e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-11, Issue-1 (February 2021)
https://doi.org/10.31033/ijemr.11.1.2

A Scheffe's Predictive Model for Modulus of Elasticity of Sawdust Ash - Sand Concrete

Kenneth Miebaka Oba[1], Onuegbu Okoronkwo Ugwu[2]
[1]Lecturer, Department of Civil Engineering, Faculty of Engineering, Rivers State University, P.M.B 5080, Port Harcourt, NIGERIA
[2]Professor, Department of Civil Engineering, Faculty of Engineering and Technology, Alex Ekwueme Federal University, P.M.B 1010, Ndufu-Alike, Abakaliki, NIGERIA

[1]Corresponding Author: kenmieoba@yahoo.com

ABSTRACT

The industrial waste, Saw Dust Ash (SDA) has been explored by several concrete related researches to achieve environmental and economic sustainability. In this study, 5% of sand was replaced with SDA to produce concrete with five different mix ratios. Scheffe's simplex theory was used for five mix ratios in a {5,2} experimental design which resulted in additional ten mix ratios. For purposes of verification and testing, additional fifteen mix ratios were generated from the initial fifteen. Concrete cubes of 150mmX150mmX150mm were formed using the thirty concrete mix ratios generated, and cured in water for 28days. The compressive strengths of cubes from each mix ratio were determined. The static moduli of elasticity were also determined with a mathematical relationship. The results of the first fifteen static moduli of elasticity values were used for the calibration of the model constant coefficients, while those from the second fifteen were used for the model verification and testing using Scheffe's simplex lattice design. A mathematical regression model was formulated from the results, with which the static moduli of elasticity were predicted. The model was then subjected to a two-tailed t-test with 5% significance, which confirms the model adequate and fit with an R^2 of 0.8536. The study also revealed that SDA can be used to replace 5% of sand and promote environmental sustainability without significantly decreasing the static modulus of elasticity.

Keywords– Saw Dust Ash, Scheffe's Simplex Lattice, Sustainability, Modulus of Elasticity of Concrete

I. INTRODUCTION

The modulus of Elasticity of an engineering material is the stress per unit strain of that material. Hardened concrete materials, as indicated by[1] undergo non-linear and non-elastic deformation which is a permanent deformation that occurs after removal of loads. A static Modulus of Elasticity is usually referred when a laboratory experiment is carried out to determine the Modulus of Elasticity. However, from the chart in Figure 1, the initial tangent modulus of elasticity, according to[1] is approximately referred to as the dynamic modulus of elasticity.

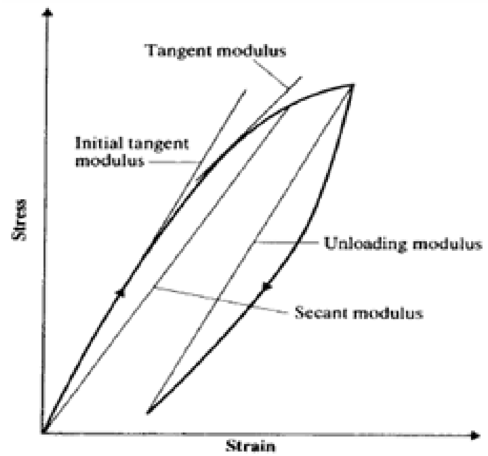

Figure1: Simplex Plot for Actual Components

The relationship between the static Modulus of Elasticity, E_c and the dynamic Modulus of Elasticity, E_d is:

$$\boldsymbol{E_c = 1.25E_d - 19} \quad (1)$$

$$E_c = 1.25E_d - (2.75 \text{ X } 10^6) \quad (2)$$

The above equations only apply when less than or equal to 500kg of cement per cubic metres is used. When otherwise, the following applies:

$$E_c = 1.04E_d - 4.1 \quad (3)$$

$$E_c = 1.04E_d - (0.59 \text{ X } 10^6) \quad (4)$$

Equations (1) and (3) are in SI units (GPa) while Equations (2) and (4) are in Imperial units (psi).

The static modulus of Elasticity E_c, according to[1] when relating it with the compressive strength can be determined by the equations:

$$E_c = 9.1f_c^{0.33} \quad (5)$$

$$E_c = 0.255f_c^{0.33} \text{ X } 10^6 \quad (6)$$

Equation (5) is in SI units (GPa) while equation (6) is in Imperial units (psi) and the density of concrete is 2320kg/m^3. When the density of concrete is between 1400kg/m^3 and 2320kg/m^3,equations (7) and (8) are used in SI units and Imperial units respectively.

$$E_c = 1.7\rho^2 f_c^{0.33} \quad (7)$$

$$E_c = 12.24\rho^2 f_c^{0.33} \text{ X } 10^6 \quad (8)$$

where, $\boldsymbol{\rho}$ is the density of concrete (kg/m^3) and $f_c$is the compressive strength (N/mm^2 or MPa).

However,[2] stated that the relationships between modulus of elasticity and compressive strength are equations (9) and (10) for SI (GPa) and Imperial (psi) units respectively.

$$E_c = 4.731\sqrt{f_c} \quad (9)$$

$$E_c = 57000\sqrt{f_c} \quad (10)$$

These equations are slightly different from those given by[1].

A study carried out by [3] revealed that 10% replacement of Portland cement with Hydrated Lime (HL) and Saw Dust Ash (SDA) gave acceptable compressive strength and modulus of elasticity. Another study on cement mortar [4] affirmed that addition of 0.5% SDA to the mixture increased the dynamic modulus of elasticity by 2-10%, while a 3% addition caused about 23% decrease in the dynamic modulus of elasticity. With reference to equations (1) to (4), these changes will also affect the dynamic modulus of elasticity.

