such that $xy \in E(H)$ and $xv \in E(H)$ for some $v \in S_v$. Thus, for every $x \in V(H) \setminus S_v \subset V(G) \setminus S$, there exists $y \in V(G) \setminus S$ such that $xy \in E(H) \subset E(G)$ and $xv \in E(G)$ for some $v \in S_v \subset S$. Hence, S is a restrained dominating set of G. Accordingly, S is a fair restrained dominating set of G. Suppose that statement (iii) is satisfied. Then $S = S_v$ where S_v is an $|S_v|$-fair dominating set of H. This means

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that S is a k-fair dominating set of H where $k = |S|$. By Theorem 2.5, S is a fair restrained dominating set of G. ■

We need the following definition for the characterization of a fair restrained dominating set in the corona of two graphs.

Definition 2.7 *Let G and H be graphs of order m and n, respectively. The corona of two graphs G and H is the graph $G \circ H$ obtained by taking one copy of G and m copies of H, and then joining the ith vertex of G to every vertex of the ith copy of H. The join of vertex v of G and a copy H^v of H in the corona of G and H is denoted by $v + H^v$.*

Remark 2.8 *Let G and H be nontrivial connected graphs. Then $\gamma(G \circ H) = |V(H)|$.*

Theorem 2.9 *Let G and H be nontrivial connected graphs. Then a nonempty subset S of $V(G \circ H)$ is a fair restrained dominating set if and only if one of the following is satisfied:*

(i) $S = V(G)$.

(ii) $S = V(G) \cup (\cup_{v \in X} S_v) \cup (\cup_{x \in V(G) \setminus X} V(H^x))$ where $\emptyset \neq X \subseteq V(G)$ and S_v is fair restrained dominating set of H^v for all $v \in X$ with $|V(H)| \geq 3$.

(iii) $S = \cup_{v \in V(G)} S_v$ where $S_v = V(H^v)$ or S_v is an $|S_v|$-fair dominating set of H^v for all $v \in V(G)$.

Proof: Suppose that a nonempty subset S of $V(G \circ H)$ is a fair restrained dominating set. Consider the following cases:

Case 1. Suppose that $V(G) \subseteq S$. If $S = V(G)$, then we are done with the proof of statement (i). Suppose that $S \neq V(G)$. Then $V(G) \subset S$. Let $S = V(G) \cup (\cup_{v \in X} S_v) \cup (\cup_{v \in V(G) \setminus X} V(H^x))$ where $\emptyset \neq X \subseteq V(G)$ and $\emptyset \neq S_v \subset V(H^v)$ for all $v \in X$. We will show that S_v is fair restrained dominating set of H^v for all $v \in X$ with $|V(H)| \geq 3$. First, since H is nontrivial, suppose that $|V(H)| = 2$. Since $\emptyset \neq S_v \subset V(H^v)$ for all $v \in X$, it follows that $|S_v| = 1$. Since $|V(H^v)| = 2$ for all $v \in X$, $|V(H^v) \setminus S_v| = 1$. Thus, for all $v \in X$ there exists $u \in V(H^v) \setminus S_v \subset V(G \circ H) \setminus S$ such that for all $z \in V(G \circ H) \setminus S$ where $z \neq u$, $uz \notin E(G \circ H)$. This contradict to our assumption that $S = V(G) \cup (\cup_{x \in X} S_v) \cup (\cup_{x \in V(G) \setminus X} V(H^x))$ is a restrained dominating set of $G \circ H$. Hence, $|V(H)|$ must be greater than 2, that is, $|V(H)| \geq 3$. Next, suppose that S_v is not a restrained dominating set of H^v for all $v \in X$. Then for each $v \in X$, there exists $u \in V(H^v) \setminus S_v$ such that $uz \notin E(H^v)$ for some $z \in V(H^v) \setminus S_v$ where $u \neq z$. Thus, for all $v \in X$ there exists $u \in V(G \circ H) \setminus S$ such that for all $z \in V(G \circ H) \setminus S$ where $z \neq u$, $uz \notin E(G \circ H)$. This contradict to our assumption that
$S = V(G) \cup (\cup_{v \in X} S_v) \cup (\cup_{x \in V(G) \setminus X} V(H^x))$ is a restrained dominating set of $G \circ H$. Hence, S_v must be a restrained dominating set of H^v for all $v \in X$. Further, suppose that S_v is not a fair dominating set of H^v for all $v \in X$. Note that $|V(H)| \geq 3$. Let $u, z \in V(H^v) \setminus S_v$ for all $v \in X$ such that $|N_{H^v}(u) \cap S_v| \neq |N_{H^v}(z) \cap S_v|$. Thus,