A. Saw Dust Ash in Concrete

Saw Dust Ash is the pulverised form of saw dust produced as waste from saw mills. It has been used in concrete construction for over 30 years [5]. In addition, cement production has been a major source of environmental degradation, as about 400kg of CO_2 are emitted from every 600kg of cement produced [6]. Investigations [7] showed that SDA has a specific gravity of 2.5, water absorption of 0.56%, fineness modulus of 1.78, and bulk dry density of 1300kg/m^3, while sand has specific gravity of 2.65, water absorption of 0.45%, fineness modulus of 2.21, and bulk dry density of 1512 kg/m^3. A 10% replacement of sand with SDA, modified the properties to 2.67, 0.5%, 2.2, and 1436kg/m^3 for specific gravity, water absorption, fineness modulus, and bulk dry density respectively. This is a strong indication that sand and SDA mixture did not significantly change the physical properties of sand, making the mixture adequate for a fine aggregate.

The chemical properties of SDA [7] by mass are: 65.3% SiO_2, 4% Al_2O_3, 2.23% Fe_2O_3, 9.6% CaO, 5.8% MgO, 0.01% MnO, 0.07% Na_2O, 0.11% K_2O, 0.43% P_2O_5, and 0.45% SO_2. The summation of SiO_2, Al_2O_3, and Fe_2O_3 gives 71.53%. A similar investigation by [8] found 67.95% SiO_2, 4.29% Al_2O_3, 2.15% Fe_2O_3, 9.47% CaO, 5.84% MgO, 0.01% MnO, 0.06% Na_2O, 0.11% K_2O, and 0.56% SO_3. The summation of SiO_2, Al_2O_3, and Fe_2O_3 gave 74.39. These, in accordance with[9] indicate that SDA is a good pozzolanic material. The chemical compositions of SDA as found by[7], [8] all indicate that SDA has a high percentage of SiO_2 and small percentages of Al_2O_3 and Fe_2O_3, which are similar to those of sand with high percentage of about 95% SiO_2. Hence SDA can be used with sand as fine aggregate.

B. Scheffe's Simplex Theory

Several authors [10], [11], [20]–[23], [12]–[19] have carried out concrete mixture researches with development of mathematical models, most of which were based on Scheffe's Simplex theory.

Scheffe's model is based on the simplex lattice and simplex theory or approach [24]. The simplex approach considers a number of components, *q*, and a degree of polynomial, *m*. The sum of all the i^{th} components is not greater than 1. Hence,

$$\sum_{i=1}^{q} x_i = 1 \quad (11)$$

$$x_1 + x_2 + \cdots + x_q = 1 \quad (12)$$

with $0 \le x \le 1$, the factor space becomes S_{q-1}. According to [24] the {*q*,*m*} simplex lattice design is a symmetrical arrangement of points within the experimental region in a suitable polynomial equation representing the response surface in the simplex region.

The number of points $C_m^{(q+m-1)}$ has (m+1) equally spaced values of $x_i = 0$, $^1/_m$, $^2/_m$, …. $^m/_m$. For a 3-component mixture with degree of polynomial 2, the corresponding number of points will be $C_2^{(3+2-1)}$ which gives 6 (eq. 13 or eq. 14 below) with number of spaced values, 2+1 = 3, that is x_i = 0, 1/2, and 1 and design points of (1,0,0), (0,1,0), (0,0,1), (1/2,1/2,0), (1/2,01/2), and (0,1/2,1/2). Similarly, for a {*5*,*2*} simplex, there will be 15 points with x_i = 0, 1/2, and 1 as spaced values.

The 15 design points are (1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (0,0,0,0,1), (1/2,1/2,0,0,0), (1/2,0,1/2,0,0), (1/2,0,0,1/2,0), (1/2,0,0,0,1/2), (0,1/2,1/2,0,0), (0,0,1/2,1/2,0), (0,0,0,1/2,1/2), (0,1/2,0,1/2,0), (0,0,1/2,0,1/2), (0,1/2,0,0,1/2).

$$N = C_n^{(q+n-1)} \quad (13)$$

$$N = \frac{(q+n-1)!}{(q-1)!(n)!} \quad (14)$$

For a polynomial of degree *m* with *q*component variables where eq. (12) holds, the general form is:

$$Y = b_0 + \sum b_i\, x_i + \sum b_{ij}\, x_i x_j + \sum b_{ijk}\, x_i x_j x_k + \cdots + \sum b_{i1,i2\ldots in}\, x_{i1} x_{i2} x_{in} \quad (15)$$

Where $1 \le i \le q$, $1 \le i \le j \le q$, $1 \le i \le j \le k \le q$, and b_0 is the constant coefficient.

x is the pseudo component for constituents *i*, *j*, and *k*.

When {*q*,*m*} = {*5*,*2*}, eq. (15) becomes:

$$Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{25}x_2x_5 + b_{34}x_3x_4 + b_{35}x_3x_5 + b_{45}x_4x_5 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2 + b_{55}x_5^2 \quad (16)$$

and eq. (12) becomes

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1 \quad (17)$$

Multiplying eq. (17) by b_0 gives

$$b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 = b_0 \quad (18)$$

Multiplying eq. (17) successively by x_1, x_2, x_3, x_4, and x_5 and making x_1, x_2, x_3, x_4, and x_5 the subjects of the respective formulas:

$$\left.\begin{aligned}
x_1^2 &= x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5 \\
x_2^2 &= x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5 \\
x_3^2 &= x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5 \\
x_4^2 &= x_4 - x_1x_4 - x_2x_4 - x_3x_4 - x_4x_5 \\
x_5^2 &= x_5 - x_1x_5 - x_2x_5 - x_3x_5 - x_4x_5
\end{aligned}\right\} \quad (19)$$

e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-11, Issue-1 (February 2021)
https://doi.org/10.31033/ijemr.11.1.2

Substituting eq. (18) and eq. (19) into eq. (16) we have:

$$Y = b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{15}x_1x_5 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{25}x_2x_5 + b_{34}x_3x_4 + b_{35}x_3x_5 + b_{45}x_4x_5 + b_{11}(x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5) + b_{22}(x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5) + b_{33}(x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5) + b_{44}(x_4 - x_1x_4 - x_2x_4 - x_3x_4 - x_4x_5) + b_{55}(x_5 - x_1x_5 - x_2x_5 - x_3x_5 - x_4x_5)$$

$$Y = (b_0 + b_1 + b_{11})x_1 + (b_0 + b_2 + b_{22})x_2 + (b_0 + b_3 + b_{33})x_3 + (b_0 + b_4 + b_{44})x_4 + (b_0 + b_5 + b_{55})x_5 + (b_{12} - b_{11} - b_{22})x_1x_2 + (b_{13} - b_{11} - b_{33})x_1x_3 + (b_{14} - b_{11} - b_{44})x_1x_4 + (b_{15} - b_{11} - b_{55})x_1x_5 + (b_{23} - b_{22} - b_{33})x_2x_3 + (b_{24} - b_{22} - b_{44})x_2x_4 + (b_{25} - b_{22} - b_{55})x_2x_5 + (b_{34} - b_{33} - b_{44})x_3x_4 + (b_{35} - b_{33} - b_{55})x_3x_5 + (b_{45} - b_{44} - b_{55})x_4x_5 \quad (20)$$

Let

$$\left.\begin{aligned} \beta_1 &= b_0 + b_1 + b_{11} \\ \beta_2 &= b_0 + b_2 + b_{22} \\ \beta_3 &= b_0 + b_3 + b_{33} \\ \beta_4 &= b_0 + b_4 + b_{44} \\ \beta_5 &= b_0 + b_5 + b_{55} \\ \beta_{12} &= b_{12} - b_{11} - b_{22} \\ \beta_{13} &= b_{13} - b_{11} - b_{33} \\ \beta_{14} &= b_{14} - b_{11} - b_{44} \\ \beta_{15} &= b_{15} - b_{11} - b_{55} \\ \beta_{23} &= b_{23} - b_{22} - b_{33} \\ \beta_{24} &= b_{24} - b_{22} - b_{44} \\ \beta_{25} &= b_{25} - b_{22} - b_{55} \\ \beta_{34} &= b_{34} - b_{33} - b_{44} \\ \beta_{35} &= b_{35} - b_{33} - b_{55} \\ \beta_{45} &= b_{45} - b_{44} - b_{55} \end{aligned}\right\} \quad (21)$$

Substituting eq. (21) into eq. (20) gives

$$Y = \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_5x_5 + \beta_{12}x_1x_2 + \beta_{13}x_1x_3 + \beta_{14}x_1x_4 + \beta_{15}x_1x_5 + \beta_{23}x_2x_3 + \beta_{24}x_2x_4 + \beta_{25}x_2x_5 + \beta_{34}x_3x_4 + \beta_{35}x_3x_5 + \beta_{45}x_4x_5 \quad (22)$$

This can be rewritten as:

$$Y = \sum_{i=1}^{5} \beta_i x_i + \sum_{1 \le i \le j \le 5} \beta_{ij} x_i x_j \quad (23)$$

Where the response, Y is a dependent variable (Water Absorption of concrete). Eq. (22) is the general equation for a {*5,2*} polynomial, and it has 15 terms, which conforms to Scheffe's theory in eq. (13).

Let Y_i denote response to pure components, and Y_{ij} denote response to mixture components in *i* and *j*. If x_i =1 and x_j = 0, since $j \neq i$, then

$$Y_i = \beta_i \quad (24)$$

Which means

$$\sum_{i=1}^{5} \beta_i x_i = \sum_{i=1}^{5} Y_i x_i \quad (25)$$

Hence, from eq. (24)

$$\left.\begin{aligned} Y_1 &= \beta_1 \\ Y_2 &= \beta_2 \\ Y_3 &= \beta_3 \\ Y_4 &= \beta_4 \\ Y_5 &= \beta_5 \end{aligned}\right\} \quad (26)$$

According to [24],

$$\beta_{ij} = 4Y_{ij} - 2\beta_i - 2\beta_j \quad (27)$$

Substituting eq. (24)

$$\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \quad (28)$$

II. MATERIALS AND METHODS

The materials used for the production of the concrete for the study were water, cement, sand, SDA, and granite. These are the five components in the concrete mix, with SDA used to partially replace 5% of the fine aggregate (sand).