$$\begin{aligned}
|N_{G \circ H}(u) \cap S| &= |(N_{H^v}(u) \cup \{v\}) \cap S| \\
&= |(N_{H^v}(u) \cap S) \cup (\{v\} \cap S)| \\
&= |(N_{H^v}(u) \cap S_v) \cup \{v\}| \text{ since } \{v\} = \{v\} \cap S \\
&\neq |(N_{H^v}(z) \cap S_v) \cup \{v\}| \\
&= |(N_{H^v}(z) \cap S) \cup (\{v\} \cap S)| \\
&= |(N_{H^v}(z) \cup \{v\}) \cap S| \\
&= |N_{G \circ H}(z) \cap S|,
\end{aligned}$$

that is, $|N_{G \circ H}(u) \cap S| \neq |N_{G \circ H}(z) \cap S|$ contrary to our assumption that S is a fair dominating set of $G \circ H$. Thus, S_v must be a fair dominating set of H^v for all $v \in X$. This proves statement (ii).

Case 2. Suppose that $V(G) \nsubseteq S$. Then $S \subseteq V(G \circ H) \setminus V(G)$. Let $S = \cup_{v \in V(G)} S_v$ where $S_v \subseteq V(H^v)$ for all $v \in V(G)$. If $S_v = V(H^v)$ for all $v \in V(G)$, then the proof of the first part of statement (iii) is done. Suppose that $S_v \neq V(H^v)$ for all $v \in V(G)$. Since H is nontrivial, if $|V(H^v)| = 2$ for all $v \in V(G)$, then $|S_v| = 1$. Let $u, v \in V(v + H^v) \setminus S_v$ for all $v \in V(G)$. Clearly, $N_{H^v}(u) \cap S_v = N_{v+H^v}(u) \cap S_v$ and $N_{H^v}(v) \cap S_v = N_{v+H^v}(v) \cap S_v$ for all $v \in V(G)$. Thus,

$$\begin{aligned}
|N_{H^v}(u) \cap S_v| &= |N_{v+H^v}(u) \cap S_v| \\
&= |N_{G \circ H}(u) \cap S_v|,\ N_{v+H^v}(u) = N_{G \circ H}(u) \\
&= |N_{G \circ H}(u) \cap S| \text{ since } S_v \subset S \\
&= |N_{G \circ H}(v) \cap S| \text{ , } S \text{ is fair dominating set} \\
&= |N_{v+H^v}(v) \cap S| \\
&= |N_{v+H^v}(v) \cap S_v| \\
&= |N_{H^v}(v) \cap S_v| = 1 = |S_v|.
\end{aligned}$$

This means that S_v is a 1-fair dominating set of of H^v for all $v \in V(G)$. If $|V(H^v)| \neq 2$ for all $v \in V(G)$, then $|V(H^v)| \geq 3$. Let $u, z \in V(H^v) \setminus S_v$ for all $v \in V(G)$. Then

$$\begin{aligned}
|N_{H^v}(u) \cap S_v| &= |(N_{H^v}(u) \cap S_v) \cup (\{v\} \cap S_v)| \\
&\quad \text{since } (\{v\} \cap S_v) = \emptyset \\
&= |(N_{H^v}(u) \cup \{v\}) \cap S_v)| \\
&= |N_{G \circ H}(u) \cap S_v| \\
&= |N_{G \circ H}(u) \cap S| \text{ since } S_v \subset S \\
&= |N_{G \circ H}(v) \cap S| \text{ for all} \\
&\quad v \in V(G) \subset V(G \circ H) \setminus S, \text{ since} \\
&\quad S \text{ is a fair dominating set of } G \circ H \\
&= |N_{v+H^v}(v) \cap S| \\
&= |N_{v+H^v}(v) \cap S_v| \\
&= |S_v|.
\end{aligned}$$

Similarly,

$$\begin{aligned}
|N_{H^v}(z) \cap S_v| &= |(N_{H^v}(z) \cap S_v) \cup (\{v\} \cap S_v)| \\
&\quad \text{since } (\{v\} \cap S_v) = \emptyset \\
&= |(N_{H^v}(z) \cup \{v\}) \cap S_v)| \\
&= |N_{G \circ H}(z) \cap S_v| \\
&= |N_{G \circ H}(z) \cap S| \text{ since } S_v \subset S \\
&= |N_{G \circ H}(v) \cap S| \text{ for all } v \in V(G) \subset V(G \circ H) \setminus S \\
&\quad \text{since } S \text{ is a fair dominating set of } G \circ H
\end{aligned}$$

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$= |N_{v+H^v}(v) \cap S|$
$= |N_{v+H^v}(v) \cap S_v|$
$= |S_v|$.