The first five concrete mix ratios derived from different mix design methods [17], [18] are given as:

BRE 12 = [0.54 11.9475 0.1025 2.95];
BRE 22 = [0.58 1 2.1185 0.1115 3.21];
USBR 22 = [0.58 12.2515 0.1185 3.29];
BIS 12 = [0.43 11.2065 0.0635 2.88];
ACI 12 = [0.55 11.8335 0.0965 3.09]

These can be put in matrix form as follows:

$$S = \begin{bmatrix} 0.54 & 0.58 & 0.58 & 0.43 & 0.55 \\ 1 & 1 & 1 & 1 & 1 \\ 1.9475 & 2.1185 & 2.2515 & 1.2065 & 1.8335 \\ 0.1025 & 0.1115 & 0.1185 & 0.0635 & 0.0965 \\ 2.95 & 3.21 & 3.29 & 2.88 & 3.09 \end{bmatrix} \quad (29)$$

Their corresponding pseudo components are given as:

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (30)$$

with centre points

X_{12} = [0.5 0.5 0 0 0];X_{13} = [0.5 0 0.5 0 0];
X_{14} = [0.5 0 0 0.5 0];X_{15} = [0.5 0 0 0 0.5];
X_{23} = [0 0.5 0.5 0 0];X_{24} = [0 0.5 0 0.5 0];
X_{25} = [0 0.5 0 0 0.5];X_{34} = [0 0 0.5 0.5 0];
X_{35} = [0 0 0.5 0 0.5];X_{45} = [0 0 0 0.5 0.5]

According to [24],

$$S_{ij} = XS_i \quad (31)$$

Substituting,

$$\begin{bmatrix} S_{12} \\ S_{13} \\ S_{14} \\ S_{15} \\ S_{23} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0.5 & 0 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0.5 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0.54 \\ 0.58 \\ 0.58 \\ 0.43 \\ 0.55 \end{bmatrix} \quad (32)$$

This process is repeated for S_{24}, S_{25}, S_{34}, S_{35}, and S_{45}. Similarly, this process is repeated for an additional 15 (control) points that will be used for the verification of the formulated model. The regular

e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-11, Issue-1 (February 2021)
https://doi.org/10.31033/ijemr.11.1.2

pentagons for the actual components with their corresponding pseudo components are given in Figures2 and 3 respectively. Tables 1 and 2 mix ratio data were generated for the main and verification purposes respectively from [25].

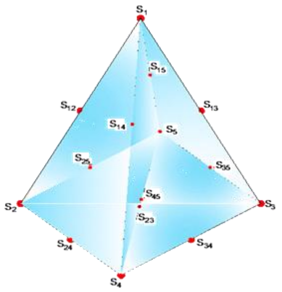

Figure2: Simplex Plot for Actual Components

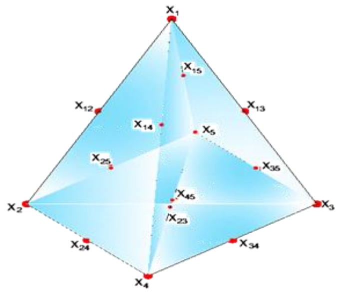

Figure3: Simplex Plot for Pseudo Components

Table 1: Model Mix Ratios

Sample Points	Actual Components: w-c ratio S_1	Cement S_2	Sand S_3	SDA S_4	Granite S_5	Response Y_{exp}	Pseudo Components: w-c ratio X_1	Cement X_2	Sand X_3	SDA X_4	Granite X_5
BRE12	0.54	1	1.9475	0.1025	2.95	Y_1	1	0	0	0	0
BRE22	0.58	1	2.1185	0.1115	3.21	Y_2	0	1	0	0	0
USBR22	0.58	1	2.2515	0.1185	3.29	Y_3	0	0	1	0	0
BIS12	0.43	1	1.2065	0.0635	2.88	Y_4	0	0	0	1	0
ACI12	0.55	1	1.8335	0.0965	3.09	Y_5	0	0	0	0	1
N1	0.56	1	2.033	0.107	3.08	Y_{12}	0.5	0.5	0	0	0
N2	0.56	1	2.0995	0.1105	3.12	Y_{13}	0.5	0	0.5	0	0
N3	0.485	1	1.577	0.083	2.915	Y_{14}	0.5	0	0	0.5	0
N4	0.545	1	1.8905	0.0995	3.02	Y_{15}	0.5	0	0	0	0.5
N5	0.58	1	2.185	0.115	3.25	Y_{23}	0	0.5	0.5	0	0
N6	0.505	1	1.6625	0.0875	3.045	Y_{24}	0	0.5	0	0.5	0
N7	0.565	1	1.976	0.104	3.15	Y_{25}	0	0.5	0	0	0.5
N8	0.505	1	1.729	0.091	3.085	Y_{34}	0	0	0.5	0.5	0
N9	0.565	1	2.0425	0.1075	3.19	Y_{35}	0	0	0.5	0	0.5
N10	0.49	1	1.52	0.08	2.985	Y_{45}	0	0	0	0.5	0.5

Table 2: Control Points

Sample Points	Actual Components: w-c ratio S_1	Cement S_2	Sand S_3	SDA S_4	Granite S_5	Response Y_{exp}	Pseudo Components: w-c ratio X_1	Cement X_2	Sand X_3	SDA X_4	Granite X_5
C1	0.558	1	2.0463	0.1077	3.114	YC_1	0.4	0	0.4	0	0.2
C2	0.52	1	1.7537	0.0923	3.078	YC_2	0	0.6	0	0.4	0
C3	0.548	1	2.0083	0.1057	3.018	YC_3	0.8	0	0.2	0	0
C4	0.49	1	1.5713	0.0827	3.012	YC_4	0	0.4	0	0.6	0
C5	0.544	1	1.9019	0.1001	3.006	YC_5	0.6	0	0	0	0.4
C6	0.55	1	2.0425	0.1075	3.208	YC_6	0	0	0.8	0.2	0
C7	0.55	1	1.9589	0.1031	3.03	YC_7	0.6	0.2	0	0	0.2
C8	0.514	1	1.6967	0.0893	3.054	YC_8	0	0.4	0	0.4	0.2
C9	0.548	1	1.8563	0.0977	3.062	YC_9	0.2	0	0	0	0.8
C10	0.46	1	1.4155	0.0745	2.962	YC_{10}	0	0	0.2	0.8	0
C11	0.566	1	2.1071	0.1109	3.182	YC_{11}	0.2	0	0.6	0	0.2

e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-11, Issue-1 (February 2021)
https://doi.org/10.31033/ijemr.11.1.2

Sample Points	Actual Components					Response Y_{exp}	Pseudo Components				
	w-c ratio	Cement	Sand	SDA	Granite		w-c ratio	Cement	Sand	SDA	Granite
	S_1	S_2	S_3	S_4	S_5		X_1	X_2	X_3	X_4	X_5
C12	0.544	1	1.9323	0.1017	3.152	YC_{12}	0	0.2	0.4	0.2	0.2
C13	0.58	1	2.1451	0.1129	3.226	YC_{13}	0	0.8	0.2	0	0
C14	0.532	1	1.7651	0.0929	3.072	YC_{14}	0	0.2	0	0.2	0.6
C15	0.536	1	1.8715	0.0985	3.084	YC_{15}	0.2	0.2	0.2	0.2	0.2

A. Static Modulus of Elasticity of Concrete

Two replicate concrete cubes were made for each of the thirty mix ratios in 150mmX150mmX150mm moulds and allowed to harden. The concrete cubes were removed from the moulds after 24hours and cured in water for 28days after which the compressive strengths were determined with results from [18]. The static modulus of Elasticity E_c, was determined by equation (33) for each mix ratio as follows:

$$E_c = 9.1 f_c^{0.33} \text{X } 10^6 \quad (\text{KN/mm}^2) \qquad (33)$$

f_c= the compressive strength (N/mm^2)

The average was taken and recorded.

Sieve analysis was carried out on the fine aggregate mixed with 5% SDA as a preliminary investigation. The particle size distribution of the 5% replacement of sand with SDA is shown in Figure4, and the fineness modulus calculated below.

Fineness modulus,

$$FM = \frac{0.73 + 4.24 + 14.08 + 43.61 + 80.48 + 97.88}{100} = 2.41$$

This value indicates that the material is a fine aggregate as it ranges between 2.3 and 3.1 [2].

III. RESULTS AND DISCUSSIONS

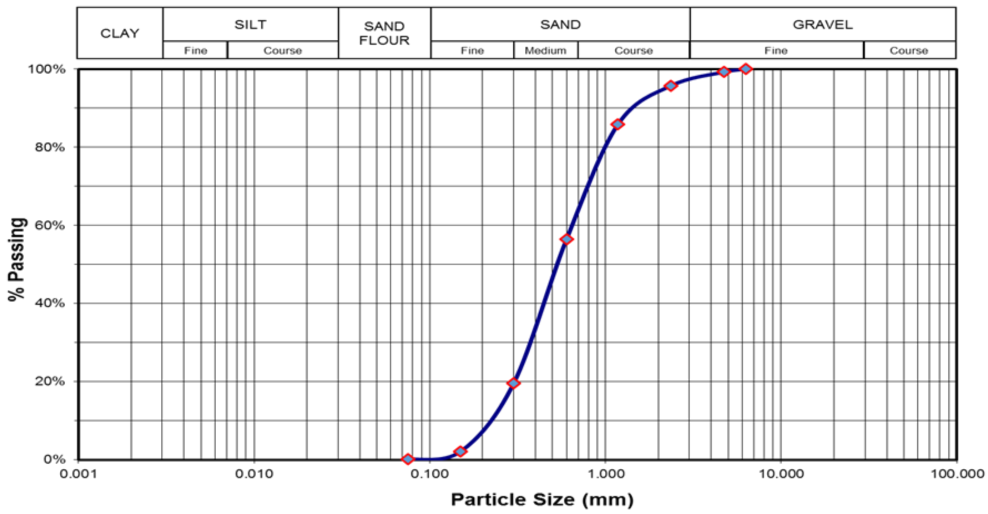

Figure 4: Particle Size Distribution for Fine Aggregate with 5% SDA replacement

The results of the 28days Static Modulus of Elasticity are shown in Table 3 below.

Table 3:28 Days Static Modulus of Elasticity of Concrete

Sample	Compressive Strength (N/mm^2)		Modulus of Elasticity (KN/mm^2)		
	A	B	A	B	Average
BRE12	28.444	28.444	27.470	27.470	27.470
BRE22	23.111	24.444	25.651	26.130	25.890
USBR22	26.191	26.313	26.732	26.773	26.753
BIS12	36.222	35.778	29.751	29.630	29.691
ACI12	31.564	31.956	28.430	28.546	28.488
N1	29.333	28.489	27.750	27.484	27.617
N2	21.956	22.489	25.220	25.421	25.321

Sample	Compressive Strength (N/mm^2)		Modulus of Elasticity (KN/mm^2)		
	A	B	A	B	Average
N3	26.222	26.667	26.743	26.891	26.817
N4	26.667	29.733	26.891	27.875	27.383
N5	29.378	27.733	27.764	27.242	27.503
N6	28.978	29.467	27.639	27.792	27.716
N7	24.800	29.067	26.255	27.667	26.961
N8	31.022	29.689	28.268	27.861	28.064
N9	22.889	24.622	25.569	26.193	25.881
N10	28.222	27.956	27.399	27.313	27.356
C1	23.333	24.356	25.732	26.099	25.915
C2	28.667	28.844	27.541	27.597	27.569
C3	24.711	24.667	26.224	26.208	26.216
C4	31.111	32.044	28.295	28.572	28.433
C5	28.889	28.400	27.611	27.456	27.534
C6	27.689	27.600	27.227	27.198	27.213
C7	28.000	28.667	27.328	27.541	27.434
C8	30.000	29.556	27.957	27.820	27.888
C9	28.533	27.200	27.498	27.068	27.283
C10	31.778	31.822	28.493	28.506	28.500
C11	23.200	22.889	25.683	25.569	25.626
C12	27.778	28.133	27.256	27.370	27.313
C13	26.133	25.289	26.712	26.425	26.569
C14	29.733	28.933	27.875	27.625	27.750
C15	26.089	26.267	26.698	26.758	26.728