Thus, $|N_{H^v}(u) \cap S_v| = |S_v| = |N_{H^v}(z) \cap S_v|$ for every $u, z \in V(H^v) \setminus S_v$ for all $v \in V(G)$. Hence S_v is an $|S_v|$-fair dominating set of H^v for all $v \in V(G)$. This complete the proof of statement (iii).

For the converse, suppose that statement (i) is satisfied. Then $S = V(G)$. Let $v \in V(G)$. Since H is a nontrivial connected graph, $\{v\}$ is a fair restrained dominating set of $v + H^v$ by Corollary 2.6. Clearly, $\bigcup_{v\in V(G)}\{v\} = V(G)$ is a fair restrained dominating set of $\langle \bigcup_{v\in V(G)} V(v + H^v)\rangle = G \circ H$. Hence, S is a fair restrained dominating set of $G \circ H$. Suppose that statement (ii) is satisfied. Then $S = V(G) \cup (\bigcup_{v\in X} S_v) \cup (\bigcup_{x\in V(G)\setminus X} V(H^x))$ where $\emptyset \neq X \subseteq V(G)$ and S_v is fair restrained dominating set of H^v for all $v \in X$ with $|V(H)| \geq 3$. If $X = V(G)$, then $S = V(G) \cup (\bigcup_{v\in V(G)} S_v)$. By Corollary 2.6, $\{v\} \cup S_v$ where S_v is fair restrained dominating set of $v + H^v$ for all $v \in V(G)$. Clearly, $\bigcup_{v\in V(G)}(\{v\} \cup S_v) = V(G) \cup (\bigcup_{v\in V(G)} S_v)$ is a fair restrained dominating set of $\langle \bigcup_{v\in V(G)} V(v + H^v)\rangle = G \circ H$. Hence, S is a fair restrained dominating set of $G \circ H$. Similarly, if $X \neq V(G)$, then $S = V(G) \cup (\bigcup_{v\in X} S_v) \cup (\bigcup_{v\in V(G)\setminus X} V(H^x))$ is a fair restrained dominating set of $G \circ H$. Suppose that statement (iii) is satisfied. Then $S = \bigcup_{v\in V(G)} S_v$ where $S_v = V(H^v)$ or S_v is an $|S_v|$-fair dominating set of H^v for all $v \in V(G)$. First, if $S_v = V(H^v)$ for all $v \in V(G)$, then $S = \bigcup_{v\in V(G)} V(H^v)$. Let $v \in V(G) = V(G \circ H) \setminus S$. Then there exists $x \in S$ such that $vx \in E(G \circ H)$. Since G is a nontrivial connected graph, there exists $u \in V(G)$ (where $u \neq v$) such that $uv \in E(G)$. Thus, for every $v \in V(G \circ H) \setminus S$, there exists $x \in S$ such that $vx \in E(G \circ H)$ and there exists $u \in V(G \circ H) \setminus S$ such that $uv \in E(G \circ H)$. Thus, S is a restrained dominating set of $G \circ H$. Further, $|N_{G\circ H}(u) \cap S| = |V(H^u)| = |V(H)| = |V(H^v)| = |N_{G\circ H}(v) \cap S|$. Hence, S is a fair restrained dominating set of $G \circ H$. Second, if $S_v \neq V(H^v)$, then $S = \bigcup_{v\in V(G)} S_v$ where S_v is an $|S_v|$-fair dominating set of H^v for all $v \in V(G)$. By Corollary 2.6, S_v is a fair restrained dominating set of $v + H^v$. Clearly, $\bigcup_{v\in V(G)} S_v$ is a fair restrained dominating set of $\langle \bigcup_{v\in V(G)} V(v + H^v)\rangle = G \circ H$. Hence S is a fair restrained dominating set of $G \circ H$. ■

The next result is an immediate consequence of Theorem 2.8.

Corollary 2.9 Let G and H be nontrivial connected graphs. Then $\gamma_{frd}(G \circ H) = |V(G)|$.

Proof: In view of Theorem 2.8, $S = V(G)$ is a fair restrained dominating set of $G \circ H$. This implies that $\gamma_{frd}(G \circ H) \leq |V(G)|$. By Remark 2.7 and Remark 2.3, $|V(G)| = \gamma(G \circ H) \leq \gamma_{frd}(G \circ H)$. This shows that $\gamma_{frd}(G \circ H) = |V(G)|$. ■

III. CONCLUSION AND RECOMMENDATIONS

In this work, we extend the concept of - the fair restrained domination of graphs. The fair restrained domination in the corona of two graphs was characterized. The exact fair restrained domination number resulting from this binary operation of two graphs was computed. This study will lead us to new researches such as bounds and other binary operations of two graphs. Other parameters involving fair restrained domination in graphs may also be explored. Finally, the characterization of a fair restrained domination in graphs and its bounds is a promising extension of this study.

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