A. Model Formulation

The coefficients of polynomial from Table 3, equation (26), and equation (28) are:

β_1 = 27.47, β_2 = 25.89, β_3 = 26.753, β_4 = 29.691, β_5 = 28.488, $\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2$

$\beta_{12} = 4 * 27.617 - 2 * 27.47 - 2 * 25.89 = 3.748$

Similarly, β_{13} = -7.162, β_{14} = -7.054, β_{15} = -2.384, β_{23} = 4.726, β_{24} = -0.298, β_{25} = -0.912, β_{34} = -0.632, β_{35} = -6.958, β_{45} = -6.934.

Substituting the above coefficients into equation (22) gives

$$\boldsymbol{Y = 27.47x_1 + 25.89x_2 + 26.753x_3 + 29.691x_4 + 28.488x_5 + 3.748x_1x_2 - 7.162x_1x_3 - 7.054x_1x_4 - 2.3844x_1x_5 + 4.726x_2x_3 - 0.298x_2x_4 - 0.912x_2x_5 - 0.632x_3x_4 - 6.958x_3x_5 - 6.934x_4x_5} \quad \textbf{(34)}$$

Equation (34) is the mathematical model to predict the 28 days static modulus of elasticity of concrete using SDA to replace 5% of fine aggregate. Table 4 shows the predictions, while Figure 5 shows the comparison between the predicted and experimented values of 28days static modulus of elasticity using the control (verification) data.

Table 4: Experimental and Predicted values of 28days Static Modulus of Elasticity of Concrete

Sample Points	Response Y	Pseudo Components					Static Mod. of Elasticity Y_{exp} (KN/mm^2)	Static Mod. of Elasticity Y_{pred} (KN/mm^2)
		w-c ratio	Cement	Sand	SDA	Granite		
		X_1	X_2	X_3	X_4	X_5		
BRE12	Y1	1	0	0	0	0	27.470	27.470
BRE22	Y2	0	1	0	0	0	25.890	25.890
USBR22	Y3	0	0	1	0	0	26.753	26.753
BIS12	Y4	0	0	0	1	0	29.691	29.691

e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-11, Issue-1 (February 2021)
https://doi.org/10.31033/ijemr.11.1.2

ACI12	Y5	0	0	0	0	1	28.488	28.488
N1	Y12	0.5	0.5	0	0	0	27.617	27.617
N2	Y13	0.5	0	0.5	0	0	25.321	25.321
N3	Y14	0.5	0	0	0.5	0	26.817	26.817
N4	Y15	0.5	0	0	0	0.5	27.383	27.383
N5	Y23	0	0.5	0.5	0	0	27.503	27.503
N6	Y24	0	0.5	0	0.5	0	27.716	27.716
N7	Y25	0	0.5	0	0	0.5	26.961	26.961
N8	Y34	0	0	0.5	0.5	0	28.064	28.064
N9	Y35	0	0	0.5	0	0.5	25.881	25.881
N10	Y45	0	0	0	0.5	0.5	27.356	27.356
C1	YC1	0.4	0	0.4	0	0.2	25.915	25.494
C2	YC2	0	0.6	0	0.4	0	27.569	27.339
C3	YC3	0.8	0	0.2	0	0	26.216	26.181
C4	YC4	0	0.4	0	0.6	0	28.433	28.099
C5	YC5	0.6	0	0	0	0.4	27.534	27.305
C6	YC6	0	0	0.8	0.2	0	27.213	27.239
C7	YC7	0.6	0.2	0	0	0.2	27.434	27.485
C8	YC8	0	0.4	0	0.4	0.2	27.888	27.255
C9	YC9	0.2	0	0	0	0.8	27.283	27.903
C10	YC10	0	0	0.2	0.8	0	28.500	29.002
C11	YC11	0.2	0	0.6	0	0.2	25.626	25.454
C12	YC12	0	0.2	0.4	0.2	0.2	27.313	26.960
C13	YC13	0	0.8	0.2	0	0	26.569	26.819
C14	YC14	0	0.2	0	0.2	0.6	27.750	27.256
C15	YC15	0.2	0.2	0.2	0.2	0.2	26.728	26.704

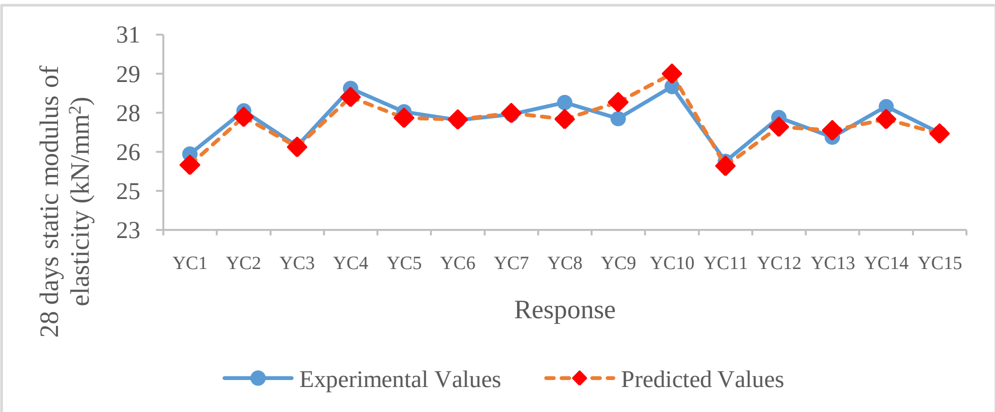

Figure 5: Comparison between Experimental and Predicted 28 days Static Modulus of Elasticity

B. Test of Adequacy of the Model

A two-tailed student t-test was carried out at 95% confidence level, which implies 100 – 95 = 5% significance. Since it is a two-tailed, significance = 5/2 = 2.5%

Hence significance level = 100 – 2.5 = 97.5%

e-ISSN: 2250-0758 | p-ISSN: 2394-6962
Volume-11, Issue-1 (February 2021)
https://doi.org/10.31033/ijemr.11.1.2

Let D be difference between the experimental and predicted responses

The mean of the difference,

$$D_a = \frac{1}{n}\sum_{i=1}^{n} D_i \quad (35)$$

The variance of the difference,

$$S^2 = \left(\frac{1}{n-1}\right)\sum_{i=1}^{n}(D - D_a)^2{}_i \quad (36)$$

$$t_{calculated} = \frac{D_a\sqrt{n}}{S} \quad (37)$$

Where n = number of observations with degree of freedom n – 1. Table 5 shows the details of the t-test results.

Table 5: Student t-test for 28 days Static Modulus of Elastcity of Concrete

Sample	Static Moduli of Elasticity (kN/m^2)		t-test		
	$Y_{experimental}$	$Y_{predicted}$	$D=Y_{exp}-Y_{pred}$	D_a-D	$(D-D_a)^2$
C1	25.915	25.494	0.421	-0.323	0.104
C2	27.569	27.339	0.230	-0.132	0.017
C3	26.216	26.181	0.035	0.063	0.004
C4	28.433	28.099	0.334	-0.236	0.056
C5	27.534	27.305	0.229	-0.131	0.017
C6	27.213	27.239	-0.026	0.124	0.015
C7	27.434	27.485	-0.051	0.149	0.022
C8	27.888	27.255	0.633	-0.535	0.286
C9	27.283	27.903	-0.620	0.718	0.516
C10	28.500	29.002	-0.502	0.600	0.360
C11	25.626	25.454	0.172	-0.074	0.005
C12	27.313	26.960	0.353	-0.255	0.065
C13	26.569	26.819	-0.250	0.348	0.121
C14	27.750	27.256	0.494	-0.396	0.156
C15	26.728	26.704	0.024	0.074	0.006
TOTAL			1.476		1.752
AVERAGE D_a			0.098		

$$S^2 = \frac{1.752}{15-1}$$
$$S^2 = 0.125$$
$$S = \sqrt{0.233} = 0.354$$
$$t_{calculated} = \frac{0.098\sqrt{15}}{0.354}$$
$$t_{calculated} = 1.077$$

From the t-table, with $v = 15 - 1 = 14$, and β = significance level. $t_{(0.975,14)} = 2.145$

Since $t_{calculated} < t_{(0.975,14)}$, and lies between -2.145 and 2.145, therefore there is no significant difference between the experimental and predicted responses, H_0 is accepted, and H_a is rejected. The model is confirmed to be adequate.

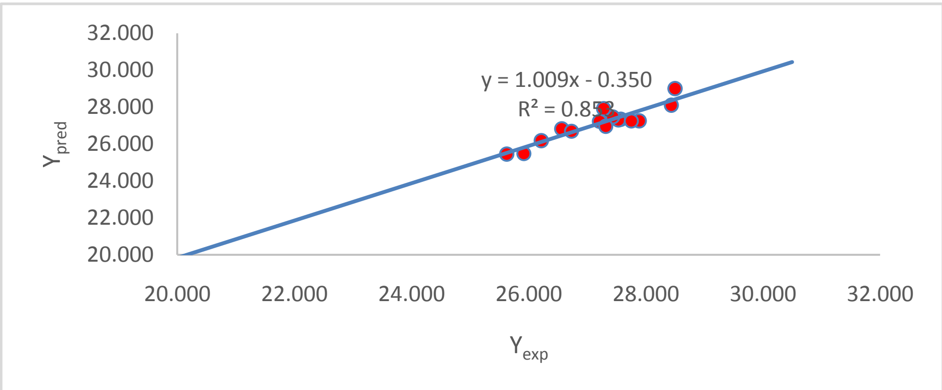

Figure 6: Scatterplot of Predicted vs. Experimental Water Absorption

The R^2 value of 0.8536 in Figure 6 indicates that the experimental results are highly correlated to the predicted results. This is also an indication that the model is fit and adequate.

IV. CONCLUSIONS

Partial replacement of fine aggregate (sand) with 5% SDA was carried out to produce concrete in which cement, granite, and water were the other ingredients. Five different concrete mix ratios were used for the batching. The static moduli of elasticity determined from the 28days compressive strengths were between 25.321KN/mm^2 and 29.691KN/mm^2. A multiple regression model was generated from the resulting 28days static moduli of elasticity values determined from the compressive strength experiments, using Scheffe's simplex theory for a {5,2} simplex lattice. A two-tailed student t-test was carried out at 5% significance level, which confirmed the model adequate with an R^2 of 0.8536. The results from the study show that Scheffe's simplex approach is very effective and has a high predictive accuracy for modulus of elasticity of concrete having about 5% sand replaced with SDA. The results also confirmed that SDA is a suitable material to replace up to 5% of sand in the production of concrete in a bid to promote environmental sustainability. However, further research is recommended with different percentages of SDA-sand replacement.

REFERENCES

[1] A. M. Neville & J. J. Brooks. (1987). *Concrete technology*. (2nd ed.). New Delhi: Pearson Education.
[2] M. S. Mamlouk & J. P. Zaniewski. (2011). *Materials for Civil and Construction Engineers*. (3rd ed.) New Jersey: Pearson Prentice Hall.
[3] T. G. C. Awodiji, J. O. Dimo, C. Nwurumibe, O. O. Awodiji, & I. J. Arimanwa. (2019). Reactions of hydrated lime-saw dust ash blend on the strength properties of cement concrete. *Glob. J. Res. Eng.*, *19*(3), 28–39.
[4] H. Gil, A. Ortega, & J. Pérez. (2017). Mechanical behavior of mortar reinforced with sawdust waste. In: *3rd International Conference on Natural Fibers: Advanced Materials for a Greener World*, *ICNF*, *200*, pp. 325–332.
[5] C. Marthong. (2012). Sawdust Ash (SDA) as partial replacement of cement. *Int. J. Eng. Res. Appl.*, *2*(4), 1980–1985.
[6] S. Chowdhury, M. Mishra, & O. Suganya. (2015). The incorporation of wood waste ash as a partial cement replacement material for making structural grade concrete : An overview. *Ain Shams Eng. J.*, *6*, 429–437.
[7] M. Mageswari & B. Vidivelli. (2009). The use of sawdust ash as fine aggregate replacement in concrete. *J. Environ. Res. Dev.*, *3*(3), 720–726.
[8] S. T. Tyagher, J. T. Utsev, & T. Adagba. (2011). Suitability of saw dust ash-lime mixture for production of sandcrete hollow blocks. *Niger. J. Technol.*, *30*(1), 1–6.
[9] ASTM.C618. (2000). *Standard specification for coal fly ash and raw or calcined natural pozzolan for use as a mineral admixture in concrete*. West Conshohocken: American Society for Testing and Materials.
[10] M. E. Onyia. (2017). Optimisation of the cost of lateritic soil stabilized with quarry dust. *Int. J. Sci. Eng. Res.*, *8*(9), 1400–1413.
[11] C. U. Anya. (2015). *Models for predicting the structural characteristics of sand-quarry dust blocks*. Ph.D Thesis, Department of Civil Engineering, University of Nigeria, Nsukka.
[12] O. M. I., bearugbulem, L. O. Ettu, J. C. Ezeh, & U. C. Anya. (2013). A new regression model for optimizing concrete mixes. *Int. J. Eng. Sci. Res. Technol.*, *2*(7), 1735–1742.
[13] Y. M. Gamil & I. H. Bakar. (2015). The development of mathematical prediction model to predict resilient modulus for natural soil stabilised by POFA-OPC additive for the use in unpaved road design. In: *Soft Soil Engineering International Conference*, pp. 1–11.
[14] P. N. Onuamah. (2015). Development-and-optimization-of-mechanical-strength-model-of-cement-laterite-sand-hollow-sandcrete-blocks.docx. *Int. J. Sci. Eng. Res.*, *6*(5), 645–655.
[15] P. N. Onuamah. (2015). Prediction-of-the-compressive-strength-of-concrete-with-palm-kernel-aggregate-using-the-artificial-neutral-networks-approach.docx. *Int. J. Sci. Eng. Res.*, *6*(6), 962–969.
[16] N. N. Osadebe, C. C. Mbajiorgu, & T. U. Nwakonobi. (2007). An optimization model development for laterized-Concrete mix proportioning in building constructions. *Niger. J. Technol.*, *26*(1), 37–46.
[17] K. M. Oba, O. O. Ugwu, & F. O. Okafor. (2019). Development of a model to predict the flexural strength of concrete using SDA as partial replacement for fine aggregate. *Int. J. Sci. Eng. Res.*, *10*(5), 1216–1224.
[18] K. M. Oba, O. O. Ugwu, and F. O. Okafor. (2019). Development of scheffe's model to predict the compressive strength of concrete using SDA as partial replacement for fine aggregate. *Int. J. Innov. Technol. Explor. Eng.*, *8*(8), 2512–2521.
[19] K. M. Oba. (2019). A mathematical model to predict the tensile strength of asphalt concrete using quarry dust filler. *Int. J. Sci. Eng. Res.*, *10*(2), 1491–1498.
[20] C. E. Okere, D. O. Onwuka, S. U. Onwuka, & J. I. Arimanwa. (2013). Simplex-based concrete mix design. *IOSR J. Mech. Civ. Eng.*, *5*(2), 46–55.
[21] E. M. Mbadike & N. N. Osadebe. (2014). Five component concrete mix optimization of aluminum waste using Scheffe's theory. *Int. J. Comput. Eng. Res.*, *4*(4), 23–31.
[22] E. M. Mbadike & N. N. Osadebe. (2013). Application of Scheffe's model in optimization of compressive strength of lateritic concrete. *J. Civ. Eng. Constr. Technol.*, *4*(9), 265–274.
[23] L. Brown, A. N. Donev, & A. T. Biessett. (2015). General blending models for data from mixture experiments. *Technometrics*, *57*(4), 449–456.
[24] H. Scheffe. (1958). Experiments with mixtures. *J. R. Stat. Ser. B*, *25*(2), 235–263.
[25] K. M. Oba & I. G. Amadi. (2020). A predictive mathematical model for water absorption of sawdust ash - Sand concrete. *Int. J. Eng. Manag. Res.*, *10*(01), 33–41.

